Basic Electrical Engineering

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Topic:

Fundamental Of Electric Circuits

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Fundamentals of Electric Circuits

Chapter - 1

Electrical circuits which are collections of circuit elements connected together are the most fundamental structure of electrical Engineering. A circuit is an interconnection of simple electrical devices that have at least one closed path in which current may flow. Circuits are important in electrical engineering because they process electrical signals, which carry energy and information. The signal can be any time varying electrical quantity. Circuit analysis is a mathematical study of some useful interconnection of simple electrical devices. The circuit model helps us to predict, mathematically, the approximate behaviour of the actually event. The model also provide insides into how to design a physical electrical circuit to perform a desired task. Electrical engineering is concerned with the analysis and design of electric circuits, systems and devices.

1.1 Electricity

Electricity is a form of energy which depends on the existence of electric charge in static or dynamic form. The electricity produced by the charges at rest is called static electricity or electro statics. It is produced by friction, hence called as frictional electricity. The electricity produced by the charges in motion is called as ***current electricity***.

1.1.1 Electric Charge

Electric charge exists on electrons and protons. The charge on a proton as positive and that on an electron as negative. This assignment of positive and negative signs to the proton charge and the electron charge is purely a convention. It does not mean that the charge on an electron is 'less' than the charge on a proton . The material which loses electrons acquires a positive charge and material which gains electrons acquires an equal negative charge. Electrons are transferred from the material whose work function is lower to the material whose work function is higher.

Electric charge is quantized. The quantization of charge is the property by virtue of which any charge exists only in discrete lumps or packets of certain minimum charge i.e. q = ne, where n = ± 1, ± 2, ±3......... and $e = 1 \cdot 6 \times 10^{-19} C$. Electric charge is conserved. It implies that charge can neither be created nor be destroyed in isolation i.e. charges can be created or destroyed only in equal and opposite pairs.

The SI unit of charge is coulomb abbreviated as C. One coulomb is the charge flowing through a wire in one second if the electric current in it is one ampere. The charge on a proton is $e = 1 \cdot 60218 \times 10^{-19}$ C and the charge on an electron is the negative of this value.

1.1.2 Electric Current

The flow of charge in a definite direction constitutes the electric current and the time rate of flow of charge through any cross-section of a conductor is the measure of current.

$$Electric\ Current = \frac{total\ charge\ flowing}{time\ taken}$$

$$\Rightarrow I = \frac{q}{t}$$

The S.I. unit of current is ampere (A)

As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called conventional current. A current is positive if it is in the direction of the arrow and negative if it is in the opposite direction . Thus in fig. 1.1 current I = +5A is a current in the direction of arrow from a to b, whereas current I = -5A is a current in the direction opposite to the arrow, that is from b to a. In effect both these currents are equal.

Fig. 1.1

The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. Valence electrons are current carriers in solid conductors. In liquid the current carriers are positively and negatively charged ions. In gases positive ions and electrons are current carriers.

1.1.3 Electric Potential

Electric Potential represents degree of electrification of a body. It determines the direction of flow of charge between two charged bodies placed in contact with each other. The charge always flows from a body at higher potential to a body at lower potential. The flow of charge stops as soon as the potentials of these two bodies become equal.

Quantitatively electric potential at any point in the electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electric force along any Path.

$$\therefore Electric\ Potential\,(V) = \frac{W}{+q}$$

The S.I. unit of electric potential is volt (V)

1.2 Basic Elements

Electric circuit consists of two types of elements. These are active elements and passive elements.

An active element is capable of producing electrical energy but a passive element consumes it or stores it.

1.2.1 Active Elements

An active element is one which supplies electrical energy to the circuit. In fig. 1.2 E_1 & E_2 are active elements because they supply energy to the circuit.

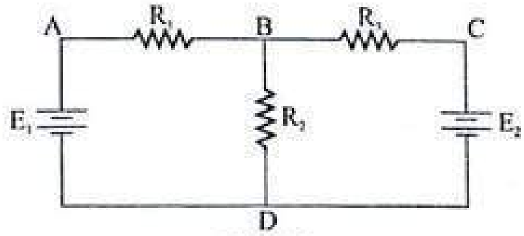


Fig 1.2

1.2.2 Passive Element

A passive element is one which receives electrical energy and then converts it into heat energy (in resistance) or stores in an electric field (in capacitance) or stores in magnetic field (in inductance) . In fig.1.2 R_1, R_2 and R_3 are three passive elements.

1.2.3 Resistance

The resistance of a conductor is the obstruction posed by the conductor to the flow of electric current through it. The resistance of a conductor is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor which in turn depends upon the arrangement of atoms in the conducting material.

The resistance (R) of a conductor

(i) is directly proportional to its length *i.e* $R \alpha \ell$

(ii) is inversely proportional to its area of cross-section i.e $R \alpha \frac{1}{A}$

(iii) depends upon the nature of the material and temperature of the conductor.

From the above, $R \alpha \ell$

$$\alpha \frac{1}{A}$$

$$\Rightarrow R \alpha \frac{\ell}{A}$$

$$\Rightarrow R = \rho \frac{\ell}{A}$$

Where ρ is a constant and is known as resistivity or specific resistance of the material of the conductor.

If $\ell = 1$, $A = 1$, then $R = \rho$

Resistivity of the material of a conductor is defined as the resistance of unit length and unit area of the cross-section of the conductor.

The S.I unit of resistance is Ohm (Ω) and resistivity is Ohm-metre $(\Omega - m)$. Resistivity depends on nature of the material only and is independent of the dimensions of the conductor, but resistance of a conductor depends upon the dimensions (i.e. length and area of cross-section) and nature of the material of the conductor. Resistivity is very low for conductors and very high for insulators. Resistivity of a conductor increases with increase in temperature.

1.2.4 Inductance

Inductance is the property of a conductor by virtue of which it opposes any change in magnitude and direction of electric current passing through the conductor.

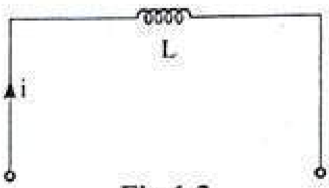


Fig 1.3

The symbol for self inductance is L . It is shown in fig. 1.3

Let us consider a coil of N turns carrying a current I which produces a flux ϕ linking with the coil. The flux linkage (ϕ) is proportional to the current (I) through the coil.

$$\therefore \phi \propto I$$

$$\Rightarrow \phi = LI$$

$$\Rightarrow L = \frac{\phi}{I}$$

As the coil has N turns, So $L = \frac{N\phi}{I}$

It S.I unit is henry.

1.2.5 Inductance EMF

Inductance is the property of a circuit element in which a voltage is induced by changing the current in it. The induced voltage is proportional to the rate of change of current.

Let, $\frac{di}{dt}$ = rate of change of current.

V_L = induced voltage.

$$\therefore V_L \propto \frac{di}{dt}$$

$$\Rightarrow V_L = L.\frac{di}{dt} \qquad (\because \text{L is inductance})$$

If $\frac{di}{dt} = 1\frac{A}{S}$ and $V_L = 1$ volt then L=1 Henry.

A circuit element has a self inductance of 1 henry when a voltage of 1 volt is induced in it by a current in the element changing at the rate of $1\frac{A}{S}$

The direction of induced voltage is given by Lenz's law. Hence induced voltage is

$$V_L = -L \cdot \frac{di}{dt}$$

The negative sign indicates that, when rate of change of current is positive then induced voltage is negative.

These are the following effects of circuit inductance are to be noted.

(i) The property of inductance comes into play only when current in the circuit changes.

(ii) When circuit current changes then inductance opposes this change.

(iii) When circuit current is constant then $\frac{di}{dt} = 0$ and there is no induced voltage.

1.2.6 Capacitance

Capacitance is the property of conductor by virtue of which it stores energy or electric charge. A device designed to have a capacitance of definite value is called a capacitor. Two conductors separated from each other by an insulating material form a capacitor. The insulating material is called dielectric.

Let Q = Charge stored by the capacitor

V = applied voltage

Experimentally it is found that, $Q \propto V$.

$$\Rightarrow Q = CV$$

Where C is a constant called capacitance of the capacitor. The S.I. unit of capacitance is farad (F).

If Q = 1 coulomb and V = 1 volt then C = 1 farad.

Thus One farad is the capacitance of a capacitor which stores a charge of 1 coulomb when a voltage of 1 volt is applied across its terminals.

1.2.7 Energy Stores in inductor

Let L = inductance of a coil.

If the current increases by di in time dt then rate of change of current = $\frac{di}{dt}$

Induced voltage, $V_L = -L \cdot \frac{di}{dt}$

Supply voltage, $V = -V_L = -\left(-L \cdot \frac{di}{dt}\right) = L \cdot \frac{di}{dt}$

Energy stored in the magnetic field of the inductor (coil) during time dt seconds.

= energy supplied in dt seconds

= voltage × current × time

$$= L \cdot \frac{di}{dt} \times i \times dt$$

$$= Li \cdot di$$

Total energy stored in the magnetic field of the inductor when current increases from zero to I amperes is,

$$W = \int_0^I Li \cdot di = L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2$$

1.2.8 Energy Stored in capacitor

Consider a capacitor of capacitance 'C'. During charging, if the voltage between the plates of the capacitor increases by dv volts in time dt seconds then stored charge dq = c.dv.

If i is the instantaneous value of charging current

then $$i = \frac{dq}{dt} = C \cdot \frac{dv}{dt}$$

The power supplied in time dt is,

$$vi = vc \cdot \frac{dv}{dt}$$

∴ Energy supplied in time dt is,

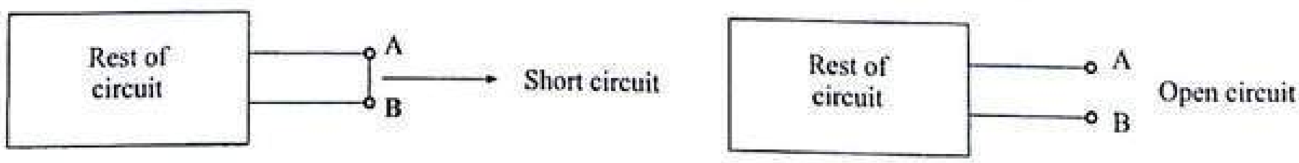
$$W = Power \times time = vc\frac{dv}{dt} \cdot dt = cv.dv$$

The total energy supplied to the capacitor when the voltage increases from zero to V volts is,

$$W = \int_0^V cv \cdot dv = C\left[\frac{V^2}{2}\right]_0^V = \frac{1}{2}CV^2$$

1.2.9 Short and open circuits

When two points of circuit are connected together by a thick metallic wire then they are said to be short-circuited. 'Short' has practically zero resistance. So no voltage can exist across it i.e. (V = IR = I × 0 = 0). The current through short (called as short circuit current) is very large.



Two Points are said to be open circuited when there is no direct connection between them. Open circuit has infinite resistance. So no current flows between two points.

1.3 Basic Laws governing electric circuits

Any electric circuit containing linear bilateral elements is governed by three basic Laws.

i) Ohm's law

ii) Kirchhoff's current law

iii) Kirchhoff's voltage law

For simpler circuit Ohm's law can be applied to find out different branch current, voltage across each element and power dissipation in each element. However for complex circuit Kirchhoff's Laws are more appropriate for solving different circuit parameters such as voltage, current and power.

1.3.1 Ohm's law

It is a fundamental law in electricity. According to this law, *if temperature and other physical conditions are constant then current (I) flowing through a conductor is directly proportional to the potential difference (V) between its two ends.*

$\therefore I \propto V$

$\Rightarrow V \propto I$ (Where R = constant of the conductor

$\Rightarrow V = IR$ = resistance of the conductor)

If we draw a graph between V and I, then it will be a straight line passing through the origin as shown in figure 1.4

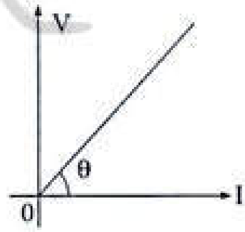


Fig 1.4

The resistance of a conductor is equal to the slope of V-I graph.

$$\text{Slope} = \tan\theta = \frac{V}{I} = R$$

The conductors are called ohmic if the V-I graph for them is a straight line.

The conductors are called non-ohmic if the V-I graph for them is a curve.

A device or circuit element whose V-I characteristics is not a straight line is said to be non-liner resistance.

1.3.2. Series Combination of resistances

Resistances are said to be connected in series if they are connected end-to-end. The extreme ends are connected to a battery as sown in fig 1.5

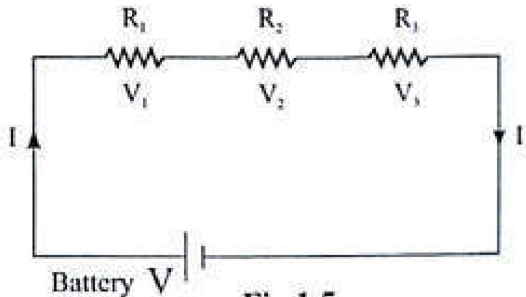


Fig 1.5

Equivalent resistance $R = R_1 + R_2 + R_3$

Mindfire is a software service provider,
with unrelenting focus on optimal software development
and delivery.

In series combination current is same in every part of circuit and p.d across the combination is equal to the sum of individual p.d across different resistances.

$V = V_1 + V_2 + V_3$

Applied p.d is divided among the resistance directly in their ratios.

i.e $V_1 : V_2 : V_3 = R_1 : R_2 : R_3$

Equivalent resistance (R) is high in this combination. It is because length of the system increases in series combination. As $R \propto l$, so equivalent resistance is high.

1.3.3. Parallel Combination of resistances

Resistances are said to be connected in parallel if they are connected in between two ends. These two ends are connected to a battery as shown in fig. 1.6

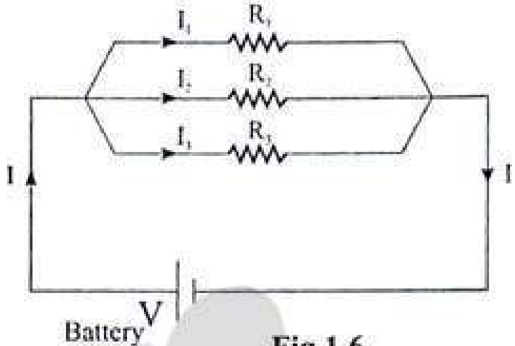


Fig 1.6

If R_1, R_2, R_3 are connected in parallel then equivalent resistance,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The current is different in different resistance . The sum of currents in the different resistances is equal to the main current of the circuit

$I = I_1 + I_2 + I_3$

The p.d is same across each resistance. In parallel combination of resistances the currents are shared in the inverse ratio of the resistances.

Equivalent resistance (R) is low in this combination. It is because in this combination area of the system increases. As $R \propto \frac{1}{A}$, so equivalent resistance (R) decreases.

1.3.4 Current division rule

Let us consider two resistances R_1 and R_2 connected in parallel to a battery of voltage V as shown in fig. 1.7

Total resistance of the circuit is, $R = \frac{R_1 R_2}{R_1 + R_2}$

As R_1 and R_2 are connected in parallel, then $V = I_1 R_1$

$$\Rightarrow I_1 = \frac{V}{R_1} = \frac{IR}{R_1} \qquad \{\because V = IR\}$$

$$\Rightarrow I_1 = I \cdot \frac{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_1}$$

$$\Rightarrow I_1 = I \cdot \frac{R_2}{R_1 + R_2} = \text{Current flowing through } R_1$$

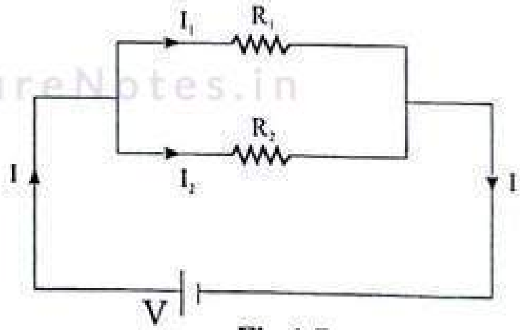


Fig 1.7

Similarly current flowing through $I_2 = I \cdot \dfrac{R_1}{R_1 + R_2}$

Where I is the total current flowing through the circuit.

1.3.5 Voltage division rule

Let us consider two resistances R_1 and R_2 connected in series with a battery of voltage V as shown in fig 1.8

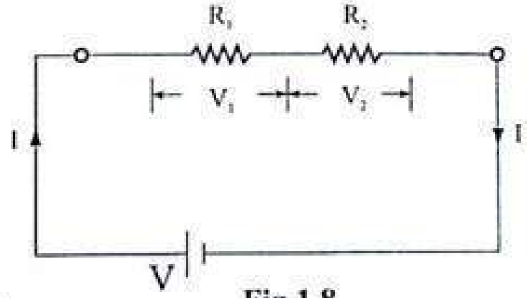


Fig 1.8

I = current flowing through the circuit .

$R = R_1 + R_2$ = total resistance of the circuit

$\therefore I = \dfrac{V}{R} = \dfrac{V}{R_1 + R_2}$

P.d across R_1 is, $V_1 = IR_1 = \dfrac{V}{R_1 + R_2} \cdot R_1 = V\left(\dfrac{R_1}{R_1 + R_2}\right)$

Similarly p.d across R_2 is, $V_2 = V\left(\dfrac{R_2}{R_1 + R_2}\right)$

Example 1.1 : For the network shown in fig 1.9 determine

(i) the voltage drop in each resistor

(ii) the current in each resistor.

Solution :

Total resistance of the circuit is $R = \dfrac{4 \times 12}{4 + 12} + 5 = 8\Omega$

Total current flowing through the circuit

$I = \dfrac{V}{R} = \dfrac{80}{8} = 10A$

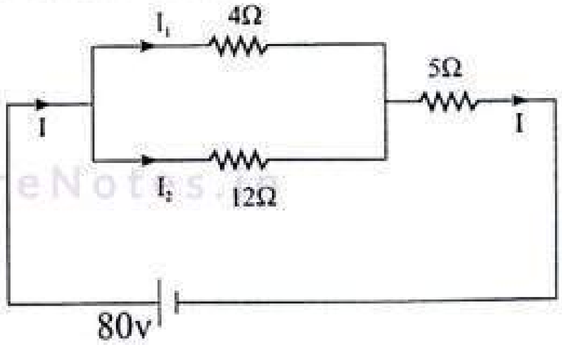


Fig 1.9

According to current division rule

$I_1 = I \cdot \dfrac{R_2}{R_1 + R_2} = 10 \times \dfrac{12}{12 + 4} = 7 \cdot 5A$

$I_2 = I \cdot \dfrac{R_1}{R_1 + R_2} = 10 \times \dfrac{4}{12 + 4} = 2 \cdot 5A$

Voltage drop across 4Ω = 7.5 x 4 = 30 volt

Voltage drop across 12Ω = 2.5 x 12 = 30 volt

Voltage drop across 5Ω = 10 x 5 = 50 volt

1.4 Ideal and Practical sources

An ideal source is a source that can provide an arbitrary amount of energy. Ideal sources are divide into two types : voltage sources and current sources. It is not possible to construct an ideal source because all voltage or current sources (practical sources) have finite internal resistances.

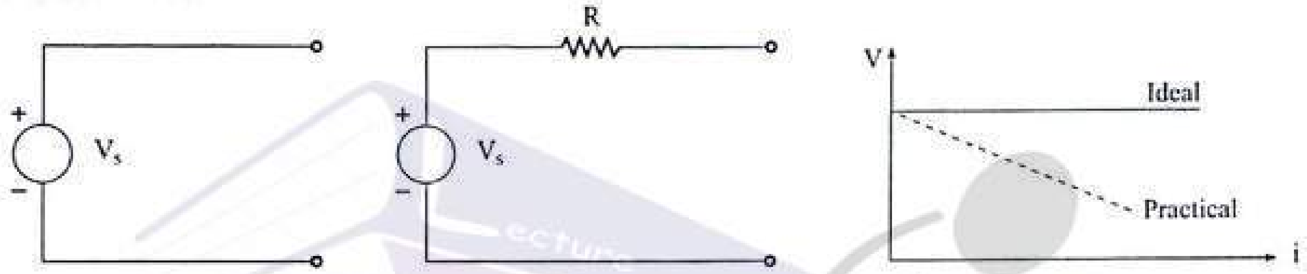
1.4.1 Ideal and Practical Voltage sources.

An ideal voltage source is a source that maintains a constant terminal voltage no matter how much current is drawn from it. It has the following characteristics.

(i) It is a voltage source whose output voltage remains absolutely constant whatever be the value of the output current.

(ii) It has zero internal resistance so that voltage drop in the source is zero.

(iii) The power drawn by the source is zero.



(a) Ideal voltage source (b) Practical voltage source (c) V-i characteristic

Fig - 1.10

A practical voltage source may be represented by an ideal voltage source of voltage V_s in series with a resistance R, equal to the internal resistance of the source. Such a voltage source is shown in fig 1.10 (b).

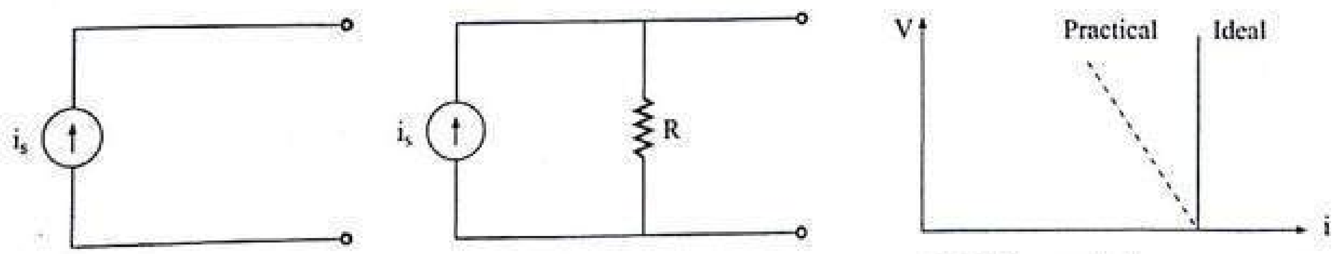
1.4.2 Ideal and Practical current sources

An ideal current source is a source that will supply the same current to any resistance connected across its terminals. The current supplied by the current source is independent of the voltage at the source terminals. It has the following characteristics.

(i) It produces a constant current irrespective of the value of the voltage across it.

(ii) It has infinite resistance

(iii) It is capable of suppling infinite power.



(a) Ideal current source (b) Practical current source (c) V-i characteristic

Fig - 1.11

A practical current source may be represented by an ideal current source of current i_s in parallel with a resistance R, equal to the internal resistance of the source. Such a current source is shown in fig 1.11(b)

1.4.3 Source Transformation

A practical voltage source can be transformed to an equivalent practical current source and vice versa. Replacing one source by an equivalent source is called a source transformation.

A voltage source of voltage V with a series resistance R can be converted into a current source of current $I = \frac{V}{R}$ and a resistance R in parallel with it as shown in figure 1.12.

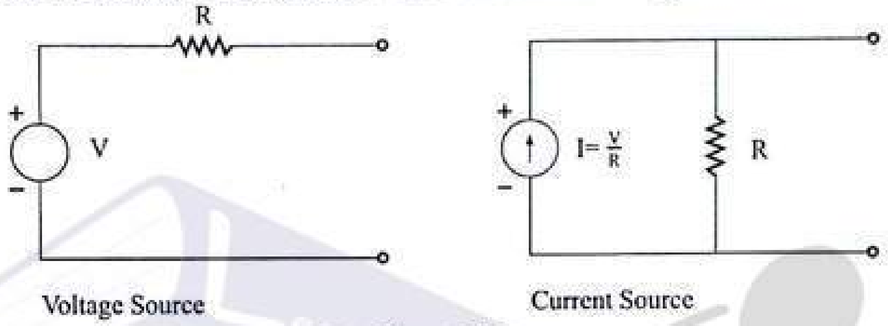


Fig - 1.12

Similarly a current source of current I with a parallel resistance R can be converted into a voltage source of voltage V=IR and a resistance R in series with it.

A practical voltage source and a practical current source are said to be equivalent if their v-i relations are same for all terminal conditions. For equivalence of the two sources the following conditions should be satisfied.

(i) The open circuit voltages at their terminals are equal

(ii) The short circuit currents at their terminals are equal.

Example 1.2 : Convert the voltage source in fig. 1.13 to a current source.

Solution : Here V = 5 volt and

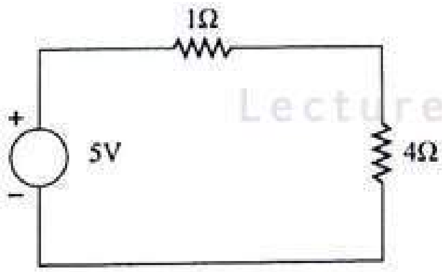


Fig - 1.13

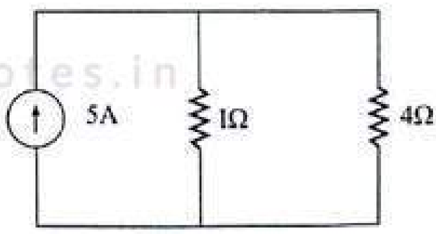


Fig - 1.14

$$R = 1\Omega$$

$$\therefore I = \frac{V}{R} = \frac{5}{1} = 5A$$

Hence the equivalent current source is shown in fig.1.14.

Example 1.3 : Convert the current source in fig.1.15 to a voltage source.

Solution : Here I=5A and R=2Ω

$$\therefore V = IR = 5 \times 2 = 10 \text{ volt}$$

Hence the equivalent voltage source is shown fig. 1.16

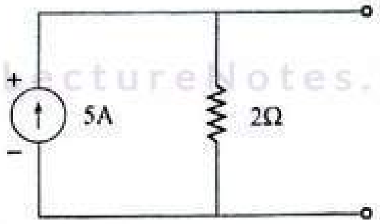


Fig - 1.15

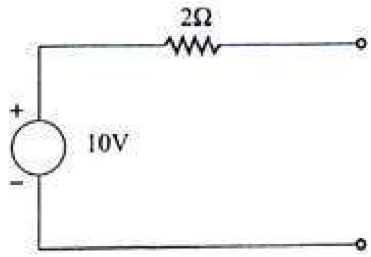


Fig - 1.16

1.4.4 Dependent (controlled sources)

If the magnitude of voltage and current source depends on other branch voltage and current then it is called dependent source. There are four possible dependent or controlled sources :

(i) Voltage controlled voltage source (VCVS)

(ii) Current controlled voltage source (CCVS)

(iii) Voltage controlled current source (VCCS)

(iv) Current controlled current source (CCCS)

Controlled sources are represented by diamond shaped symbols as shown in fig. 1.17

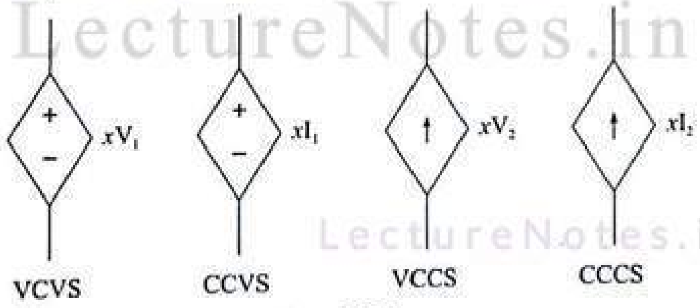


Fig - 1.17

Source can be recognised from inside symbol (i.e voltage source ± and current source ↑) and dependence can be recognised from outside like xV_1 volt or xI_1 volt.

1.5 Electric Power

The rate of doing electrical work is called electric power.

Electric power = rate of doing electric work

$$P = \frac{work}{time} = \frac{W}{t} = \frac{Vq}{t} \qquad \left\{ \because V = \frac{W}{q} \right\}$$

$$P = VI$$

$$P = V\left(\frac{V}{R}\right) = \frac{V^2}{R} \qquad \left\{ \because I = V/R \right\}$$

Also $P = \frac{(IR)^2}{R} = I^2 R$

Thus electric power $(P) = VI = I^2 R = \frac{V^2}{R}$

The unit of power is watt(w). The higher units of power are killowatt (KW) and mega watt (MW). The other units are horse power (hp)

1hp (British) = 746 watt

1hp (Metric) = 735.5 watt

The units horse power (hp) are widely used to specify the output power of electric motors.

1.6 Network Terminology

An electrical network is a collection of elements through which current flows . The following definitions introduce some important elements of a network.

Active Network : An active network is that which contains active elements (i.e voltage sources and current sources).

Passive Network : A passive network is that which contains passive elements (i.e Resistance, Inductance and Capacitance).

Node : A node of a network is an equipotential surface at which two or more circuit elements are joined.

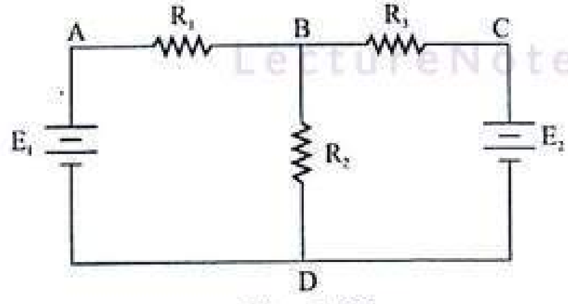


Fig. 1.18

In Fig. 1.18 A, B, C and D are nodes.

Junction : A junction is that point in a network where three or more circuit elements are joined. In fig. 1.18, B and D are junctions.

Branch : A branch is that part of a network which lies between two junctions. In fig. 1.18 BAD, BCD and BD are branches.

Loop : A loop is any closed path of a network. In fig. 1.18 ABDA, BCDB and ABCDA are loops.

Mesh : It is a loop and can not be further divided into other loops. In fig. 1.18 ABDA and BCDB are meshes.

Linear circuit : If the parameters of the circuit are independent of voltage and current then the circuit is called as linear circuit.

Non linear circuit : If the parameters of the circuit are change with voltage and current then the circuit is called non-linear circuit.

Bilateral circuit : A circuit whose characteristics are the same in either direction of current flow is called a bilateral circuit.

Transmission line is a bilateral circuit because it can be made to perform its function equally well in either direction.

Unilateral circuit : A circuit whose characteristics change with the direction of current flow is called unilateral circuit.

A diode rectifier is a unilateral circuit because it can not perform rectification in both directions.

1.7 Ammeter and Voltmeter

Ammeter is a device to measure an electric current and voltmeter is a device to measure a potential difference. In both the instruments there is a coil suspended between the poles of a magnet. When a current is passed through the coil, it deflects. The angle of deflection is proportional to the current going through the coil. A needle is fixed to the coil. When the coil deflects, the needle moves on a graduated scale as shown in fig. 1.19.

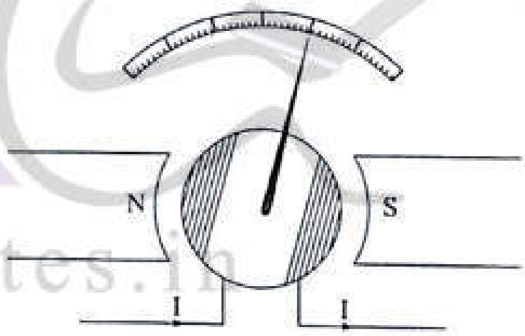


Fig. 1.19

1.7.1 Ammeter

In an ammeter, a resistor having a small resistance is connected in parallel with the coil. This resistor is called the shunt. The current to be measured is passed through the ammeter by connecting it in series with the segment which carries the current.

Let R_C = resistance of the coil

r = small resistance (shunt)

I = current to be measured

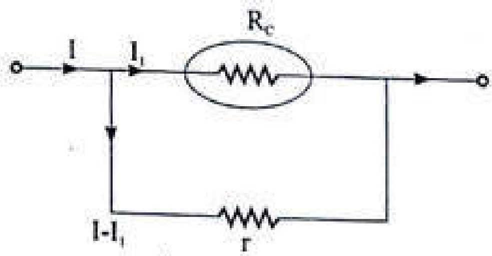


Fig 1.20

I_1 = Current goes through the coil.

$I - I_1$ = Current goes through the shunt.

As potential difference across R_C is same as that across r,

$$I_1 R_C = (I - I_1) r$$

or, $$I_1 R_C + I_1 r = Ir$$

or, $$I_1 (R_C + r) = Ir$$

or, $$I_1 = \left(\frac{r}{R_C + r}\right) I$$

The deflection is proportional to I_1 and hence to I. The scale is graduated to read the value I directly.

The small resistance (shunt) is given by,

$$r = \frac{I_1 R_C}{I - I_1}$$

Also $$\frac{I}{I_1} = \frac{R_C + r}{r}$$

or $$\frac{I}{I_1} = \frac{R_C}{r} + 1$$

$$\text{Multiplying power} = \frac{I}{I_1} = \frac{R_C}{r} + 1$$

The ratio of maximum current (I) to this full scale deflection current (I_1) is known as the multiplying power or multiplying factor of the shunt.

1.7.2 Voltmeter

In voltmeter, a resistor having a high resistance R is connected in series with the coil . When the potential difference is applied, a current passes through the coil and the high resistance.

Let R_C = resistance of the coil

V = potential difference to be measured.

I = circuit current to give full scale deflection

$$\therefore I = \frac{V}{R_C + R}$$

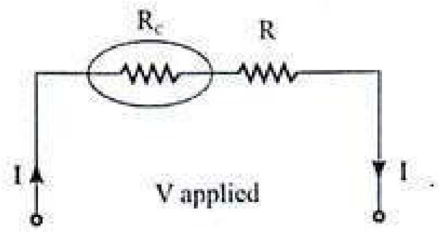


Fig 1.21

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The deflection is proportional to the current I and hence to V. The scale is graduated to read the potential difference directly.

The high resistance $R = \dfrac{V - IR_C}{I}$

or $\quad R = \dfrac{V - v}{I} \qquad (\because\ v = IR_C = Full\ scale\ p.d)$

Also $\quad IR = V - v$

Dividing both sides by v we get,

$$\frac{IR}{v} = \frac{V - v}{v}$$

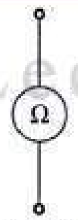
or, $\quad \dfrac{IR}{IR_C} = \dfrac{V}{v} - 1$

or, $\quad \dfrac{R}{R_C} + 1 = \dfrac{V}{v}$

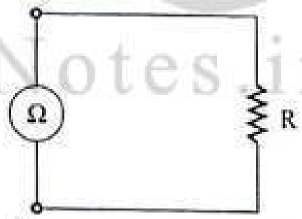
or, $\quad \dfrac{R}{R_C} + 1 = \dfrac{V}{v} =$ Voltage multiplication.

1.8 Ohmmeter :

The Ohmmeter is a device that when connected across a circuit element, can measure the resistance of the element. Fig. 1.22 shows the circuit connection of an ohmmeter to a resistor.



Symbol for Ohmmeter



Circuit for measurement of resistance R

Fig. 1.22

The resistance of an element can be measured only when the element is disconnected from any other circuit.

In case of ammeter and voltmeter the measured quantities such as current (I) and voltage (V) are responsible for production of deflection in the pointer. However in case of a ohmmeter as the measured quantity does not involve any active source, the ohmmeter contain a conventional measuring instrument along with internal source in the form of either a hand driven generator or a dry cell.

1.9 Kirchhoff's Laws

To solve complex electric circuit Gustav Kirchhoff gave two laws known as Kirchhoff's Laws. These are (i) KCL (ii) KVL.

(i) Kirchhoff's Current law (KCL) :

This law deals with law of conservation of charge.

This law states that, *the algebraic sum of the currents meeting at a junction in an electric circuit is zero.*

Sign convention :

a. Current flowing towards the junction is taken as positive.

b. Current flowing away from the junction is taken as negative.

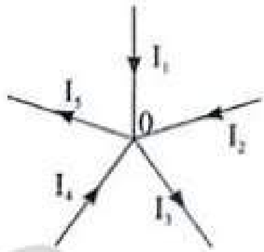


Fig. 1.23

From Figure 1.23, $I_1 + I_2 - I_3 + I_4 - I_5 = 0$

$\Rightarrow I_1 + I_2 + I_4 = I_3 + I_5$

$\Rightarrow$ Total incoming currents = Total outgoing currents.

(ii) Kirchhoff's Voltage law (KVL) :

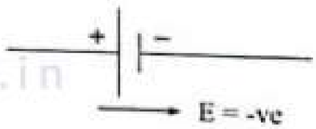
This law deals with law of conservation energy.

This law states that *in any closed circuit or mesh the algebraic sum of all the emfs and the voltage drops equal to zero.*

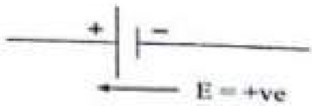
Algebraic sum of emfs + Algebraic sum of voltage drops = 0

Sign convention :

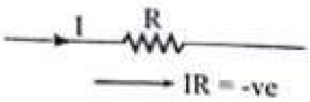
a. If we go from +ve terminal of battery to negative terminal then there is a fall in potential and emf is taken as -ve.



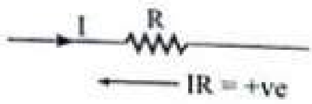
b. If we go from -ve terminal of battery to +ve terminal then there is a rise in potential and emf is taken as +ve



c. If we go through the resistor in the same direction as current then there is a fall in potential. Hence this voltage drop is taken as -ve.



d. If we go through the resistor against the direction of current then there is a rise in potential. Hence this voltage drop is taken as +ve.



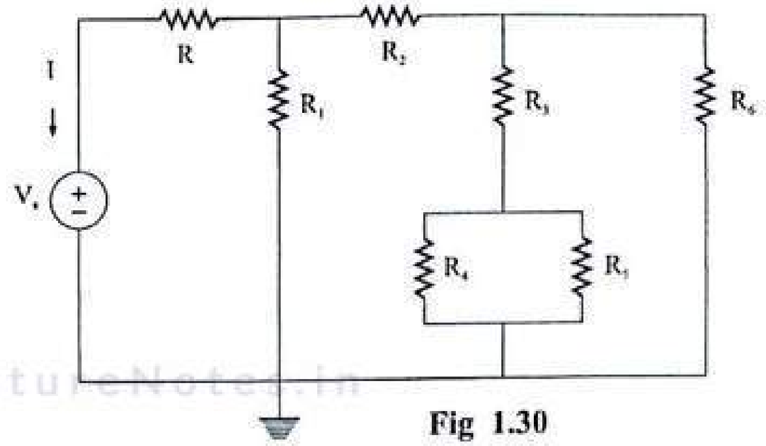
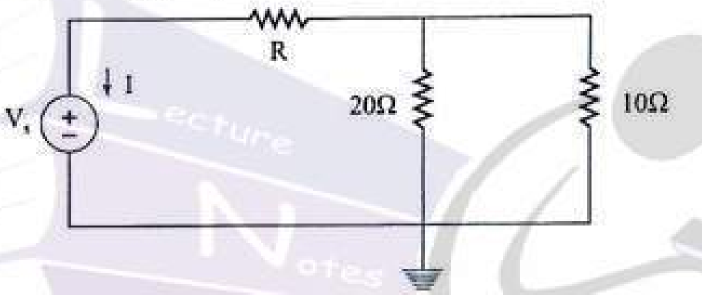


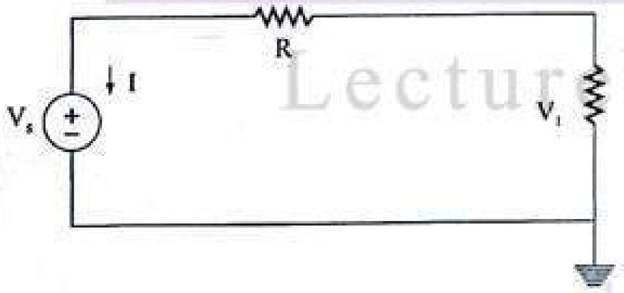
Fig 1.30

Solution : Starting from the right hand side we can replace resistors R_2, R_3, R_4, R_5 and R_6 with a single equivalent resistors.

$$R_{eq} = R_2 + \left[R_3 + \left(R_4 \,||\, R_5\right)\right] || R_6 = 10\Omega$$



The same voltage appears across both R_1 and R_{eq} and therefore, these elements are in parallel.



$$\frac{20\times10}{20+10} = \frac{200}{30} = 6\cdot67\Omega$$

According to voltage divider rule

$$V_1 = V_s \times \frac{6\cdot67}{R+6\cdot67} = 50\times\frac{6\cdot67}{R+6\cdot67}$$

As R_1 and R_{eq} are parallel so voltage V_1 appears across R_1.

The power absorbed by R_1 is,

$$P = \frac{V_1^{\,2}}{R_1} = \left[50\times\frac{6\cdot67}{R+6\cdot67}\right]^2 \times \frac{1}{20}$$

Consider a circuit as shown in fig. 1.24

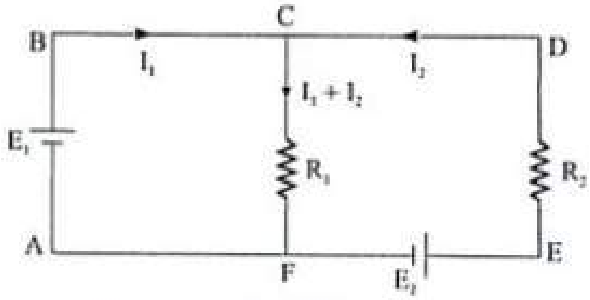


Fig 1.24

Loop ABCFA, $E_1 - (I_1 + I_2)R_1 = 0$

Loop CDEFC $I_2R_2 - E_2 + (I_1 + I_2)R_1 = 0$

Loop ABCDEFA, $E_1 + I_2R_2 - E_2 = 0$

Example 1.4 : Use KCL to determine the unknown currents in the circuit of fig. 1.25. Assume that

$I_0 = -2A,$ $I_1 = -4A,$ $I_s = 8A$ and $V_s = 12V$

Solution : Applying KCL to node ⓐ, we get ,

$I_0 + I_1 + I_2 = 0$

$\Rightarrow I_2 = -I_0 - I_1 = -(I_0 + I_1) = -(-2-4) = 6A$

Applying KCL to node ⓑ we get,

$I_0 + I_s + I_1 = I_3$

$\Rightarrow I_3 = -2 + 8 - 4 = 2A$

So the unknown currents are $I_2 = 6A$ and $I_3 = 2A$

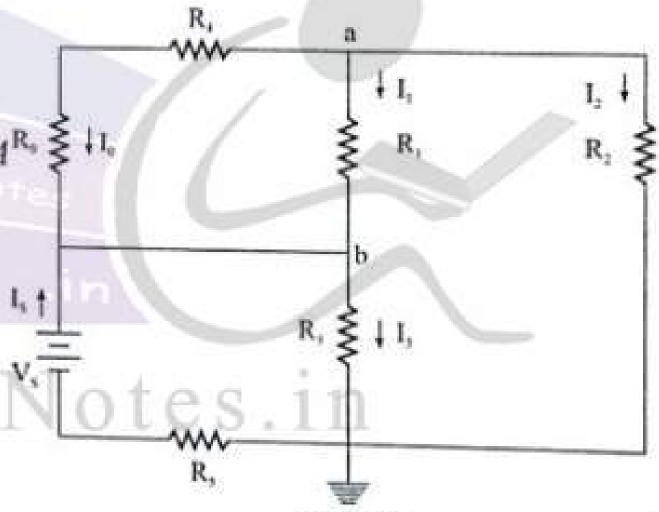


Fig 1.25

Example 1.5 : A 24 volt automotive battery is connected to two headlights, such that the two loads are in parallel; each of the headlights is intended to be a 75 w load, however, a 100 w headlight is mistakenly installed. What is the resistance of each headlight, and what is the total resistance seen by the battery ?

Solution : Headlights no. 1:

$$P = Vi = 100 = \frac{V^2}{R}$$

$$\Rightarrow R = \frac{V^2}{100} = \frac{576}{100} = 5 \cdot 76\Omega$$

But given P = 20 watt.

Therefore $$20 = \left(50 \times \frac{6 \cdot 67}{R + 6 \cdot 67}\right)^2 \times \frac{1}{20}$$

$$\Rightarrow R = 10\Omega$$

Example 1.11 : Determine the voltage between nodes A and B in the circuit shown in fig. 1.31

Given. $R_1 = 2 \cdot 2k\Omega$, $R_2 = 18k\Omega$, $R_3 = 4 \cdot 7k\Omega$, $R_4 = 3 \cdot 3k\Omega$

$V_s = 5V$

Solution: The same current flows through R_1 and R_3. Therefore they are connected in series. Similarly R_2 and R_4 are connected in series.

Specify the assumed polarity of the voltage between nodes A and B. This will have to be a wild guess at this point.

Specify the polarities of the voltage across R_3 and R_4 which will be determined using voltage division. The actual polarities are not difficult to determine.

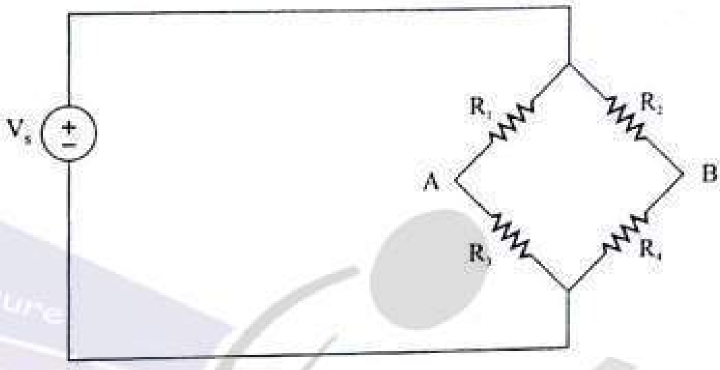


Fig 1.31

$$V_{R3} = \frac{V_s R_3}{R_1 + R_3} = \frac{5 \times 4 \cdot 7 \times 10^3}{(2 \cdot 2 + 4 \cdot 7)10^3} = 3 \cdot 406 \text{ volt.}$$

$$V_{R4} = \frac{V_s R_4}{R_2 + R_4} = \frac{5 \times 3 \cdot 3 \times 10^3}{(18 + 3 \cdot 3)10^3} = 0 \cdot 775 \text{ volt}$$

Applying KVL, $V_{R3} - V_{AB} - V_{R4} = 0$

$$\Rightarrow V_{AB} = V_{R3} - V_{R4} = 3 \cdot 406 - 0 \cdot 917 = 2 \cdot 63 \text{ volt}$$

Example 1.12 : Determine the voltage across R_3 in fig. 1.32

$V_s = 12V$, $R_1 = 1 \cdot 7m\Omega$, $R_2 = 3k\Omega$, $R_3 = 10k\Omega$

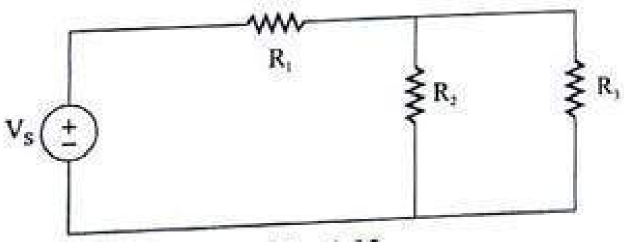


Fig. 1.32

Headlights no. 2:

$$P = Vi = 75 = \frac{V^2}{R}$$

$$\Rightarrow R = \frac{V^2}{75} = \frac{576}{75} = 7\cdot 68\Omega$$

The total resistance is given by the parallel combination :

$$\frac{1}{R_{total}} = \frac{1}{5\cdot 76} + \frac{1}{7\cdot 68}$$

$$\Rightarrow R_{total} = 3\cdot 29\Omega$$

Example 1.6 : For the circuit shown in fig. 1.26

Find (a) The equivalent resistance seen by the source

(b) The current i

(c) The power delivered by the source

(d) The voltage V_1 and V_2

(e) The minimum Power rating required for R_1

Given V = 24 volt

$R_0 = 8\Omega$, $R_1 = 10\Omega$, and $R_2 = 2\Omega$

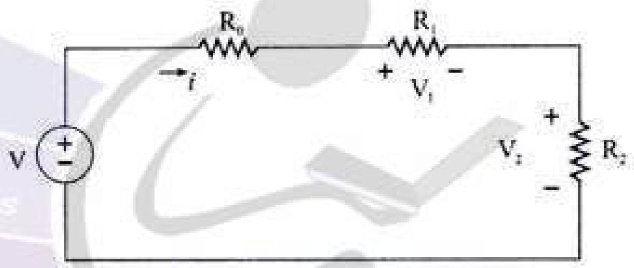


Fig - 1.26

Solution :

(a) The equivalent resistance of the circuit, $R = R_0 + R_1 + R_2 = 8 + 10 + 2 = 20\Omega$

(b) Applying KVL, $V - iR = 0$

So $i = \frac{V}{R} = \frac{24}{20} = 1\cdot 2A$

(c) Power delivered by the source $P = Vi = 24 \times 1\cdot 2 = 28\cdot 8$ watt

(d) $V_1 = R_1 i = 10 \times 1\cdot 2 = 12$ volt

$V_2 = R_2 i = 2 \times 1\cdot 2 = 2\cdot 4$ volt

(e) Power $P_1 = i^2 R_1 = (1\cdot 2)^2 10 = 14\cdot 4$ watt

Therefore the minimum power rating for R_1 is 16 watt.

Solution : The same voltage appears across both R_2 and R_3. Therefore these elements are in parallel.

The equivalent resistance of R_2 and R_3 is

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{3\times 10}{3+10} = \frac{30}{13} = 2\cdot 307 k\Omega$$

According to voltage divider rule voltage across

R_{eq} is $V = V_s \cdot \frac{R_{eq}}{R_1 + R_{eq}}$

$$\Rightarrow V = 12 \times \frac{2\cdot 3\times 10^3}{1\cdot 7\times 10^{-3} + 2\cdot 3\times 10^3} = 11\cdot 999 \text{ volt}$$

$$\Rightarrow V \cong 12 \text{ volt}$$

As R_2 and R_3 are parallel, so voltage across R_{eq} is same as voltage across R_3. Therefore voltage across R_3 = 12 volt.

Example 1.13 : Consider the practical ammeter, described in Fig. 1.33, consisting of an ideal ammeter in series with a 1KΩ resistor. The meter sees a full scale deflection when the current through it is 30μA. If we desire to construct a multirange ammeter reading full scale values of 10mA, 100 mA and IA, depending on the setting of a rotary switch, determine appropriate values of R_1, R_2 and R_3.

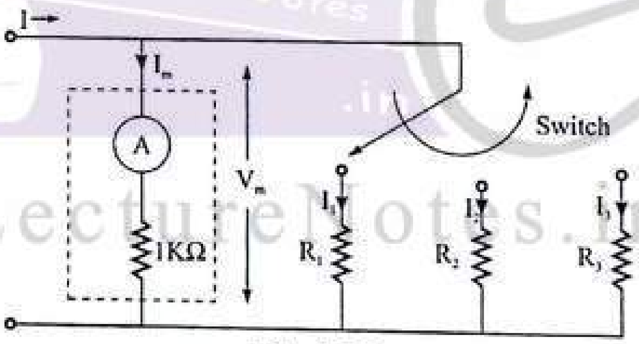


Fig 1.33

Solution : Given $I_m = 30\times 10^{-6} A$

Voltage across the meter is $V_m = I_m R_m$

$$= 30\times 10^{-6} \times 1000$$

$$= 30\times 10^{-3} \text{ volt.}$$

(i) For 10mA :

$I = 10mA = 10^{-2} A$

Current through $R_1 = I_1 = I - I_m = 10^{-2} - 30\times 10^{-6} = 9\cdot 97\times 10^{-3} A$

Example 1.7 : For the circuit shown in figure 1.27

Find (a) The currents i_1 and i_2

(b) The Power delivered by 3A current source and by the 12V voltage source.

(c) The total Power dissipated by the circuit.

Let $R_1 = 25\Omega$, $R_2 = 10\Omega$, $R_3 = 5\Omega$

$R_4 = 7\Omega$ and express i_1 and i_2 as functions of V.

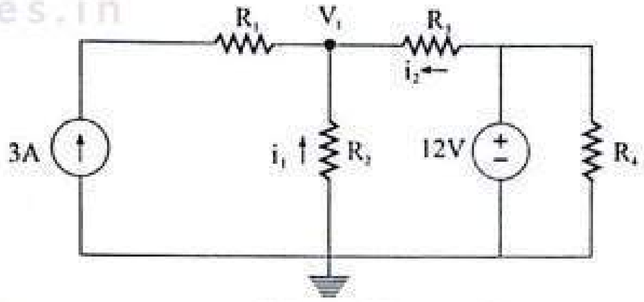


Fig - 1.27

Solution : (a) Apply KCL at node-1,

$$3+\frac{0-V_1}{R_2}+\frac{12-V_1}{R_3}=0$$

$$\Rightarrow 3-\frac{V_1}{R_2}+\frac{12-V_1}{R_3}=0$$

$$\Rightarrow 3-\frac{V_1}{10}+\frac{12-V_1}{5}=0$$

$$\Rightarrow \frac{30-V_1+24-2V_1}{10}=0$$

$$\Rightarrow 54-3V_1=0$$

$$\Rightarrow V_1 = 18 \text{ volt.}$$

Therefore $i_1 = \frac{0-V_1}{R_2} = \frac{0-18}{10} = -1\cdot 8A$

and $i_2 = \frac{12-V_1}{R_3} = \frac{12-18}{5} = \frac{-6}{5} = -1\cdot 2A$

(b) We can compute the voltage across the 3A source as, $V_2 = 3R_1 + V_1 = 3\times 25 + 18 = 93$ V

The power delivered by 3A source is, $P_1 = V_2(3) = 93\times 3 = 279$ watt.

Similarly we can compute current across 12V source as $I_3 = i_2 + \frac{12}{R_4} = -1\cdot 2 + \frac{12}{7} = 0\cdot 514A$

The power supplied by 12V source is

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$$\therefore\ R_1 = \frac{V_m}{I_1} = \frac{30\times10^{-3}}{9\cdot97\times10^{-3}} = 3\cdot00902\Omega$$

(ii) For 100 mA :

I = 100 mA = 0.1A

Current through $R_2 = I_2 = I - I_m = 0\cdot1 - 30\times10^{-6} = 99\cdot97\times10^{-3}\,A$

$$\therefore R_2 = \frac{V_m}{I_2} = \frac{30\times10^{-3}}{99\cdot97\times10^{-3}} = 0\cdot30009\Omega$$

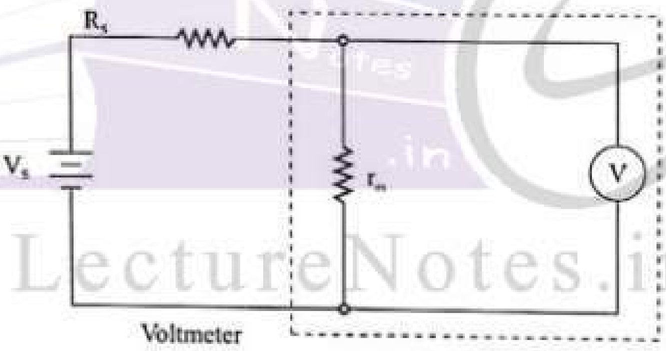
(iii) For 1A :

I = 1A

Current through $R_3 = I_3 = I - I_m = 1 - 30\times10^{-6} = 999\cdot97\times10^{-3}\,A$

$$\therefore R_3 = \frac{V_m}{I_m} = \frac{30\times10^{-3}}{999\cdot97\times10^{-3}} = 0\cdot0300009\,\Omega$$

Example 1.14 : A practical voltmeter has an internal resistance r_m. What is the value of r_m if the meter reads 11.81 V when connected as shown in fig. 1.34.



$R_s = 25k\Omega$

Fig. 1.34

$V_s = 12V$

Solution : Given $R_s = 25k\Omega$, $V_s = 12$ volt.

Voltmeter reads voltage $V_m = 11\cdot81$ volt

Voltage across $r_m = 11\cdot81$ volt

Voltage across $R_s = V = V_s - 11\cdot81 = 12 - 11\cdot81 = 0\cdot19$ volt

Current through $R_s = I_s = \frac{V}{R_s} = \frac{0\cdot19}{25\times10^3} = 7\cdot6\times10^{-6}\,A$

$P_2 = (12)(0\cdot5142) = 6\cdot171$ watt.

(c) Since the power dissipated equals the total power supplied,

$P_{diss} = P_1 + P_2 = 279 + 6\cdot17 = 285\cdot17$ watt

Example : 1.8 Determine the Power delivered by the dependent source in the circuit of fig. 1.28

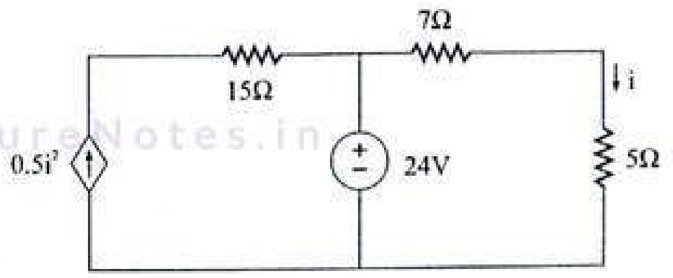


Fig. 1.28

Solution : From the circuit, $i = \dfrac{24}{7+5} = 2A$

Current of dependent source $= 0\cdot5i^2 = (0\cdot5)(2)^2 = 2A$

Let V_D = Voltage across the dependent source.

Applying KVL, $V_D - 15(0\cdot5i^2) - 24 = 0$

$\Rightarrow\ V_D - 15(2) - 24 = 0$

$\Rightarrow\ V_D = 54$ volt.

Power delivered by the dependent source is,

$P_D = V_D(0\cdot5i^2) = 54(2) = 108$ watt

Example 1.9 : Find the equivalent resistance of the circuit of fig. 1.29 by combining resistors in series and in parallel.

$R_0 = 4\Omega$, $R_1 = 12\Omega$, $R_2 = 8\Omega$, $R_3 = 2\Omega$, $R_4 = 16\Omega$ $R_5 = 5\Omega$

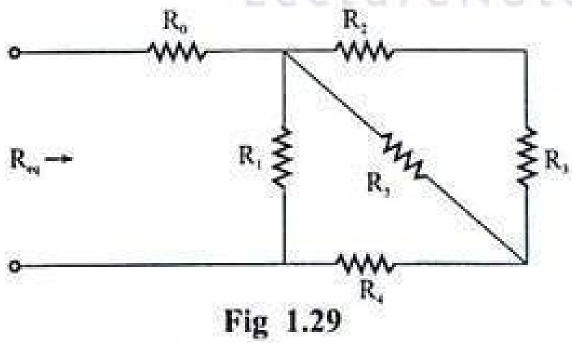


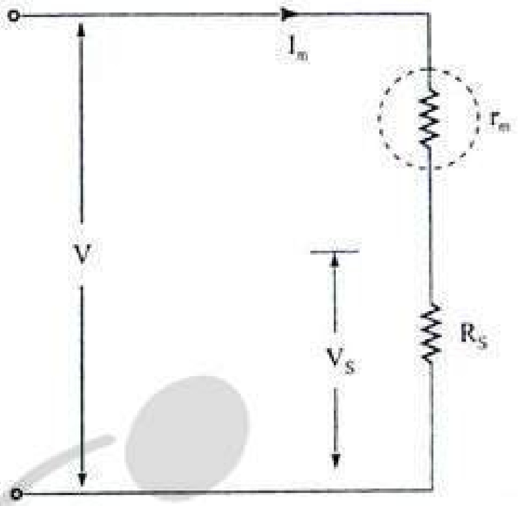
Fig 1.29

$$\therefore r_m = \frac{V_m}{I_s} = \frac{11 \cdot 81}{7 \cdot 6 \times 10^{-6}} = 1 \cdot 55\, M\Omega$$

Assuming the current through the voltmeter is zero.

Example 1.15 : A moving coil instrument has a resistance of 10Ω and gives a full scale deflection when carrying 50mA, Show how it can be adapted to measure voltages up to 750 V, and currents up to 100 A.

Solution : Given, $r_m = 10\Omega,\ I_m = 50 \times 10^{-3}\, A$



(i) As a voltmeter :

V = 750 volt

Voltage across the meter

$$V_m = I_m r_m = 50 \times 10^{-3} \times 10 = 0 \cdot 5V$$

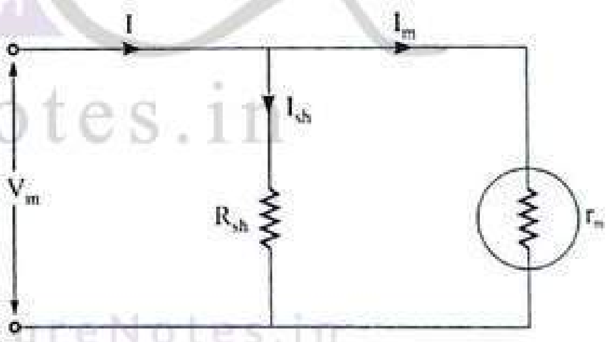
Voltage across $R_s = V_s = V - V_m$

$= 750 - 0 \cdot 5$

$= 749 \cdot 5$ volt

Therefore $$R_s = \frac{V_s}{I_m} = \frac{749 \cdot 5}{50 \times 10^{-3}} = 14990\,\Omega$$

(ii) As an ammeter :



I = 100 A

Current through shunt, $I_{sh} = I - I_m$

$$= 100 - 50 \times 10^{-3}$$

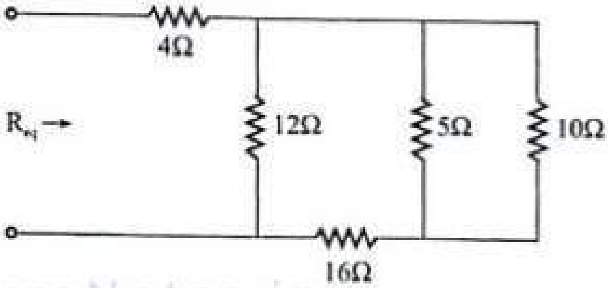
$$= 99 \cdot 095\, A$$

Therefore

$$R_{sh} = \frac{V_m}{I_{sh}} = \frac{I_m r_m}{I_{sh}} = \frac{50 \times 10^{-3} \times 10}{99 \cdot 095} = 0 \cdot 005\,\Omega$$

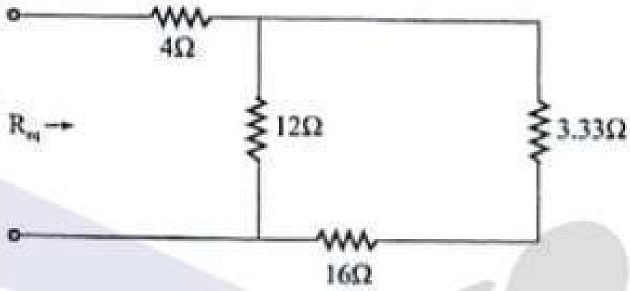
Solution : Starting from the right side, we combine the two resistors in series i.e

$R_2 + R_3 = 8 + 2 = 10\Omega$

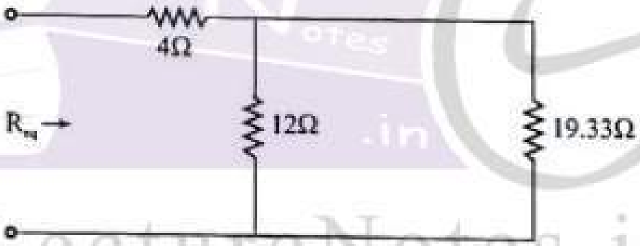


Then we combine the two parallel resistors , namely 5Ω and 10Ω.

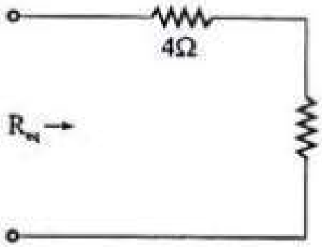
$\frac{1}{R} = \frac{1}{5} + \frac{1}{10} \Rightarrow R = \frac{10}{3} = 3 \cdot 33\Omega$



Then we can combine the two resistors in series, namely 3.33Ω and 16Ω.



Then we can combine the two parallel resistors, namely 12Ω and 19.33Ω.



$$\frac{12 \times 19 \cdot 33}{12 + 19 \cdot 33} = 7 \cdot 4\Omega$$

Therefore $R_{eq} = 4 + 7 \cdot 4 = 11 \cdot 4\Omega$

Example 1.10 : In the circuit of figure 1.30 the power absorbed by the 20Ω resistor is 20watt.

Find R Given $V_s = 50$ volt,

$R_1 = 20\Omega$, $R_2 = 5\Omega$, $R_3 = 2\Omega$, $R_4 = 8\Omega$ $R_5 = 8\Omega$ and $R_6 = 30\Omega$

BPUT PREVIOUS YEAR QUESTIONS SOLVED

1. A moving coil instrument gives a full scale reading of 24 mA when the potential difference across its terminals is 72mV.

Calculate :

(i) the value of shunt resistance for a full scale deflection corresponding to 100 A.

(ii) the series resistance for a full scale reading with 500 V.

(1st semester 2003)

Solution :

$$R_C = \text{resistance of coil} = \frac{72}{24} = 3\,\Omega$$

$$I_1 = \text{full scale current} = 24\text{ mA} = 0.024\text{ A}$$

(i) Shunt resistance $r = \dfrac{I_1 R_C}{I - I_1} = \dfrac{0\cdot024\times3}{100-0\cdot024} = 7\cdot201\times10^{-4}$ ohm

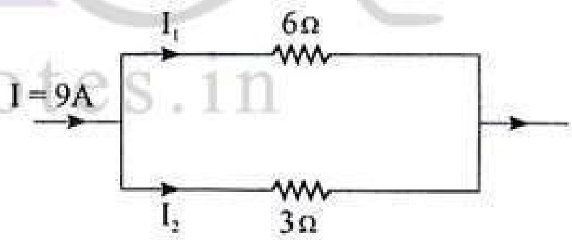
(ii) Series resistance $R = \dfrac{V - IR_C}{I} = \dfrac{500-0\cdot024\times3}{0\cdot024} = 20830\cdot33$ ohm

2. 9 A flows through the Parallel combination of 2 resistances of value 6 ohm and 3 ohm. What is the current in each ?

(Supplementary exam 2004)

Solution : Current through $6\,\Omega$ resistance is

$$I_1 = I\cdot\frac{R_2}{R_1+R_2} = 9\cdot\frac{3}{6+3} = 3A.$$



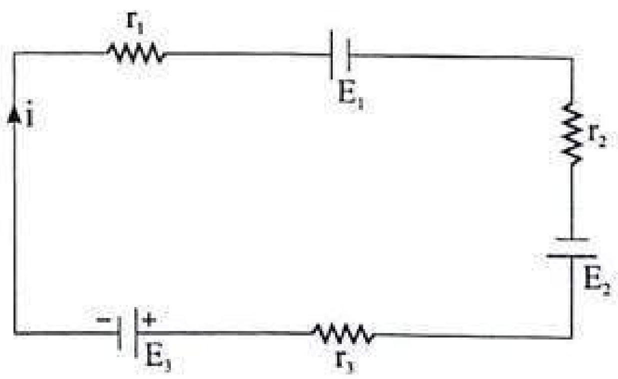
Current through $3\,\Omega$ resistance is,

$$I_2 = I\cdot\frac{R_1}{R_1+R_2} = 9\cdot\frac{6}{6+3} = 6A$$

3. Write the Loop equation for the given circuit.

(Supplementary exam 2004)

Solution :



Apply KVL to the loop,

$$-ir_1 - E_1 - ir_2 + E_2 - ir_3 - E_3 = 0$$

$$\Rightarrow E_1 - E_2 + E_3 = -ir_1 - ir_2 - ir_3$$

$$\Rightarrow E_1 - E_2 + E_3 = -i(r_1 + r_2 + r_3)$$

Applying KVL to loop-5, we get

$$-0\cdot01(I_5+I_4)-0.05I_5-0\cdot05(I_5-I_1)=0$$

$$\Rightarrow 0\cdot01I_4+0\cdot11I_5=2 \text{ .... } (2)$$

Solving equation (1) and (2) we get $I_4=94\cdot34A$ and $I_5=9\cdot6A$

Current through 0.01 ohm resistor connected between nodes A and B is $=I_4+I_5=103\cdot94\approx104A$

14. Three resistances of 10 ohms, 20 ohms and 25 ohms magnitude are connected in parallel across a 200 volts dc source. Compute the current and power drawn from the source.

(1st semester 2008)

Solution :

Total resistance of the circuit is,

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$\Rightarrow\frac{1}{R_{eq}}=\frac{1}{10}+\frac{1}{20}+\frac{1}{25}=\frac{19}{100}$$

$$\Rightarrow R_{eq}=\frac{100}{19}=5\cdot263\,\Omega$$

Current flowing through the circuit is,

$$I=\frac{200}{5\cdot263}=38A$$

Power drawn from the source $P=VI=200\times38=7600$ watts.

15. Four resistances of 5 ohms, 10 ohms, 15 ohms and 20 ohms, magnitude are connected in series and the series combination is connected across a 200 volts dc source. Compute the voltage across each resistor.

(1st semester 2008)

Solution : Total resistance of the circuit is,

$$R=R_1+R_2+R_3+R_4=5+10+15+20=50\text{ ohm.}$$

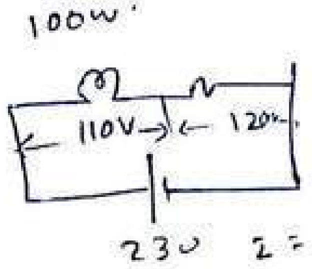
Current flowing through the circuit, $I=\frac{200}{50}=4A$

Voltage across 5 ohm is $V_1=4\times5=20$ volts.

4. A 100 W, 110 V filament lamp is to be operated on a 230 V DC supply. Determine the value of resistance to be connected in series with this lamp.

(1st semester 2005)

Solution :



Resistance of lamp $R = \frac{V^2}{P} = \frac{(110)^2}{100} = 121\Omega$

When lamp operated on 230 volt DC supply then resistance

$$R^1 = \frac{V^2}{P} = \frac{(230)^2}{100} = 529\Omega$$

$I = \frac{100}{110} = 0.909$ A

∴ $R^1 - R = R_s$ = resistance connected in series with the lamp

$\Rightarrow R_s = 529 - 121 = 408\,\Omega$ X

$R_s = \frac{120}{0.909} = 132\ \Omega$

5. A moving coil instrument gives full scale deflection with 15mA and has a resistance of 5 ohm. How the instrument may be used as (i) 2A ammeter (ii) a 100 V voltmenter

(1st semester 2005)

Solution : R_C = resistance of the coil = 5 ohm

I_1 = full scale current = 15 mA = 0.015A

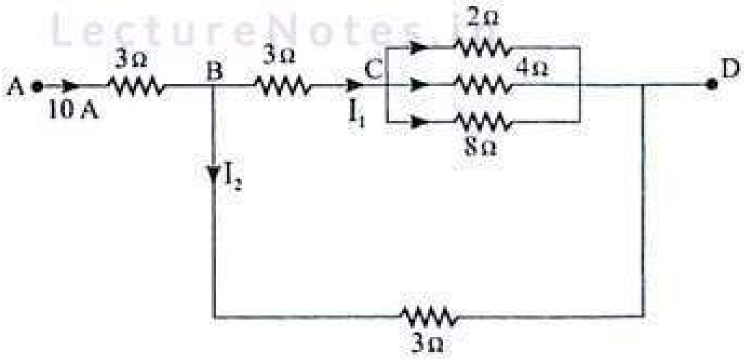
(i) Shunt resistance $r = \frac{I_1 R_C}{I - I_1} = \frac{0 \cdot 015 \times 5}{2 - 0 \cdot 015} = 0 \cdot 0377$ ohm.

If a resistance of 0.0377 ohm connected in parallel with the moving coil instrument then it represents 2A ammeter.

(ii) Series resistance $R = \frac{V - IR_C}{I} = \frac{100 - 0 \cdot 015 \times 5}{0 \cdot 015} = 6661 \cdot 66$ ohm

If a high resistance 6661.66 ohm connected in series with the moving coil instrument then it represents 100 V voltmeter.

6. Current flowing through branch AB is 10A. Find the Currents through the resistors of 2 Ω, 4 Ω and 8 Ω.



(2nd semester 2005)

Voltage across 10 ohm is $V_2 = 4 \times 10 = 40$ volts.

Voltage across 15 ohm is $V_3 = 4 \times 15 = 60$ volts.

Voltage across 20 ohm is $V_4 = 4 \times 20 = 80$ volts.

16. An inductor of inductance 75 milli-henries carries a current of 10A which reverses in 20 milliseconds. What is the average voltage induced in the inductor because of this current reversal ?

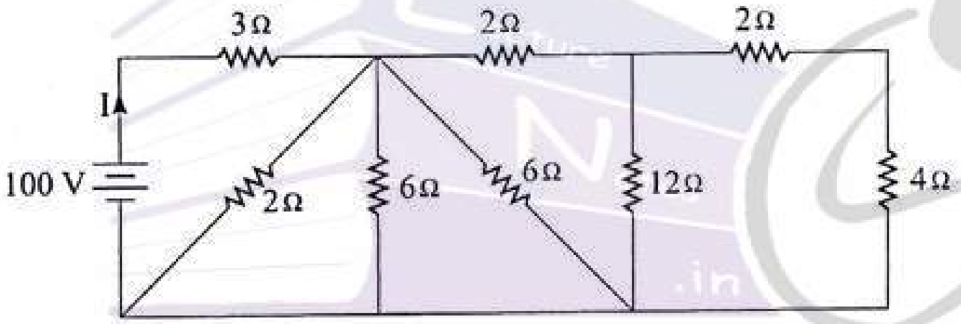
(1st semester 2008)

Solution : Average voltage induced in the inductor is

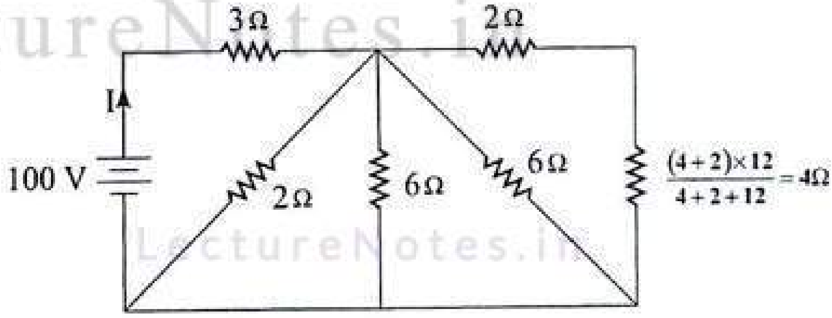
$$V = L \cdot \frac{di}{dt} = 75 \times 10^{-3} \times \frac{10-(-10)}{20 \times 10^{-3}} = 75 \text{ volts.}$$

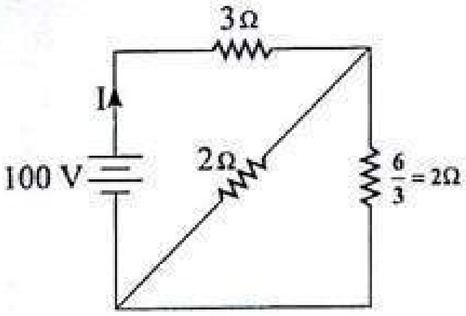
17. Find current in the circuit given below marked as ' I '.

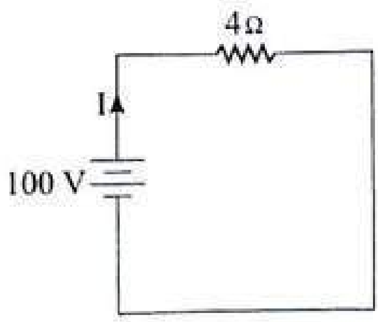
(1st semester 2009)



Soution :







$\therefore \quad I = \frac{100}{4} = 25A$

BURN IGNORANCE

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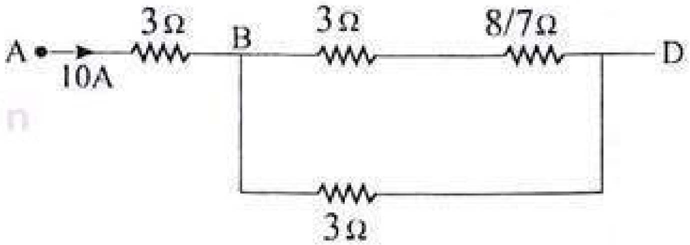
with unrelenting focus on optimal software development
and delivery.

Solution :

The equivalent resistance of 2Ω, 4Ω and 8Ω is 8/7 ohm.

According to current division rule,

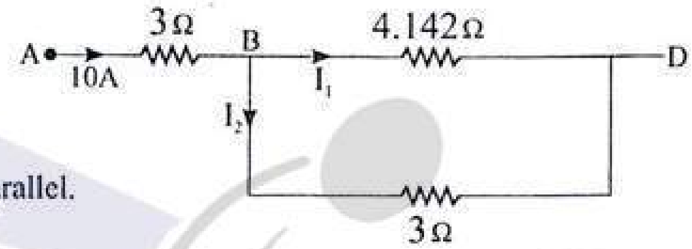


$$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

$$= 10 \cdot \frac{3}{3 + 4 \cdot 142}$$

$$= 4 \cdot 2A$$

Current through 8/7 ohm resistance is 4.2 A.



∴ P.d across CD $= 4 \cdot 2 \times \frac{8}{7} = 4 \cdot 8$ volt.

As 2Ω, 4Ω and 8Ω are connected in parallel.

So current through 2Ω $= \frac{4 \cdot 8}{2} = 2 \cdot 4A$

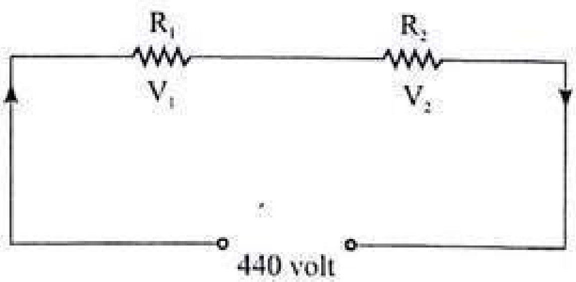
Current through 4Ω $= \frac{4 \cdot 8}{4} = 1 \cdot 2A$

Current through 8Ω $= \frac{4 \cdot 8}{8} = 0 \cdot 6A$

7. A 60w, 220V lamp is connected in series with a 100 W, 220 V lamp and the series combination (of thses two lamps) is connected across a 440V supply. Which lamp will experience a voltage more than its rated value ?

(1st semester 2006)

Solution :



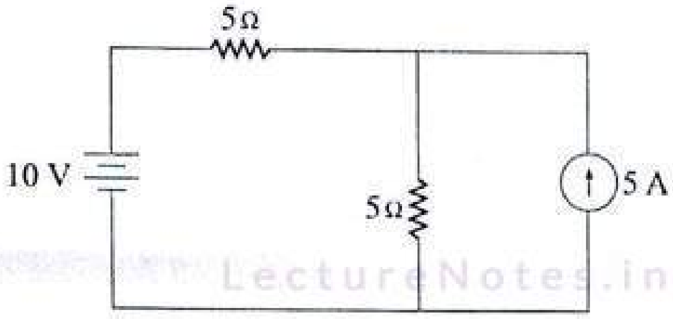
$P_1 = 60W, V_1 = 220$ volt.

∴ $R_1 = \frac{V_1^2}{P_1} = \frac{(220)^2}{60} = 806 \cdot 66\Omega$

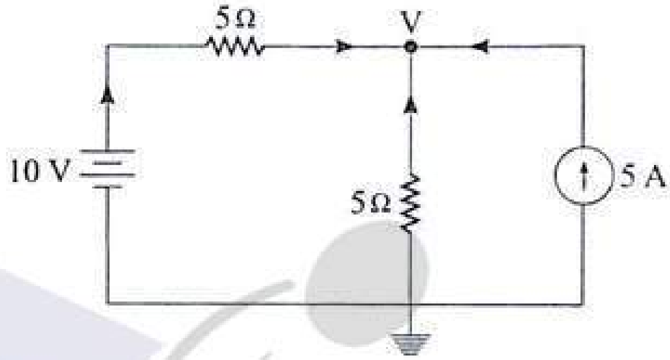
$P_2 = 100\ W, V_2 = 220V$

18. Find the voltage across 5A source in the circuit shown below.

(1st semester 2009)



Solution :



Applying KCL to the node,

$$\frac{10-V}{5}+\frac{0-V}{5}+5=0$$

$$\Rightarrow 10-V-V+25=0$$

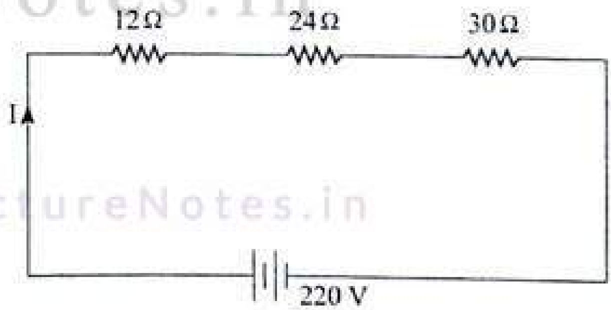
$$\Rightarrow V=17\cdot 5 \text{ volts.}$$

19. Three resistances of 12 ohms, 24 ohms and 30 ohms magnitude are connected in series and the series combination is connected across a 220 volts DC source. Compute the power dissipated in each resistor and the total power drawn from the source

(2nd semester 2009)

Solution :

Total resistance of the circuit is R=12+24+30=66 Ω



Current flows through the circuit, $I=\frac{220}{66}=3\cdot 33A$

Total power drawn from the source = I^2R =

$$=(3\cdot 33)^2(66)$$

$$=731\cdot 867 \text{ watt}$$

Power dissipated across 12 Ω resistor $=(3\cdot 33)^2(12)$

$$=133\cdot 06 \text{ watt.}$$

Power dissipated across 24 Ω resistor $=(3\cdot 33)^2(24)$

$$=266\cdot 13 \text{ watt.}$$

Power dissipated across 30 Ω resistor $=(3\cdot 33)^2(30)$

$$=332\cdot 66 \text{ watt.}$$

$$\therefore \quad R_2 = \frac{V_2^2}{P_2} = \frac{(220)^2}{100} = 484\,\Omega$$

Applying voltage division rule,

$$V_1 = V \cdot \frac{R_1}{R_1 + R_2} = 440 \times \frac{806 \cdot 66}{806 \cdot 66 + 484} = 275 \text{ volt}$$

and $$V_2 = V \cdot \frac{R_2}{R_1 + R_2} = 440 \times \frac{484}{806 \cdot 66 + 484} = 165 \text{ volt.}$$

The first lamp will experience a voltage (275 V) more than its rated value.

8. A moving coil instrument gives full scale deflection with 20 mA and has a resistance of 5 ohm. Calculate the resistance required in parallel to enable the instrument to read upto 1A.

(1st semester 2006)

Solution : I_1 = full scale current = 0.020 A

R_C = resistance of the coil = 5 ohm

Shunt resistance required $r = \frac{I_1 R_C}{I - I_1} = \frac{0 \cdot 020 \times 5}{1 - 0 \cdot 020} = 0 \cdot 10$ ohm

9. A moving coil having a resistance of 5 ohm gives full scale deflection with 20 mA current. Calculate the resistance required in series to enable it to read upto 20 V.

(1st semester 2006)

Solution : R_C = resistance of the coil = 5 ohm.

I = full scale current = 0.020 A

Series resistance required $R = \frac{V - IR_C}{I} = \frac{20 - 0 \cdot 020 \times 5}{0 \cdot 020} = 995\ \Omega$

10. A voltmeter V of 25 kilo-ohm resistance connected across a load resistance R reads 250 volts. What is the value of R if the total current supplied to V and R combination is 0.0725 ampere ?.

(1st semester 2007)

20. Four resistances of 15 ohms, 30 ohms, 35 ohms, and 50 ohms magnitude are connected in parallel across a 230 volts DC source. Compute the power dissipated in each resistor and the total power drawn from the source. What is the total current ?

(Second semester 2009)

Solution :

Total resistance of the circuit, is

$$\frac{1}{R}=\frac{1}{15}+\frac{1}{30}+\frac{1}{35}+\frac{1}{50}$$

$$\Rightarrow R=6\cdot 734\,\Omega$$

Total Power drawn from source $=\frac{V^2}{R}=\frac{(230)^2}{6\cdot 734}=7855\cdot 66$ watt.

Power dissipated across 15 Ω resistance $=\frac{(230)^2}{15}=3526\cdot 66$ watt.

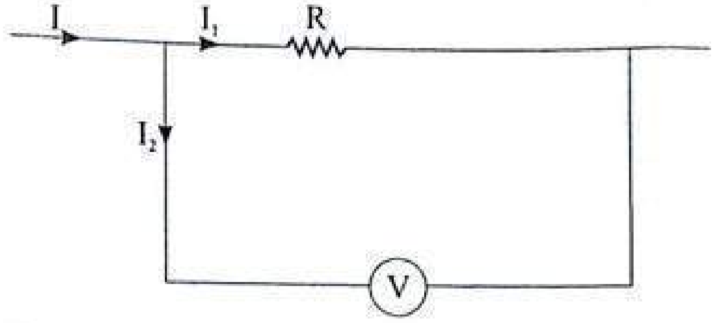
Power dissipated across 30 Ω resistance $=\frac{(230)^2}{30}=1763\cdot 33$ watt.

Power dissipated across 35 Ω resistance $=\frac{(230)^2}{35}=1511\cdot 4$ watt.

Power dissipated across 50 Ω resistance $=\frac{(230)^2}{50}=1058$ watt.

Total current flows through the circuit, $I=\frac{V}{R}=\frac{230}{6\cdot 734}=34\cdot 155\,A$

Solution :



Given, I=0.0725 A

Voltmeter reads 250 volts.

$\therefore \quad I_2 \times 25 \times 10^3 = 250$

$$\Rightarrow I_2 = \frac{250}{25 \times 10^3} = 0 \cdot 01 A$$

But $\quad I = I_1 + I_2$

$$\Rightarrow I_1 = I - I_2 = 0 \cdot 0725 - 0 \cdot 01 = 0 \cdot 0625 A$$

But R and 25×10^3 are connected in parallel.

So $\quad I_1 R = I_2 \times 25 \times 10^3$

$$\Rightarrow R = \frac{I_2 \times 25 \times 10^3}{I_1} = 4000\, ohm$$

11. A resistor of 5 ohms is connected across a potential difference of 20 volts. Calculate the power dissipated and the energy transferred to heat in 3 minutes.

(1st semester 2007)

Solution :

Given $\quad R = 5\, ohm, \qquad V = 20\, volts. \qquad t = 3$ minutes = 180 sec.

$$\text{Power, } P = \frac{V^2}{R} = \frac{(20)^2}{5} = 80 \text{ watt.}$$

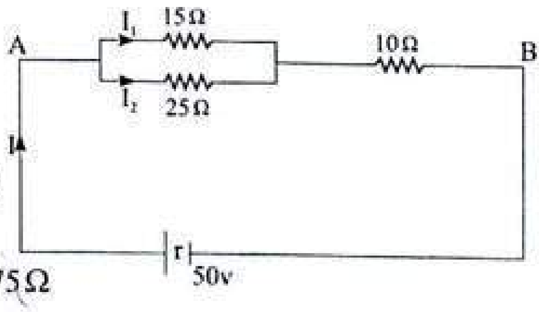
Heat energy transferred, H = W = Pt = 80×180 = 14400 Joule

12. A coil of 15 ohms resistance is in parallel with a coil of 25 ohms resistance. This combination is connected in series with a third coil of 10 ohms resistance. If the whole circuit is connected across a battery having an emf of 50 V and an internal resistance of 1.5 ohm, calculate

(i) the terminal voltage of the battery

(ii) the power dissipated in the 15 ohm coil. *(1st semester 2007)*

Solution :



Total resistance of the circuit is, $R = \frac{15 \times 25}{15 + 25} + 10 + 1 \cdot 5$

$= 20.875\ \Omega$

Current flowing through the circuit is, $I = \frac{50}{20 \cdot 875} = 2 \cdot 395 A$

Resistance between terminals A and B $= \frac{15 \times 25}{15 + 25} + 10 = 19 \cdot 375 \Omega$

(i) Terminal voltage, $V = 2\cdot395 \times 19\cdot375 = 46\cdot4V$

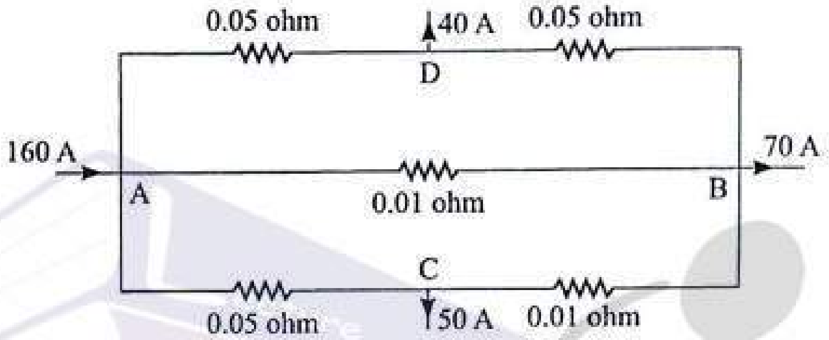
(ii) According to current division rule,

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2} = 2\cdot395 \times \frac{25}{15+25} = 1\cdot496A$$

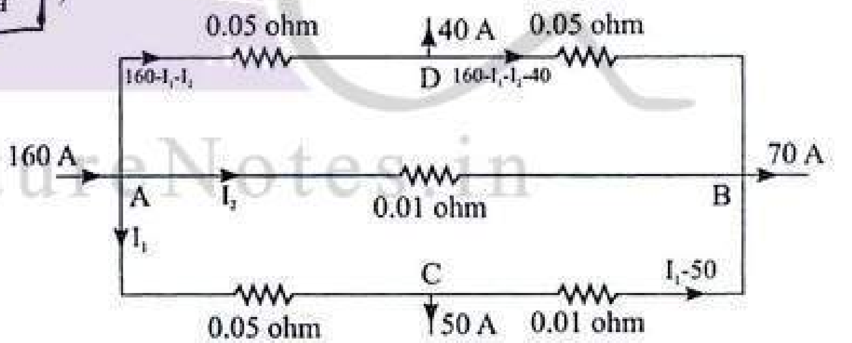
Power dissipated in 15 ohm is $P = I_1^2 R = (1\cdot496)^2 (15) = 33\cdot57$ watt.

13. Compute the current in the 0.01 ohm resistor connected between nodes A and B in fig using Kirchhoffs laws.

(2nd semester 2007)



Solution : From figure, $I_1 = 40A$, $I_2 = 70A$, $I_3 = 50A$



Apply KVL to the loop ABDA,

$$-0\cdot01\, I_2 + 0\cdot05(160 - I_1 - I_2 - 40) + 0\cdot05(160 - I_1 - I_2) = 0$$

$$\Rightarrow 0\cdot1\, I_1 + 0\cdot11\, I_2 = 6\cdot8 \text{ [14]} \quad \ldots\ldots\ldots\ldots (1)$$

Apply KVL to the loop ABCA,

$$-0\cdot01\, I_2 + 0\cdot01\,(I_1 - 50) + 0\cdot05\, I_1 = 0$$

$$\Rightarrow 0\cdot06\, I_1 - 0\cdot01\, I_2 = 0\cdot5 \quad \ldots\ldots\ldots\ldots (2)$$

Solving equation (1) and (2) we get, $I_1 = 16\cdot184A$, $I_2 = 47\cdot10A$

So Current through $0\cdot01\Omega$ is 47.10 A.

$I_1 = 25.65$ $I_2 = 103.95$

103.95A

Do Your Self

T 1.1 The hot resistance of a 250V filament lamp is 627Ω. Determine the current taken by the lamp and its power rating. **[0.4A, 100W]**

T 1.2. A current of 4A flows through a conductor and 10W is dissipated. What p.d. exists across the ends of the conductor ? **[2.5V]**

T 1.3 Find the power dissipated when :

(a) a current of 5mA flows through a resistance of 20 kΩ.

(b) a voltage of 400V is applied across a 120 kΩ resistor.

(c) a voltage applied to a resistor is 10 kV and the current flow is 4mA.

[(a) 0.5W (b) 1.33W (c)40W]

T 1.4. The p.d's measured across three resistors connected in series are 5V, 7V and 10V, and the supply current is 2A. Determine (a) the supply voltage, (b) the total circuit resistance and (c) the values of the three resistors.

[(a) 22V (b) 11Ω (c) 2.5Ω , 3.5Ω , 5Ω]

T 1.5 For the circuit shown in Figure T 1.1, determine the value of V_1. If the total circuit resistance is 36Ω determine the supply current and the value fo resistors R_1, R_2 and R_3.

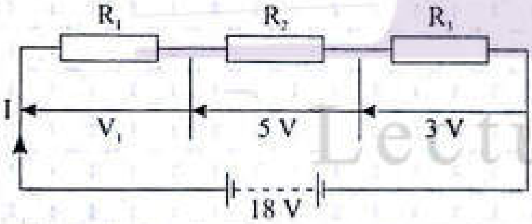


Fig T1.1

[10V, 0.5A, 20 Ω, 10 Ω, 6 Ω]

T. 1.6 When the switch in the circuit in Figure T 1.2 is closed the reading on voltmeter 1 is 30V and that on voltmeter 2 is 10V. Determine the reading on the ammeter and the value of resistor Rx.

[4A, 2.5 Ω]

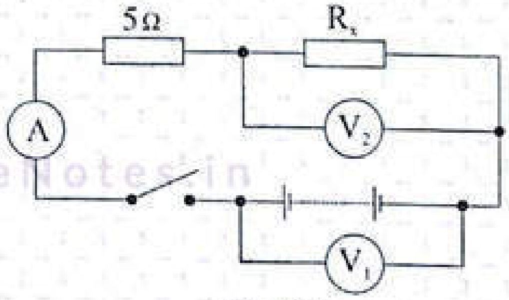


Fig. T1.2

T 1.7 Two resistors are connected in series across an18V supply and a current of 5A flows. If one of the resistors has a value of 2.4 Ω determine (a) the value of the other resistor and (b) the p.d. across the 2.4 Ω resistor.

[(a) 1.2 Ω (b) 12V]

T 1.8 An arc lamp takes 9.6A at 55V. It is operated from a 120V supply. Find the value of the stabilizing resistor to be connected in series. [6.77 Ω]

T 1.9 An oven takes 15A at 240 V. It is required to reduce the current to 12A. Find (a) the resistor which must be connected in series, and (b) the voltage across the resistor. [(a) 4Ω, (b) 48V]

T 1.10 For the circuit shown in Figure T 1.3 determine (a) The reading on the ammeter, and (b) the value of resistor R.

[2.5A, 2.5 Ω]

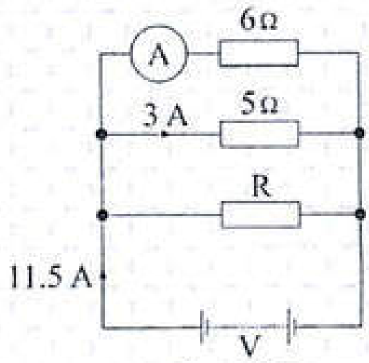


Fig. T1.3

T 1.11 Find the equivalent resistance between the terminals C and D of the circuit shown in Figure T 1.4. [27.5 Ω]

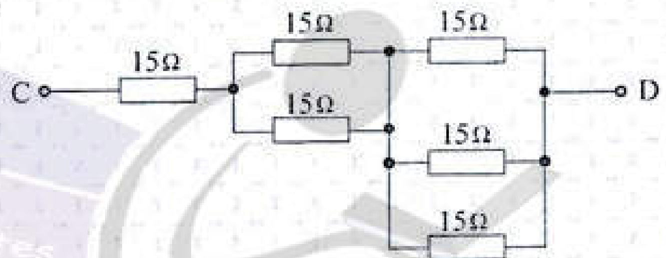


Fig. T1.4

T 1.12 Resistors of 20 Ω, 20 Ω and 30 Ω are connected in parallel. What resistance must be added in series with the combination to obtain a total resistance of 10 Ω. If the complete circuit expends a power of 0.36 KW, find the total current flowing. [2.5 Ω, 6A]

T 1.13 A resistor of 2.4 Ω is connected in series with another of 3.2 Ω. What resistance must be placed across the one of 2.4 Ω so that the total resistance of the circuit shall be 5 Ω ? [7.2 Ω]

T 1.14 A resistor of 8 Ω is connected in parallel with one of 12 Ω and the combination is connected in series with one of 4 Ω. A p.d. of 10V is applied to the circuit. The 8 Ω resistor is now placed across the 4 Ω resistor. Find the p.d. required to send the same current through the 8 Ω resistor. [30 V]

T 1.15 If four identical lamps are connected in parallel and the combined resistance is 100 Ω, find the resistance of one lamp. [400 Ω]

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T 1.16 Three identical filament lamps are connected (a) in series, (b) in parallel across a 210 V supply. State for each connection the p.d. across each lamp.

[(a) 70V (b) 210V]

T 1.17 Calculate the resistances of 110 V light bulbs rated at 25 W, 60 W, 75 W and 100 W.

[484 Ω, 161.3 Ω, 201.67 Ω, 121 Ω]

T 1.18 An electric heating pad rated at 110 V and 55 W is to be used at a 220 V source. It is proposed to connect the heating pad in series with a series - parallel combination of light bulbs, each rated at 110 V, bulbs are available having ratings of 25 W, 60 W, 75 W and 100 W. Obtain a possible scheme of the pad-bulbs combination. At what rate will heat be produced by the pad with this modification ?

[One 100 W bulb in series with a parallel combination of two 60 W bulbs, 54.54 W.]

T 1.19 Find the voltage of point A with respect to point B in the Fig T 1.5. Is it positive with respect to B ?

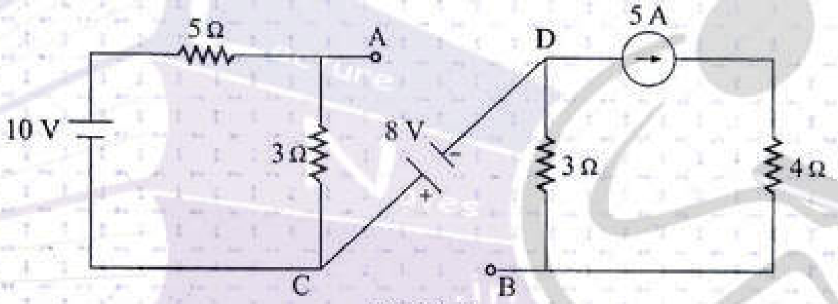

Fig. T1.5

T 1.20 An electric lamp whose resistance, when in use, is 2 ohm is connected to the terminals of a dry cell whose e.m.f is 1.5 V. If the current through the lamp is 0.5 A, calculate the internal resistance of the cell and the potential difference between the terminals of the lamp. If two such cells are connected in parallel, find the resistance which must be connected in series with the arrangement to keep the current the same as before. [1 Ω; 1V; 0.5 Ω]

T 1.21 Determine the current by the source in the circuit shown in figure T 1.6.

[28.45 A]

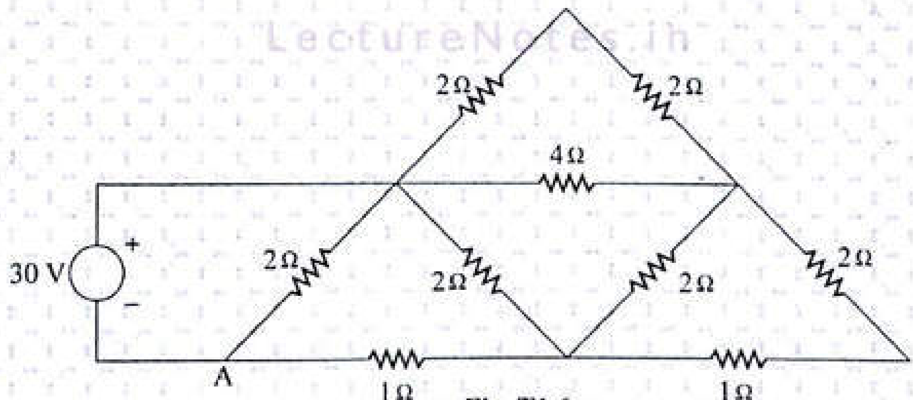

Fig. T1.6

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Basic Electrical Engineering

Topic:

Measurement System And Transducers

Contributed By:

Dr. RAMAPRASAD PANDA

Silicon Institute Of Technology

Measurement Systems and Transducers

Chapter - 6

6.1 Introduction

In every engineering application there is a need for measuring some physical quantities such as temperature, pressure, stress, forces, flows and displacements etc. The quantity to be measured also includes electric quantities like, current, voltage, resistance, inductance, capacitance, frequency, phase angles, power and magnetic quantities like flux, flux density, inductance etc.

All these quantities require a Primery detection elements Known as sensor or transducer. The output of these elements are Converted into another analogus format which in acepted by the measurement system.

The input signal to the sensor or transducer is called an information for the measurement system. The information may be in the form of a physical phenomenon or it may be an electrical signal. These informations are passed through Various Processer such as signal Conditioning, Sampling, analog to digitial Conversion and necessary Computer interface be fore using it by a digital Computer. A typical measurement system is shown in fig 6.1 .

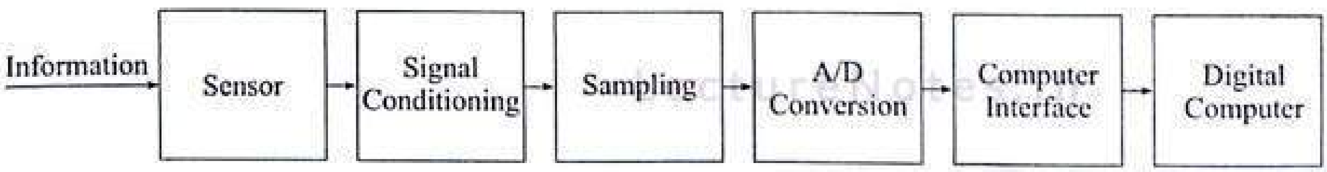


Fig 6.1

6.2 Sensor Classification

Sensors are broadly classified as below,

a) Depending on power Supply :

(i) Active transducers.

(ii) Passive transducers.

b) Depending on type of measurements :
   (i) Primary transducers.
   (ii) Secondary transducers.

c) (i) Analogous transducers.
   (ii) Digitial transducers.

Active transducers develop their own voltage or current without help of auxiliary electrical power supplies. Thermocouple, photovoltaic diode and piezoelectric crystals of self generating transducers. Passive transducers are those that require an external power source. Their examples are potentiometric device, resistance strain gauge, thermistor etc.

Primary transducers are those which sense a physical phenomenon. Thermocouple is one such transducer which sense the heat and convert it directly into an analogous electrical output. Secondary transducers first sense the input which in converted to an electrical output and then this output is again converted to electrical signal by a secondary transducer.

Analog transducers always produce an electrical analogous output due to an input physical variable which in a continous function of time. LVDT, thermistor, thermocouple, straingauge are examples of analogous transducers.

In digital transducers the output is descrete or digitally coded output of binary time.

6.3 Specification of Sensors

Each sensor is usually accompanied by a set up specification that indicate its overall effectiveness in measuring the desired physical variables.

(i) **Accuracy :** Conformity of the measurement to the true value. It is usually represented in percent of full scale reading.

(ii) **Error :** Difference between measurement and true value expressed a percentage of full scale reading.

(iii) **Precision :** Number of significant figures of the measurement.

(iv) **Resolution :** It in the Smallest measurable in crement.

(v) **Span :** It is the linear operating range over which it operates.

(vi) **Range :** It is the range of measurable values.

(vii) **Linearity :** Conformity to an ideal linear calibration curve, usually in percent of reading or full scale reading (whichever is greater).

6.4 Motion and Dimensional Measurements

These type of measurements include absolute position, relative position (displacement), velocity, acceleration and jerk (the derivative of acceleration). These can be either translational or rotational measurement.

The sensors which is used for motion and dimensional measurements are resistive potentiometer, straingauge, peizoelectric sensors, capacitive sensors etc.

(a) **Resistive Potentiometer :** In resistive potentiometer, the displacement or motion will be measured by changing the resistance at the output of the potentiometer. This will be of two types, (i) Linear potentiometer (ii) rotational potentiometer.

(i) ***Linear Potentiometer :*** In this potentiometer, the displacement will be measured by changing the resistance at the output of the potentiometer. The circuit diagram is shown in fig 6.1

Let R = total resistance of the section AB of the Potentiometer.

The resistance per unit

length of the system $= \frac{R}{AB} = \frac{R}{L} \quad \frac{\Omega}{m}$

V_{in} = input voltage to the potentiometer

V_{out} = output voltage of the potentiometer.

Resistance at the output section OB of the potentiometer

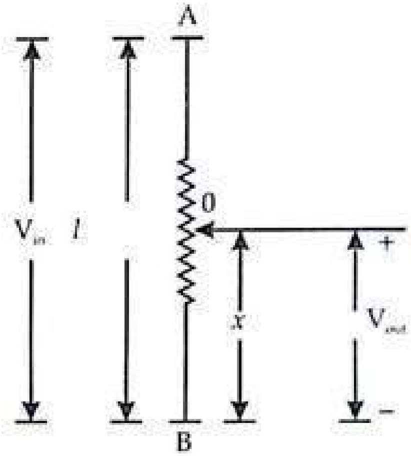


Fig 6.1

$$= \frac{R}{L} \times OB = \frac{R}{L} \; x$$

According to voltage division rule,

$$V_{out} = \textit{total voltage applied} \times \frac{\textit{resistance of the section OB}}{\textit{total resistance}}$$

$$\Rightarrow V_{out} = V_{in} \times \frac{\frac{Rx}{L}}{R} = V_{in} \times \frac{x}{L}$$

$$\Rightarrow \; x = \frac{V_{out}}{V_{in}} \times L$$

(ii) ***Rotational potentiometer :*** In this potentiometer the angular displacement will be measured by changing the resistance at the output of the potentiometer. The circuit diagram is shown is fig 6.2.

Let θ = angle subtended by the potentiometer in radian.

R = total resistance of the potentiometer in ohm.

θ_1 = angle subtended in between initial point A and wiper (sliding contact) of the potentiometer.

$\frac{R}{\theta}$ = resistance per unit angle (in radian) subtended by the potentiometer.

V_{out} = output voltage between initial point A and wiper of the potentiometer.

V_{in} = input voltage to the potentiometer.

According to voltage division rule,

$$V_{out} = \textit{total voltage applied to the potentiometer} \times \frac{\textit{resistance between initial point A and wiper}}{\textit{total resistance.}}$$

$$\Rightarrow V_{out} = V_{in} \frac{\frac{R}{\theta} \times \theta_1}{R}$$

$$\Rightarrow V_{out} = V_{in} \times \frac{\theta_1}{\theta}$$

$$\Rightarrow \theta_1 = \frac{V_{out}}{V_{in}} \times \theta$$

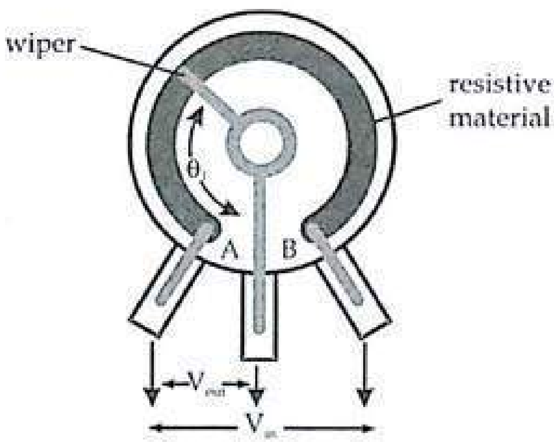


Fig. 6.2

(b) Strain Gauge : Strain gauges are well used in measurement of pressure/ force applied on a surface of a body. Strain gauges are the devices that are bonded to the surface of an object and whose resistance varies as a function of the surface strain experienced by the object. Strain gauges may be used to perform measurements of strain, stress, force and pressure. The resistance of any material is given by $R = \rho \frac{\ell}{A}$

where ℓ = length of the material

ρ = resistivity of the material

A = area of cross-section of the material.

When either length (ℓ) or area of cross-section (A) of a material changes then its resistance changes. Resistive strain gauges are based on this principle when the material is compressed then its resistance decreased (as length decreses). The relationship between the change in resistance and change in length is given by gauge factor (GF).

$$\text{Gauge factor (GF)} = \frac{\frac{\Delta R}{R}}{\frac{\Delta \ell}{\ell}}$$

Where ΔR = change in resistance.

$\Delta \ell$ = Change in length

$\frac{\Delta \ell}{\ell}$ = longitudinal strain.


Wire strain gauge bonded on elastic backing

Fig. 6.3

Schematic diagram of resistive strain gauge is shown in the fig 6.3.

(c) Peizoelectric Sensor : It is based on the principle of Piezo electric effect i.e a voltage is produced between surfaces of a solid dielectric (non conducting substance) when a mechanical stress is applied to it or when a voltage is applied across certain surfaces of a solid that exhibits the Piezoelectric effect, the solid undergoes a mechanical distortion.

(d) Capacitive Sensors :

The capacitance of a parallel plate capacitor is given by $C = \frac{A \in}{d}$.

Where A = area of each plate

d = distance between two plates.

$\in$ = Permittivity of the material in between two plates.

For small distance or proximity measurements, capacitive transducers are used. In this kind of transducers, the distance between the plates d will be used as a input parameter by which the distance/ proximity will be converted into its equivalent capacitance by the capacitive transducer.

Sensitivity of the capacitive transducer is $S = \frac{dc}{dx}$.

where dc = change in capacitance

dx = change in distance between two paralled plates.

Capcitive transducers may change its capacitance by three parameters;

(i) due to change in distance (d)

(ii) due to change in area (A)

(iii) due to change in dielectric constant ($\in$).

The three configurations are shown in fig 6.4

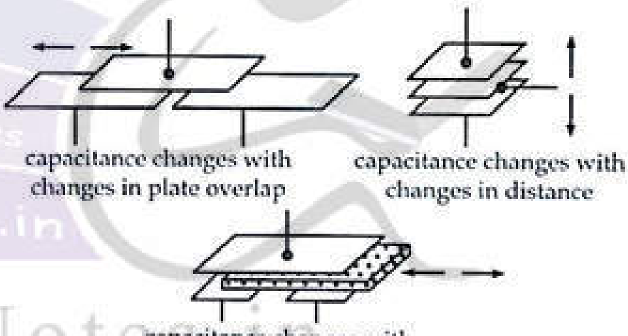


Fig. 6.4

6.5 Flow measurement

In many engineering applications it is desirable to sense the flow rate of a fluid. The measurement of flow rate is accomplished by two fluids. These are compressible (gas) and incompressible (liquid) Generally three types of sensors are used for measurement of flow rate of fluid. These sensors operate on three principles, they are named as,

(i) differential pressure measurement on a calibrated orifice.

(ii) hot wire anemometer.

(iii) turbine flow meter.

(i) ***Differential pressure measurement on a calibrated orifice :*** Fig 6.5 shows a differential pressure measurement unit on a calibrated orifice. The relationship between pressure across the orifice (P_1 - P_2) and flow rate through the orifice (q) is pre determined through proper calibration.

The difference of pressure gives the flow rate.

$\Rightarrow$ Flow rate (q) α pressure difference

$$\Rightarrow \quad q = k\,(P_1 - P_2)$$

Where K is calibration constant.

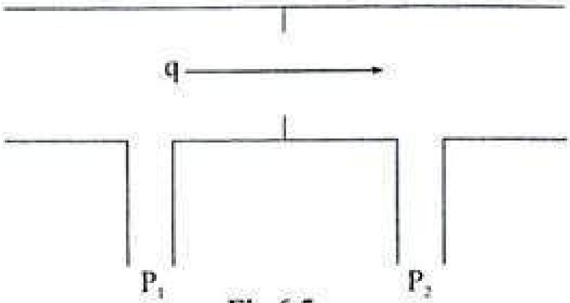


Fig 6.5

(ii) Hot wire anemometer : Hot wire anemometer works on the principle in which a heated wire is cooled by the flow of a gas. The resistance of the wire changes with temperature which is reflected in the form of change in voltage by the help of wheat stone bridge circuit as shown in fig 6.6.

When gas flows through the heated wire then its temperature falls. Due to this fall in temperature, the resistance of heated wire R decreases. As a result the voltage V_0 changes.

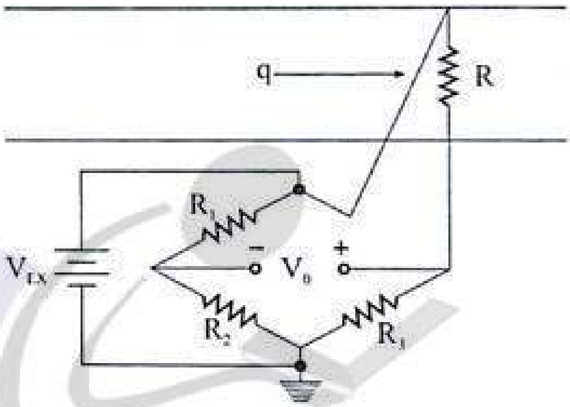


Fig 6.6

(iii) ***Turbine flow meter :*** In a turbine flow meter the fluid flow causes a turbine to rotate. The velocity of rotation ot the turbine is related to the flow velocity which is measured by a non-contact sensor as shown in fig 6.7.

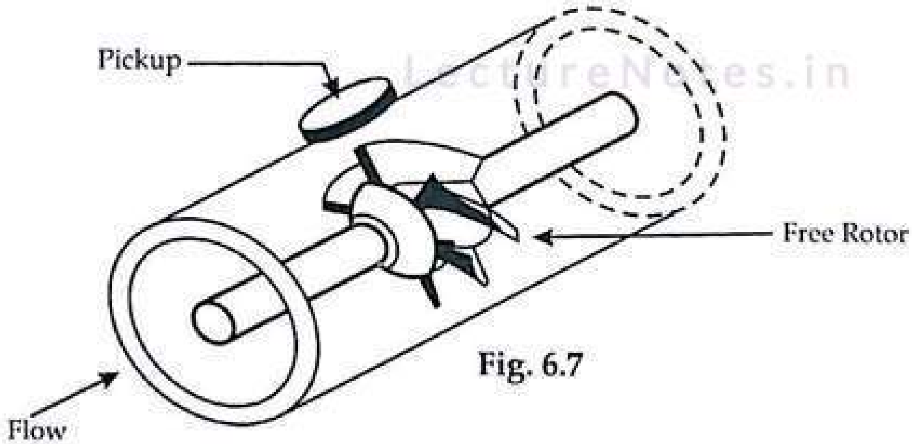


Fig. 6.7

6.6 Temperature measurement

Temperature is widely measured and control as industrial variable. In measurement of temperature two types of sensors / transducers are used. (i) thermocouple (ii) resistance temperature detector (RTD).

(i) Thermocouple : A thermocouple is working on a basic principle of "Seeback effect". According to this effect, when two different types of metal junction is exposed to hot or cold environment then there will be current at the metal junction. In fig 6.8 two metals (Nickel - chromium and Nickel aluminium) are formed a junction. When this junction is kept at 300° C then junction is generating 12.2 mv at the thermocouple terminals. So depending on this principle the temperature is normally measured.

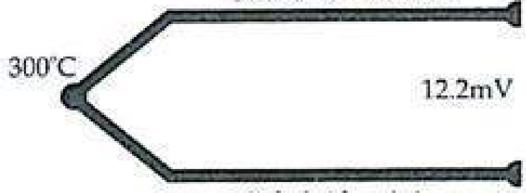


Fig. 6.8

Let T = temperature

V = generated voltage across the thermo couple.

The relation between T and V is given by,

$$T = a_0 + a_1 V + a_2 V^2 + ... + a_n V^n$$

Where a_0, a_1, a_2, a_3, a_n are metal coefficients.

(ii) Resistance temperature detector (RTD) : It is a variable resistance device whose resistance is a function of temperature. It works on the principle of piezo resistance.

Piezo resistance is a property of a metal by which the metal resistance decreases or increases depends on temperature of the metal. The schematic of RTD is shown in fig 6.9. When RTD exposed to temperature then its resistance will be varied, so that, the resistance will be measured as a unknown in a wheatstone bridge as shown in fig 6.10

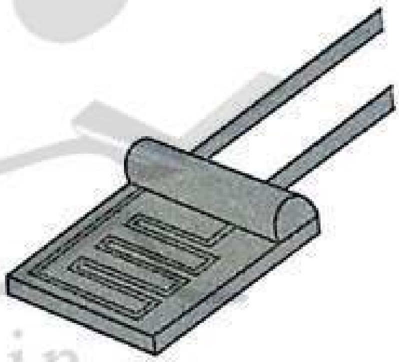


Fig. 6.9

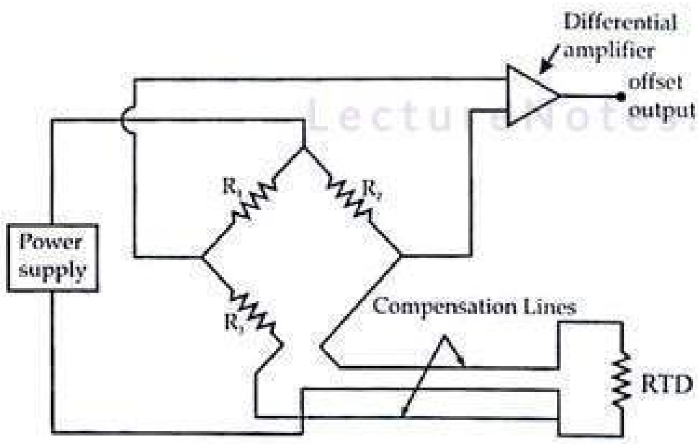


Fig. 6.10

A more accurate representation of RTD temperature dependence can be obtained by using non-linear equation and it will be described by temperature coefficients, $R_T = R_o\left(1 + AT - BT^2 - CT^3\right)$ where A, B are temperature coefficients and C will be zero for temperatures above $0°C$.

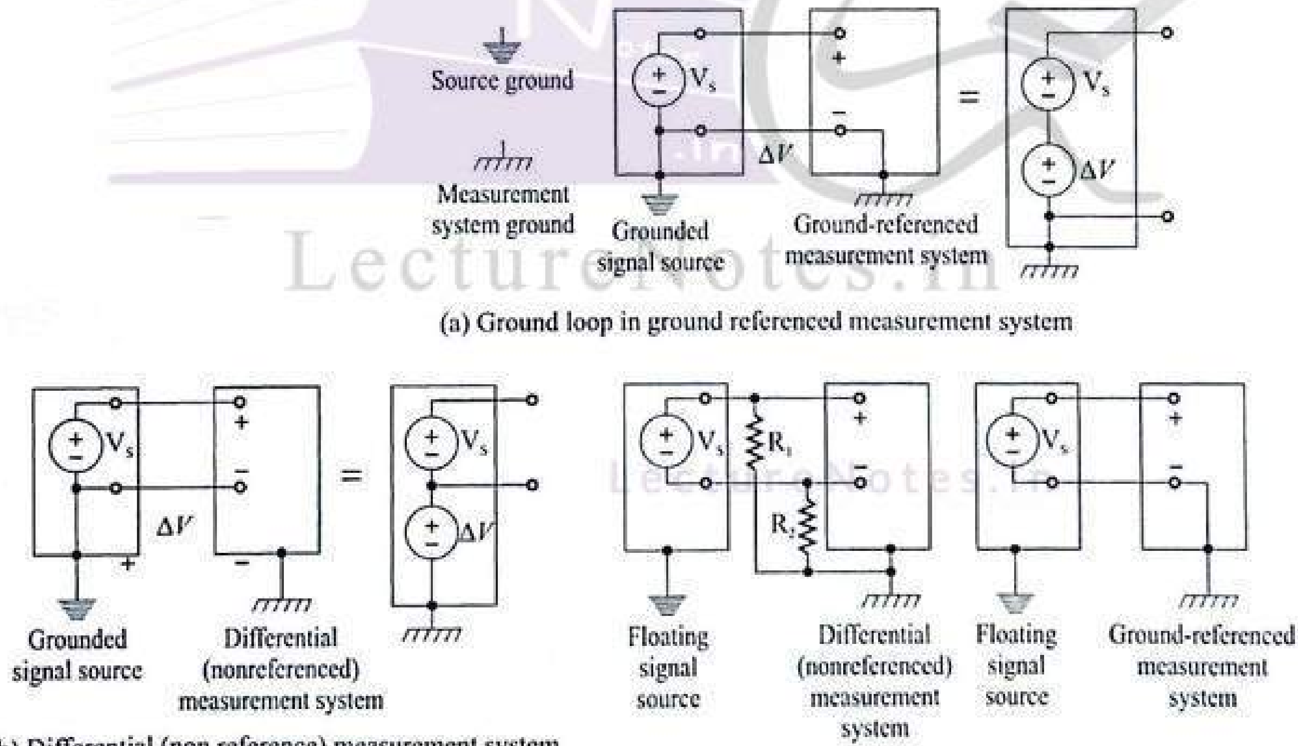
6.7 Wiring, Grounding and Noise

When a signal is generated from a transducer it should be properly connected to the measurement system without disturbing its inherent properties. This includes proper wiring, grounding and reduction of noise in the signal if any.

6.7.1 Signal and Measurement Configuration System

Ideally a sensor can be modeled as an ideal voltage source in series with a source resistance. The signal source can be a grounded type or a floating type. A grounded signal source is one in which a ground reference is established. In this case the negative point of the signal is connected to the ground. In a floating signal source non of the terminal is connected to the ground.

In analogy with a signal source a measurement system can be either ground referenced or differential. In a ground referenced system, the signal negative point is tied to the instrument case ground. In a differential system neither of the two signal connection is tied to ground. Thus a differential measurement system is well suited for measuring the difference between the two signal levels. Fig 6.11 shows a grounded system and a floating system of measurement.



(a) Ground loop in ground referenced measurement system

(b) Differential (non reference) measurement system

(c) measuring signals from a floating source

Fig 6.11

6.7.2 Noise Sources and Coupling Mechanism

The output of a sensor contains undesirable signals along with signals to be measured. These signals are often impossible to eliminate completely. This noise is mainly caused due to coupling of a noise source. Fig 6.12 shows the essential stages of a noisy measurement.

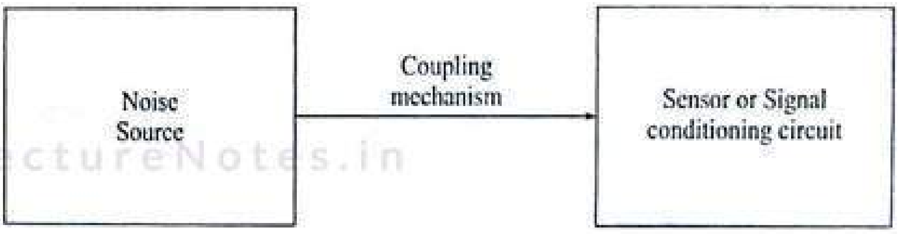


Fig 6.12

The coupling mechanism may be following types.

(i) Conductive coupling.

(ii) Capacitive coupling.

(iii) Inductive coupling.

In conductive coupling the power supply is connected to both a load and a sensor. In capacitive coupling mechanism the electric field may cause the noise. Inductive coupling cause the noise due to magnetic field generated by the current flowing through conductor. Fig 6.13 shows the various ways

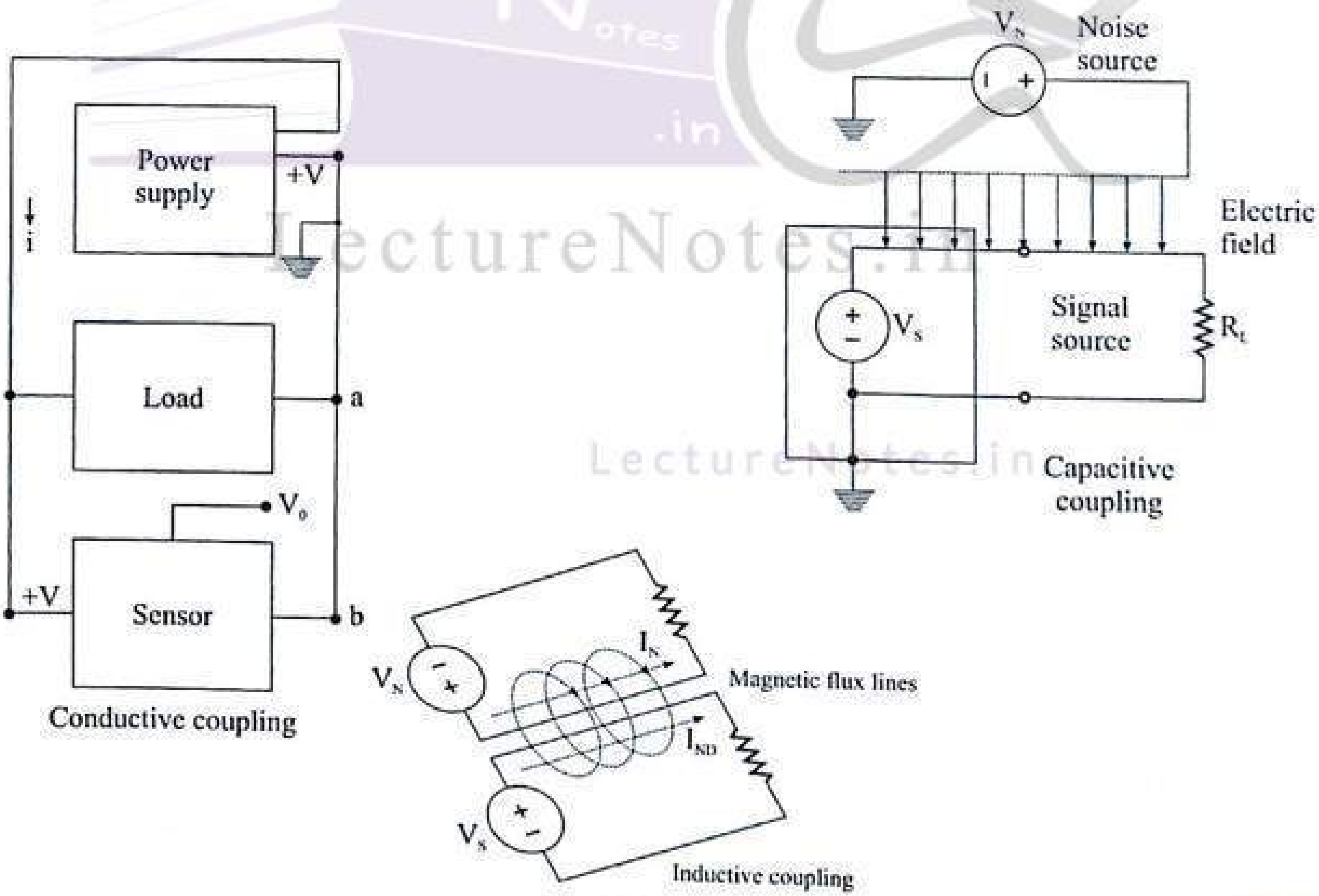


Fig 6.13

6.7.3 Noise Reduction

The noise caused in the signal before received by the measurement system needs to be completely eliminated or reduced to the maximum possible extent. The various technique are,

(i) Sheielding

(ii) Twisted pair wire.

A sheielded cable is made of a cupper braid or of foil and is usually grounded at the source end but not at the instrument end. The sheield can protect the signal from a significant amount of electromagnetic interference, especially at low frequencies. Sheielding can not prevent inductive coupling and hence twisted pair wire is used for reducing inductive coupling. Fig 6.14 shows the schematic diagram of shielding.

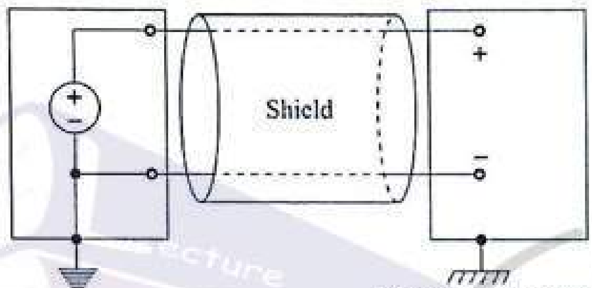


Fig 6.14

6.8 Signal Conditioning

After noise reduction the signal available is to be conditioned for effective and accurate measurement. The signal available may need the following operations known as signal conditioning.

(i) Amplification

(ii) Filtering

(iii) Isolation

Instrumentation amplifiers are usually employed to increase the strength of the measured signal without disturbing its original characteristics. During the course of amplification the noise present in the signal also get amplified. These noise are then filtered by suitable filters. The filtering may be of passive type or of active type. In passive filteres, passive elements like inductors and capacitors are used. The passive filter can filter noises of a fixed range of frequencies.

Active filters are used to effectively filter out the various noisy signals by using operational amplifiers (opamps), resistors and capacictors.

Active filters can be of following types.

(i) Low Pass

(ii) High Pass

(iii) Band Pass

(iv) Band Stop

If the noise have frequencies higher than the frequency of useful signal thus low pass filters will allow only the useful signal to the measurement system. signal to pass through above a certain frequency known as ***cut off frequency***.

The band pass filter allows only a certain range of frequencies to pass through while band stop filters blocks a particular range of frequencies. The characteristics of these filters are shown in the fig 6.15 .

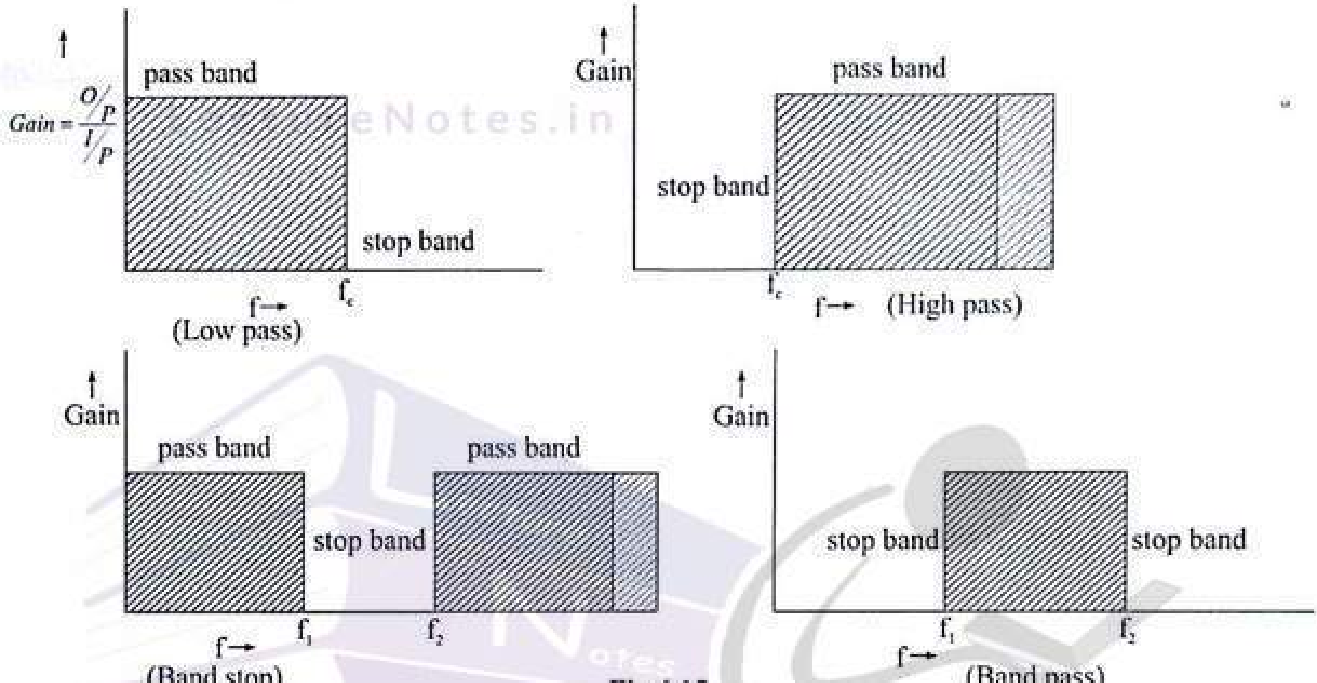


Fig 6.15

Some time the output of a sensor is used to control a system like speed cntrol of motors, voltage control of a dc-dc converter etc. In these application it is necessary to isolate the control circuit from the main power circuit. This can be achived by using pulse transformers, opto- couplers, drivers with isolation facilities (e.g IR 2110). A signal conditioning block diagram is shown in Fig 6.16 .

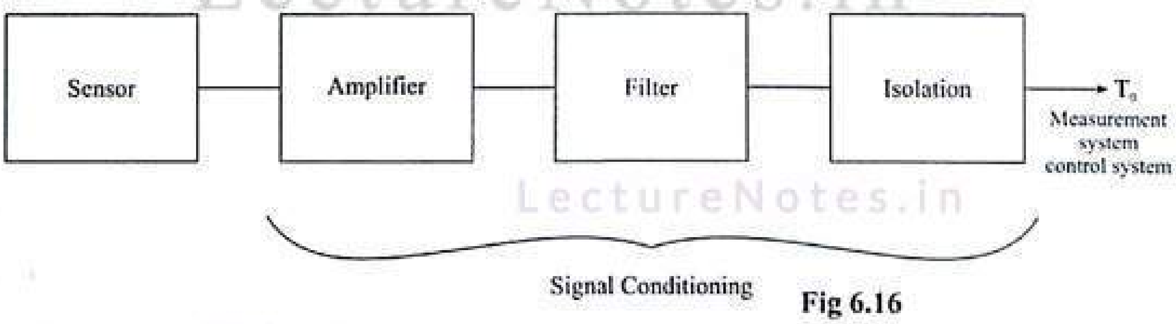


Fig 6.16

6.9 Analog to Digital Converter

An analog to digital converter, sometime abbreviated as ADC, A/D or A to D. It is an electronic device that converts an input analog voltage or current to a digital number proportional to the magnitude of the voltage of current.

By converting an analog into digital, it can be used to,

- display in a digital display devices.
- stored in a digital computer
- modified in a compatible form to other device.

Methods of ADC are (i) quantization

(ii) tracking ADC

(iii) integrating ADC.

(i) Quantization :

→ In quantization process, an analog signal will be sampled then quantized each sample.

→ It is a process of converting a real value into its equivalent binary form in one's and zero's.

→ The quantization process is shown in fig 6.17.

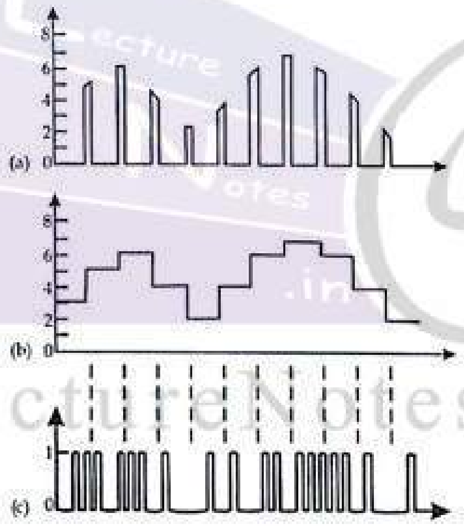


Fig. 6.17

The clock count accumulated during the discharge time is proportional to the analog voltage. Hence the clock count can be treated as a digital output for a particular analog signal.

6.10 Digital to analog converter (DAC)

A digital to analog converter (DAC) will convert a binary word to an analog output voltage. The binary word is represented in terms of one's (1) and zero's (0). DAC is working on a simple basic principle of binary to decimal conversion.

Let a given binary digit is, $B = \left(b_3\, b_2\, b_1\, b_0\right)_2$

Its equivalent decimal (analog) representation is, $\left[b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0\right]$

Depending on the above principle, a DAC shown in fig 6.18.

The given 4 bit binary are applied at the corresponding locations as shown at inputs b_3, b_2, b_1, b_0. For an operation amplifier the V_{out} is fined as,

$$V_{out} = -\left[\frac{R_f}{R_i} \cdot b_i \cdot V_{in}\right]$$

Hence $R_f = 2K$, $R_i = 1 \cdot 25K, 2 \cdot 5K, 5K, 10K$,

and $b_i = b_3, b_2, b_1, b_0$.

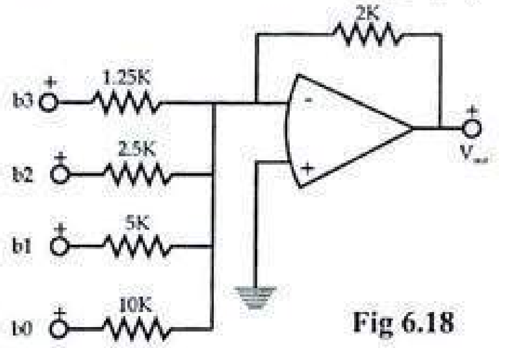


Fig 6.18

It will simply act as a summing amplifier. So the equation of converting binary to decimal will be applied here in order to convert a digital signal into analog signal the above circuit will be useful.

6.11 Types of ADC

Analog to digital converters are used to convert analog signal to digital signals. There are various type ADCS are used.

(1) Successive approximation (2) Flash

(3) Integrating ADC

Integrating ADC

The integrating ADC operates by the principles of discharging a capacitor. When a clock is used to allow the input analog voltage then the capacitor is charged for a shorter time, and determined by a fixed number of clock pulses. Then the capacitor is allowed to discharge through a known circuit and the corresponding clock count is incremented until the capacitor is fully discharged. The letter condition is verified by a comparator as shown in fig 6.19. The clock count accumulated during the discharge time is proportional to the analog voltage.

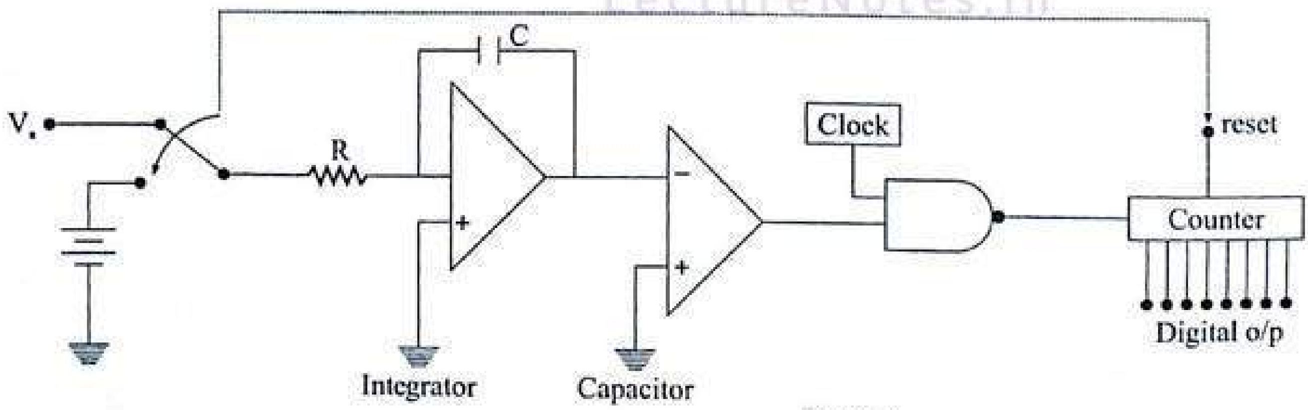


Fig 6.19

Successive Approximation ADC

Successive approximation ADCs are the most commonly used. A block diagram of the Successive approximation ADC is shown in fig 6.20 This method involvės making successive guesses at the binary code corresponding to the input voltage yi. The trial code is converted into an analogue voltage using a DAC and a comparator is used to decide whether the guess is too high or too low. On the basis of this result another guess is made, and process repeated until V_q is within half a quantization interval of y_i.

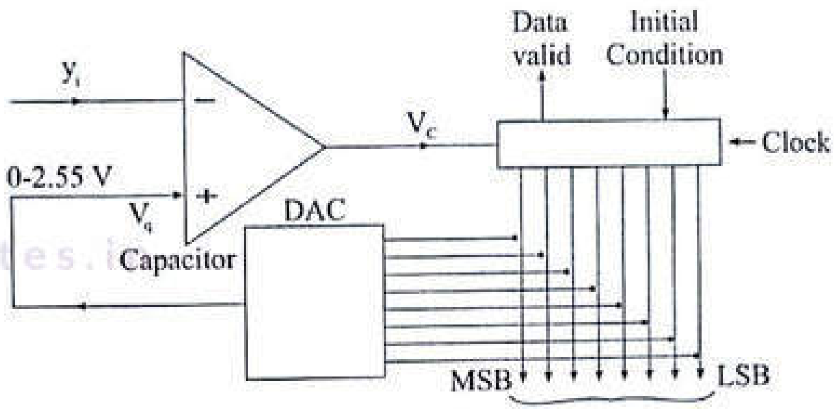


Fig 6.20

Flash ADC

The flash ADC is fully parallel and is used for high speed conversion. A resistive divider network of 2^n resistors divedes the known voltage range into that many equal increments. A network of $2^n - 1$ Comparators then compares the unknown voltage with that array of test voltages. All comparators with inputs excluding the unknown are on, all others are off. This comparator code can be converted to conventional binary by a digital priority encode circuit.

Tracking ADC

The tracking ADC is shown in fig 6.21. The input signal is applied to a positive input of a differential amplifier and it is continjuously compared with the negative input. The digital output will be achieved at the output of up-down counter.

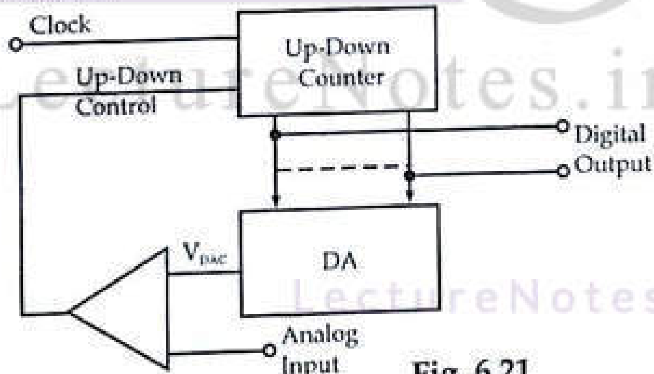


Fig. 6.21

The integrating ADC

Integrating ADC operates by charging and discharging a capacitor. It one can ensure that the capacitor charges linearly, then the time it will take for the capacitor to discharge is linearly related to the amplitude of the voltage that has charge the capacitor. A clock is used to allow the input (analog) signal to charge the capacitor for a short time and then the capacitor is allowed to discharge through a known circuit and the corresponding clock count is incremented until the capacitor is fully discharged. The later condition is verified by a comparator as shown in fig 6.22 in the block diagram.

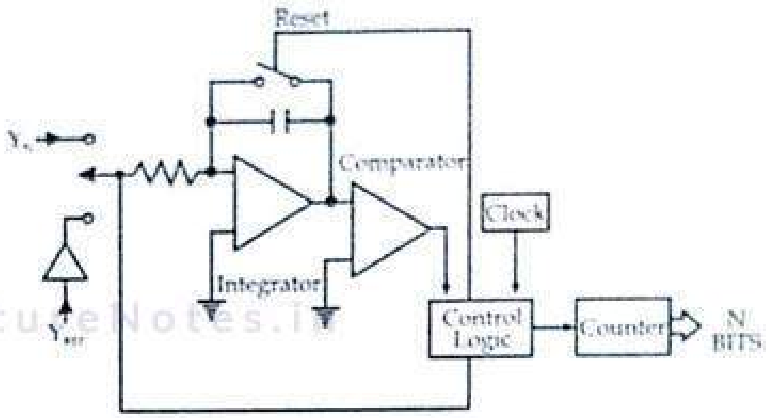


Fig 6.22

Example 6.1

How many comparator are needed in 8 bit flash ADC ?

Solution :

The number of comparators neededis $2^n - 1 = 2^8 - 1 = 255$

Example 6.2

A 4 bit DAC using a binary weighted resistor ladder has a reference sources of 10 v dc. and $R_f = R$. Find the output voltage E_0 for the input word $(1101)_2$.

Solution : $= \frac{-ER_f}{R} \sum \frac{ai}{2^{(i-1)} R}$ $\quad (a_1 = 1, a_2 = 1, a_3 = 0, a_4 = 1)$

$$= (-10v)\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2}\right] = (-10v)[1 + 05 + 0.125] = -1625v.$$

Example 6.3

A 4 bit DAC using the R-2R has a 5 v dc reference potential. Calculate to for the input word $(1010)_2$.

Solution : $E_0 = E \sum_{i=1}^{n} \frac{ai}{2^i} = 5\left[\frac{1}{2^1} + \frac{0}{2^2} + \frac{1}{2^3} + \frac{0}{2^4}\right] = 5\left[\frac{1}{2} + \frac{1}{8}\right]$

$$= 5[0 \cdot 5 + 0 \cdot 125] = 3 \cdot 125v$$

Example 6.4

If the maximum analog voltage $V_{a\,max}$ of a 12 bit digital to analog converter (DAC) is 15 V, find the essential step size 0 v by which V_a can increment. Here $n = 12, v = 15v$

Ans. $0v = \frac{V}{2^n} = \frac{15}{2^{12}} = 3.662\,mv$.

Do Your Self

Q.1. Find the full scale output potential form 8 bit DAC with a reference potential of 10 V DC.

[9.96 V]

Q2. Find the conversion time for a full scale input potential for an SA (Successive Approximation) ADC if the clock speed is 2.5 mHz.

Q3. An 8 bit DAC using a binary weighted resistor ladder has a +7.5 V DC reference potential. Find the output potential for the input word 101101112 if Rf = R.

Q4. A very fast ADC circuit that uses a bank of voltage comparators is called the .... or .... ADC.

Q5. A platinium resistance sensor is to be used to measure temperature between 0 and 200^0 C. Given the resistance $R_T\Omega$ at T^0C is given by $R_T = R_0\left(1+\alpha T+\beta T^2\right)$ and $R_0 = 100\Omega$, $R_{100} = 138\cdot 50\Omega$, $R_{200} = 175\cdot 83\Omega$. Calculate the value of α and β.

$$\left[3\cdot 91\times 10^{-3}\, i^{-1} - 5\cdot 85\times 10^{-7}\, i^{-2}\right]$$

Q6. A temperature alarm unit with a time constant of 120s is subjected to a sudden rise of temperature of 50^0 C because of fire. If an increase of 300 C is required to activate the alarm. What will be the delay in sudden temperature increase. [100δ]

Q7. A thermistor has a resistance temperature co-efficient of -5% over a temperature range of 25^0C to 50^0C. If the resistance of the thermistor is 100Ω at 25^0C. What is the resistance at 35^0C.

[50Ω]

Basic Electrical Engineering

Topic:

Transient Analysis

Contributed By:

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Silicon Institute Of Technology

Chapter - 4

Transient Analysis

4.1 Introduction

When an electric circuit containing an ohmic resistance only is switched on, the electric current acquires its maximum value almost in zero time. Similarly when such a circuit is switched off the current reduces to zero almost in zero time. When electric circuit contains an inductor or a capacitor or both, the growth as well as decay of current are opposed by emf induced. Therefore electric current takes some finite time to reach its maximum value, when the circuit is switched on. Similarly when the circuit is switched off, the current takes some finite time to decay to zero value. Electric currents which vary for a small finite time, while growing from zero to maximum value or while decaying from maximum value to zero value are called ***transient currents***

Transients are produced whenever :

(i) a circuit is shorted

(ii) there is a sudden change in applied voltage

(iii) a circuit is suddenly connected to or disconnected from the supply.

Transient currents are associated with the changes in stored energy in inductors and capacitors. There are no transients in pure resistive circuit. It is because resistors are not stored energy.

4.2 Differential equations for circuits containing inductors and capacitors

Inductor

When a wire of finite length twisted into a coil then it represents an inductor. Inductance is a property which opposes any change of magnitude or direction of electric current passing through the conductor. When a steady current (D.C) is allowed to flow in a inductor, it behaves like an inert element (i.e. presence is not felt). However, when the current changes with respect to time then inductor shows its presence and opposes to the change in current. As a result a voltage is induced across the inductor whose magnitude is given by $V_L = L\frac{di}{dt}$ and direction is opposite to the flow of current (by Lenz's law). If current through the inductor is constant then change in current is zero (i.e. di = 0). So voltage across inductor in zero (i.e. V_L=0). This means that an inductor behaves as a short circuited coil in steady state. For a minute change in current within zero time (dt = 0) the voltage across the inductor is infinite (i.e. $V_L = \infty$). This means that an inductor behaves as open circuited just after switching across d.c voltage.

Power absorbed by inductor is given by $P = Vi = L\cdot\frac{di}{dt}\cdot i = Li\cdot\frac{di}{dt}$

Energy absorbed by inductor is given by $W = \int P\cdot dt = \int Li\cdot\frac{di}{dt}\cdot dt = \frac{1}{2}Li^2$

It may be noted that the inductor can store finite amount of energy, even the voltage across it may be nil.

Capacitor

A capacitor is formed by any two conducting surfaces separated by a dielectric medium. Capacitance of the capacitor is defined as the ability to store energy in its electrostatic field. However, a capacitor has a typical characteristics which does not allow any change in voltage across it. If a change in voltage with respect to time is imposed across it then it allows a current through its dielectric medium (ie. *displacement current*). The magnitude of this current is given by

$$i = C\frac{dv}{dt} \quad \text{.....................................} \quad (1)$$

Where C in the capacitance in farad and $\frac{dv}{dt}$ is the rate of change of voltage in volt/sec.

(1) $$dv = \frac{1}{C}i\cdot dt$$

$$dt \Rightarrow V = \frac{1}{C}\int_{-\infty}^{t} i\cdot dt$$

$$\Rightarrow V = \frac{1}{C}\int_{-\infty}^{0} i \cdot dt + \frac{1}{C}\int_{0}^{t} i \cdot dt$$

$$\Rightarrow V = V_0 + \frac{1}{C}\int_{0}^{t} i \cdot dt \text{ .... (2)} \qquad \left[\because V_0 = \frac{1}{C}\int_{-\infty}^{0} i \cdot dt\right]$$

Where V_0 is the initial voltage (if any) across the capacitor at the instant of inserting in the circuit.

From equation (2) $\quad V = V_0 + \frac{1}{C}q(t) \qquad \left[\begin{array}{l} As\ i = \frac{dq}{dt} \\ \Rightarrow q(t) = \int i \cdot dt \end{array}\right]$

Power absorbed by capacitor is, $P = Vi = V \cdot C\frac{dv}{dt}$

Energy sorted by capacitor is, $W = \int_0^t p \cdot dt = \int_0^t V \cdot C\frac{dv}{dt} \cdot dt = \frac{1}{2}CV^2$

The capacitor, on application of d.c. voltage and with no initial charge first acts as short circuit but as soon as the full charge it retains, the capacitor behaves as open circuit. It may be noted that the capacitor can store finite amount of energy even the current across it may be nil.

4.2.1 Solution of Differential Equation

Differential equations are used in the treatment of transient i.e. single energy transients (involved in R-L and R-C circuits) and double -energy transients (involved in R-L-C circuit). We will consider both first - order and second - order differential equations.

1. First order Equations :

(i) Let $\frac{dy}{dx} + ay = 0$, Where a is a constant. The solution of this homogeneous differential equation is $y = Ke^{-ax}$, where K is the constant of integration whose value can be found from the boundary conditions.

(ii) Let $\frac{dy}{dx} + ay = Q$, where Q is either a function of independent variable or a constant.

The solution of this non homogeneous differential equation is

$y = e^{-ax}\int e^{ax} \cdot Q \cdot dx + Ke^{-ax}$

y = Particular integral + Complementary function

The particular integral part $\left(e^{-ax}\int e^{ax}\cdot Q\cdot dx\right)$ does not contain the arbitrary constant K and complementary function part $\left(Ke^{-ax}\right)$ does not depend on function Q.

2. Second Order Equations :

Let $A\dfrac{d^2y}{dx^2}+B\dfrac{dy}{dx}+Cy=0$, where A,B and C are constants.

The solution of this homogenous differential equation is $y=K_1\,e^{P_1x}+K_2e^{P_2x}$

Where K_1 and K_2 are the constants and P_1 and P_2 are the roots of the characteristic equation.

The characteristic equation is $AP^2+BP+C=0$

The roots of this equation $P_1,P_2=\dfrac{-B\pm\sqrt{B^2-4AC}}{2A}$

$$=\frac{-B+\sqrt{B^2-4AC}}{2A},\ \frac{-B-\sqrt{B^2-4AC}}{2A}$$

The values of K_1 and K_2 can be evaluated by using initial conditions for the original equations and derivative of the original equations, which is time. To summerise the behaviour of the basic elements at t = 0 and t = ∞ discussed elaborately in the examples.

4.2.2 Initial and steady state conditions

Every circuit when excited from a energy source the behaviour of the circuit is characterised by a differential equation, while solving the equation we get constants of integration. To evaluate these equations it is necessary to know the behaviour of the circuit (of each element) at various instant, particularly at initial (t = 0) and final (t = ∞) the following table is given.

Table

Sl.No.	Name of the element with symbol	At t = 0	At t = ∞
1	Resistance (R)	R	R
2	Inductance (L)	O.C	S.C
3	Capacitance (C)	S.C	O.C
4	Inductance with initial current Io (Io→)	Io	Io, S.C
5	Capacitance with initial voltage Vo (C, − +, V_o)	− + V_o, SC	+ − V_o, O.C

4.3. Transient response of 1st order circuit

The first order circuit contains resistance and one energy storing element i.e. one inductor or capacitor. This first order circuit, during its transient state of operation, is governed by first order linear differential equation.

4.3.1 D.C steady state solutions of R-L Circuit

Consider a R-L circuit connected in series with a battery of voltage V and a switch S. The R-L combination gets connected to the battery when switch S is connected to terminal 1 and is short circuited when switch S is connected to terminal 2.

Growth of current in RL circuit

When switch S moves to terminal 1, the battery is connected and current grows in R-L circuit. Due to self induction an induced emf is set up across inductance L. By lenz's law this induced emf opposes the growth of current. The current rises gradually and it takes a definite time to reach its steady value (i.e. maximum value). This time is called transient time.

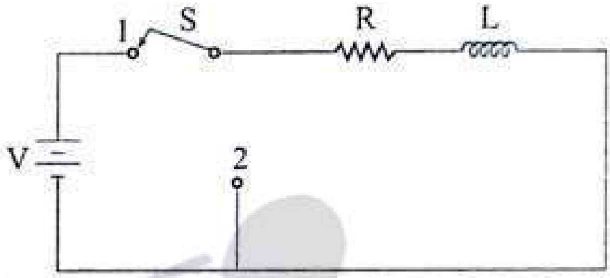

Fig. 4.1

Consider some instant t seconds after the voltage is applied,

i = Current flowing through the circuit at instant t seconds.

$\frac{di}{dt}$ = rate of growth of current at this instant.

$V_R = iR$ = Voltage across R

$V_L = L \cdot \frac{di}{dt}$ = Voltage across L

Applying Kirchhoff's Voltage law (KVL),

$$V = Ri + L \cdot \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \quad \text{.............................} \ (1)$$

Equation (1) is a non-homogeneous differential equation.

Solution of this equation is,

$$i = e^{-\frac{R}{L}t}\int e^{\frac{R}{L}t}\cdot\frac{V}{L}dt + Ke^{-\frac{R}{L}t} \text{ .... (2)}$$

$$\Rightarrow i = i_p + i_c$$

Where i_p is the particular solution that provides the steady state response and i_c is the complementary function that always goes to zero value in a short time (transient period).

we know $\frac{dy}{dx} + ay = Q$

$$\therefore y = e^{-ax}\int e^{ax}.Q.dx + Ke^{-ax}$$

$i = y$

$t = x$

here $\frac{R}{L} = a$

$\frac{V}{L} = Q$

Solution of equation (2) is,

$$i = \frac{V}{R} + Ke^{-\frac{R}{L}t} \text{ .... (3)}$$

An inductance has a property called as *"electrical inertia"* which does not allow sudden change of current through it following the laws of electromagnetic induction.

So current flowing through inductor just before switching is equal to current just after switching. But before switching there was no current through the inductor. Therefore, just after switching (i.e. at time $t = 0^+$) the current through the inductor will be zero.

At $t = 0^+$ (just after switching) i.e. with initial condition, equation (3) becomes

$$0 = \frac{V}{R} + Ke^{-\frac{R}{L}\times 0} = \frac{V}{R} + K$$

$$\Rightarrow K = -\frac{V}{R}$$

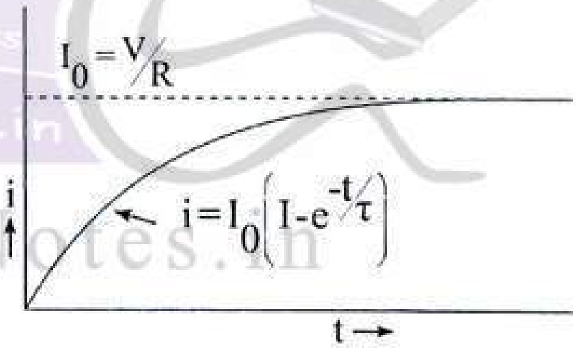

Fig. 4.2

Putting this value of K in equation (3) we get

$$i = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \text{ .... (4)}$$

Where $I_0 = \frac{V}{R}$ = maximum current or steady state current.

$\tau = \frac{L}{R}$ = time constant (or inductive time constant)

Equation (4) is called *Helmholtz equation* for growth of current in R-L circuit. This equation clearly shows the exponential rise of current i charging the inductor. The graph between i and t is shown in fig 4.2.

If we put $t = \tau = \frac{L}{R}$ in equation (4) we get $i = I_0\left(1 - e^{-1}\right) = 0\cdot632 I_0 = 63\cdot2\%$ of I_0.

Thus time constant is defined as the time during which current in an inductor rises to 63.2 percent of its maximum value.

If we put $t = \infty$ in equation (4) then we get $i = I_0\left(1 - e^{-\infty/\tau}\right) = I_0$. Thus current in R-L circuit would attain maximum value (I_0) only after infinite time. But practically current reaches its maximum value after a time which is five times the time constant.

Voltage drop across inductor is , $V_L = L\cdot\frac{di}{dt}$

$$\Rightarrow V_L = L\cdot\frac{d}{dt}\left[I_0\left(1 - e^{-\frac{t}{\tau}}\right)\right] = L\cdot\frac{d}{dt}\left[I_0 - I_0 e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_L = L\left[\frac{d}{dt}I_0 - \frac{d}{dt}I_0 e^{-\frac{t}{\tau}}\right] = L\left[0 + \frac{1}{\tau}I_0 e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_L = L\left[\frac{1}{L/R}I_0 e^{-\frac{t}{\tau}}\right] = I_0\,Re^{-\frac{t}{\tau}}$$

$$\Rightarrow V_L = Ve^{-\frac{t}{\tau}} \qquad (\because where\ V = I_0 R)$$

Voltage drop across resistor is

$$V_R = iR = I_0\left(1 - e^{-\frac{t}{\tau}}\right)R = V\left(I - e^{-\frac{t}{\tau}}\right)$$

In transient period voltage across resistor exponentially rising and voltage across inductance exponentially decaying. Once the transient dies out within a short time then steady current $\left(I_0 = \frac{V}{R}\right)$ remains in the circuit.

Decay of current in R-L circuit :

When switch S suddenly moves to terminal 2 then battery gets disconnected from the circuit and current in R-L circuit decays.

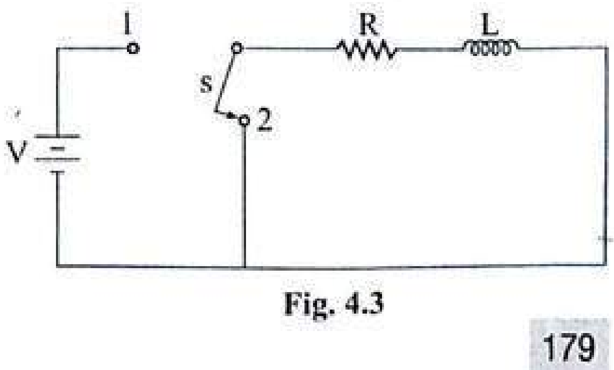

Fig. 4.3

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and delivery.

Let i = decaying current at any instant.

Applying Kirchhoff's law (KVL),

$$Ri + L\cdot\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0 \text{ .... (5)}$$

we know $\frac{dy}{dx} + ay = 0$

$\therefore y = Ke^{-ax}$

$i = y$

here $t = x$

$\frac{R}{L} = a$

Equation (5) is a homogeneous differential equation. Solution of this equation is

$$i = Ke^{-\frac{R}{L}t} \text{ .... (6)}$$

Where K is a constant whose value can be calculated from initial conditions.

With the initial conditions, at $t = 0, i = I_0 = \frac{V}{R}$

$\therefore$ equation(6) becomes.

$$\frac{V}{R} = Ke^{-\frac{R}{L}\times 0}$$

$$\Rightarrow K = \frac{V}{R}$$

Putting this value of K in equation (6) we get.

$$i = \frac{V}{R}e^{-\frac{R}{L}t} = I_0 e^{-\frac{t}{\tau}} \text{ .... (7)}$$

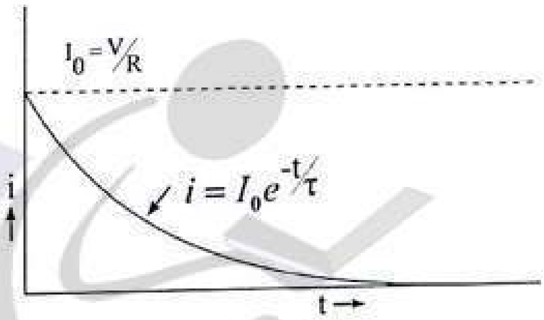

Fig. 4.4

where $\frac{V}{R} = I_0$ and $\frac{L}{R} = \tau$ = time constant.

Equation (7) is called Helmholtz equation for decay of current in R-L circuit. The graph between i and t is shown in fig.4.4.

If we put $t = \tau = \frac{L}{R}$ in equation (7) we get $i = I_0 e^{-1} = 0\cdot 37\, I_0 = 37\%$ of I_0.

Thus time constant is defined as the time during which current in an inductor falls to 37% of its maximum value.

Voltage drop across inductor, $V_L = L\frac{di}{dt}$

$$\Rightarrow V_L = L\frac{d}{dt}I_0 e^{-\frac{t}{\tau}} = L\cdot I_0\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} = L\cdot I_0\left(-\frac{1}{\frac{L}{R}}\right)e^{-\frac{t}{\tau}}$$

$$\Rightarrow V_L = -I_0 R\, e^{-\frac{t}{\tau}} = -Ve^{-\frac{t}{\tau}}$$

Voltage drop across resistance, $V_R = iR = I_0 Re^{-\frac{t}{\tau}} = Ve^{-\frac{t}{\tau}}$

4.3.2 D.C Steady state solutions of R-C circuit

Consider a R-C circuit connected in series with a battery of voltage V and a switch S. The R-C combination gets connected to the battery when switch S is connected to terminal 1 and is short circuited when switch S is connected to terminal 2.

Charging of R.C. Circuit

When switch S moves to terminal 1 then battery is connected to the circuit. Initially capacitor is uncharged and voltage across it is zero. (i.e. $V_C = 0$) . The whole of supply voltage V appears across resistance (i.e. $V_R = V$). As V_C is zero so capacitor behaves as a short circuit. According to Ohm's law the initial current in the circuit is $I_0 = \frac{V}{R}$.

As current flows the capacitor starts charging and capacitor voltage V_C increases. As a result voltage across resistance (V_R) decreases and charging current also decreases.

When capacitor becomes fully charged (i.e. $V_C = V$) then V_R becomes zero and charging current also becomes zero. As charging current is zero, the capacitor behaves as open circuit. This current takes a definite time to reach its zero value. This time is called transient time.

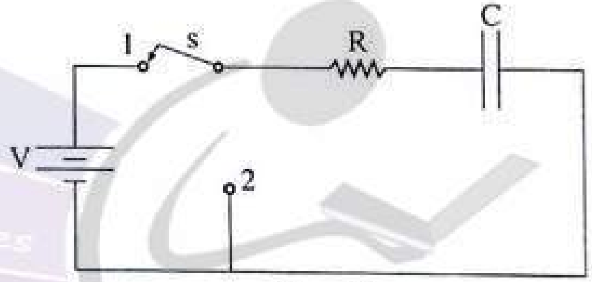

Fig. 4.5

Consider some instant t seconds after the voltage is applied.

i = current flowing through the circuit at instant t seconds.

$V_R = Ri$ = Voltage drop across R.

$V_C = \frac{1}{C}\int i \cdot dt$ = Voltage across C.

Applying Kirchhoff's voltage law (KVL)

$$Ri + \frac{1}{C}\int i \cdot dt = V$$

Differentiating both sides w.r.t time 't' we get,

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{1}{RC}i = 0 \text{ .... (1)}$$

we know $\frac{dy}{dx} + ay = 0$

$\therefore y = Ke^{-ax}$

here $i = y$

$t = x$

$\frac{1}{RC} = a$

Equation (1) is a homogeneous equation.

The solution of this equation is,

$$i = Ke^{-\frac{t}{RC}} \text{ .... (2)}$$

Where K is a constant whose value can be calculated from initial conditions.

With application of voltage and assuming no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the circuit current to be $\frac{V}{R}$

With the initial condition i.e. at $t = 0^+$,

Current $i(0^+) = \frac{V}{R}$, equation (2) becomes

$$\frac{V}{R} = Ke^{\frac{-0}{RC}}$$

$$\Rightarrow K = \frac{V}{R}$$

Putting this value of K in equation (2) we get

$$i = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$\Rightarrow i = I_0\, e^{-\frac{t}{\tau}} \text{ .... (3)}$$

Where $I_0 = \frac{V}{R}$ = maximum current (i.e initial current) in the circuit.

$\tau = RC$ = time constant (or capacitive time constant)

Equation (3) is an expression for charging current at any instant t seconds. This charging current in a decaying function whose graph is a curve as shown in fig. 4.6

As the capacitor is getting charged, the charging current dies out.

Voltage across resistor

$$V_R = iR = I_0\, Re^{-\frac{t}{\tau}} = Ve^{-\frac{t}{\tau}} \text{ .... (4)}$$

Voltage drop across capacitor

$$V_C = \frac{1}{C}\int i \cdot dt = \frac{1}{C}\int_0^t I_0 e^{-\frac{t}{\tau}}.$$

$$\Rightarrow V_C = \frac{1}{C} I_0 (-\tau)\left[-\frac{t}{\tau} \right]_0^t = \frac{1}{C} \cdot \frac{V}{R}(-RC)\left[e^{-\frac{t}{\tau}} \right]_0^t$$

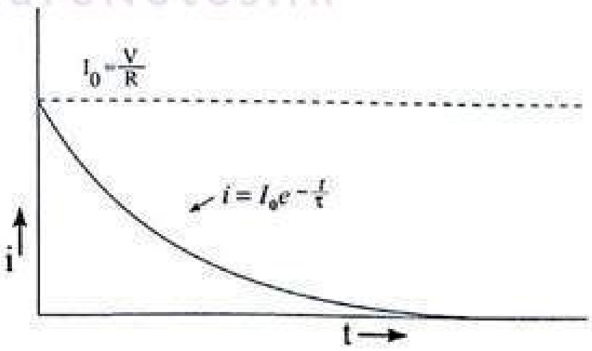

Fig. 4.6

$$\Rightarrow V_C = -V\left[e^{-\frac{t}{\tau}} - e^0\right] = V\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_C = V\left(1 - e^{-\frac{t}{\tau}}\right) \text{............................} (5)$$

The charge stored in capacitor during charging is $q = CV_C = CV\left(1 - e^{\frac{-t}{\tau}}\right) = Q\left(1 - e^{\frac{-t}{\tau}}\right)$

In transient period voltage across resistor exponentially decaying and voltage across capacitor exponentially rising as shown in fig. 4.7

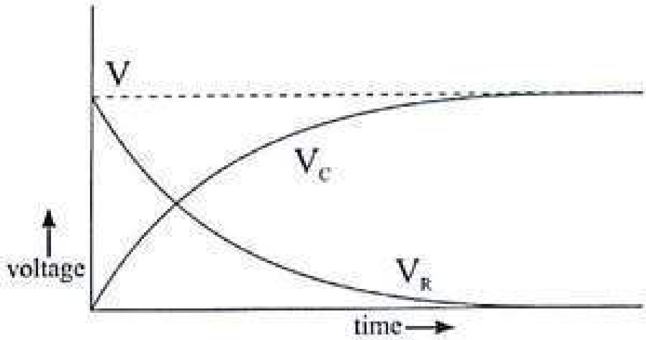

Fig. 4.7

If we put $t = \tau = RC$ in equation (5) we get

$V_C = V\left(1 - e^{-1}\right) = 0.632V = 63.2\%$ of V.

Thus time constant is defined as the time required for the voltage of capacitor to attain 63.2% of its steady state value. The time constant determines the length of transient period.

If time constant (τ) is more then transient period is also more (i.e. capacitor takes more time to attains its steady state value).

Discharging of RC Circuit :

When switch S moves to terminal 2 then battery gets disconnected from the circuit and R-C circuit is shorted as shown in fig. 4.8.

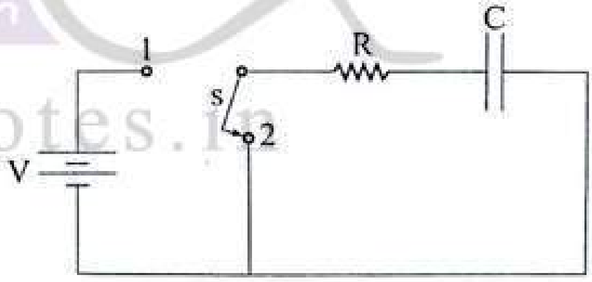

Fig. 4.8

The voltage across the capacitor was start discharging current through the resistor in opposite to the original current direction. Hence the direction of current during discharge is negative and its magnitude is V/R.

Let i = discharging current at any instant.

Applying Kirchhoff's Law (KVL)

$$Ri + \frac{1}{C}\int i.dt = 0$$

Differentiating both sides w.r.t. time t we get,

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

Equation (6) is a homogenous differential equation. Solution of this equation is,

$$i = Ke^{\frac{-t}{RC}} \quad \text{..............................} \quad (7)$$

Where K is a constant whose value can be calculated from initial conditions.

With the initial condition, at $t = 0$, $i = \frac{-V}{R}$, equation (7) becomes,

$$\frac{-V}{R} = Ke^{-\frac{0}{RC}}$$

$$\Rightarrow K = -\frac{V}{R}$$

Putting this value of K in equation (7) we get,

$$i = -\frac{V}{R}e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{\tau}} \quad \text{......................} \quad (8)$$

Equation (8) provides the decay of current in a charged capacitor. Fig. 4.9 shows the plot of decay of current of a charged capacitor with respect to time.

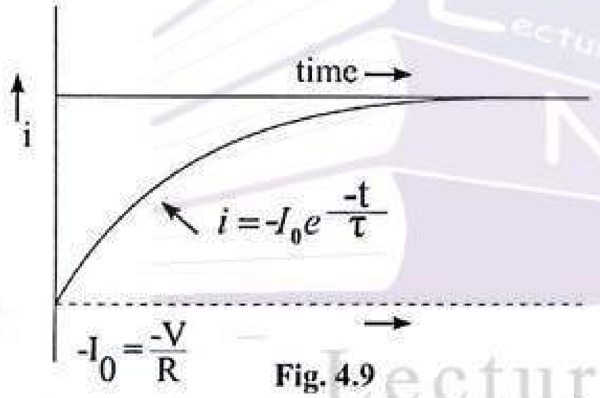

Fig. 4.9

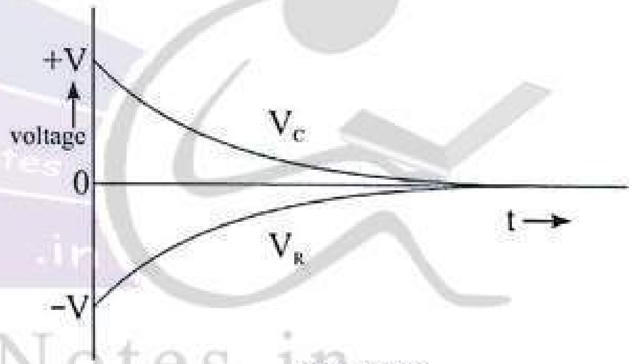

Fig. 4.10

voltage across resistor is $V_R = Ri = -I_0 \text{R}e^{-\frac{t}{\tau}} = -Ve^{-\frac{t}{\tau}}$

voltage across capacitor is $V_C = \frac{1}{C}\int i \cdot dt$

$$\Rightarrow V_C = \frac{1}{C}\int -I_0 e^{-\frac{t}{\tau}} \cdot dt = \frac{I_0}{C} \times e^{-\frac{t}{\tau}} \times \tau = \frac{I_0}{C} e^{-\frac{t}{\tau}} RC$$

$$\Rightarrow V_C = I_0 \text{R}e^{-\frac{t}{\tau}} = Ve^{-\frac{t}{\tau}}$$

Fig. 4.10 shows the plot of decay of voltage of a charged capacitor with respect to time.

The charge in capacitor during discharging is,

$$q = CV_C = CVe^{-\frac{t}{\tau}} = Qe^{-\frac{t}{\tau}}$$

where $Q = CV$ = maximum charge in capacitor.

4.4 Transient response of second order

The second order circuit contains two independent energy storage elements with or without addition to resistance. This second order circuit during its transient state of operation is governed by second order differential equation.

4.4.1 Solution of Second order circuits

Consider a second order circuit containing resistance (R),inductance (L) and capacitance (C) connected in series with d.c. source of voltage V as shown in fig. 4.11.

This circuit involves two types of energies i.e. electromagnetic and electrostatic. So any sudden change in the conditions of the circuit involves the redistribution of these two energies. The transient current produced due to this redistribution may be unidirectional or a decaying oscillatory.

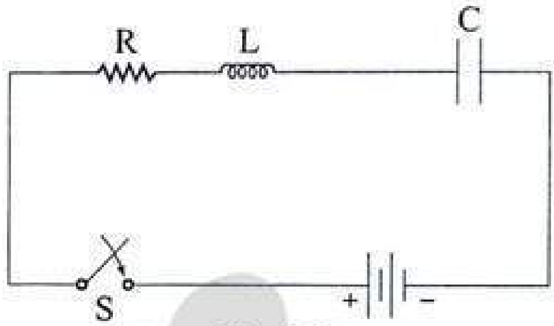

Fig. 4.11

When switch S is closed then R-L-C combination gets connected to the battery.

Let i = current flowing through the circuit at instant t seconds.

Applying Kirchhoff's voltage law (KVL)

$$iR + L\cdot\frac{di}{dt} + \frac{1}{C}\int i\cdot dt = V$$

Differentiating both sides with respect to time t we get,

$$R\cdot\frac{di}{dt} + L\cdot\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\cdot\frac{di}{dt} + \frac{1}{LC}i = 0 \text{ .... (1)}$$

Equation (1) is a second order linear homogeneous differential equation. Its characteristic equation is, $P^2 + \frac{R}{L}P + \frac{1}{LC} = 0$ .... (2) $\left[\because P = \frac{d}{dt}\right]$

The roots of this characteristic equation are, $P_1, P_2 = \dfrac{\frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= \alpha \pm \beta$$

$$P_1 = \alpha + \beta \quad \text{and} \quad P_2 = \alpha - \beta$$

where $\alpha = \frac{-R}{2L}$ and $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

The solution of the differential equation is,

$$i = K_1 e^{P_1 t} + K_2 e^{P_2 t} \quad \text{.......................} \ (3)$$

Where K_1 and K_2 are constants whose values are calculated from boundary conditions.

Case - 1 : High - loss circuit, $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ *i.e* overdamped.

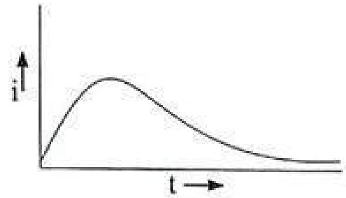

Fig. 4.12

In this case β is positive real quantity. Hence roots P_1 and P_2 are real but unequal.

$$\therefore \quad i = K_1 e^{P_1 t} + K_2 e^{P_2 t} = K_1 e^{(\alpha+\beta)t} + K_2 e^{(\alpha-\beta)t}$$

$$\Rightarrow i = K_1 e^{\alpha t} \cdot e^{\beta t} + K_2 e^{\alpha t} \cdot e^{-\beta t} = e^{\alpha t}\left[K_1 e^{\beta t} + K_2 e^{-\beta t}\right] \text{..........} \ (4)$$

Equation (4) represents over damped transient non-oscillatory current as shown in fig. 4.12.

Case - 2 : Low - Loss circuit, $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ *i.e* underdamped.

In this case β is imaginary. Hence the roots P_1 and P_2 are complex conjugates.

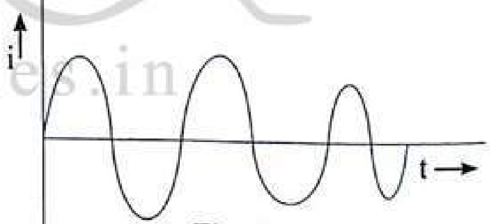

Fig. 4.13

$$P_1 = \alpha + j\beta \quad \text{and} \quad P_2 = \alpha - j\beta$$

$$\therefore \quad i = K_1 e^{P_1 t} + K_2 e^{P_2 t} = K_1 e^{(\alpha+j\beta)t} + K_2 e^{(\alpha-j\beta)t}$$

$$\Rightarrow i = K_1 e^{\alpha t} \cdot e^{j\beta t} + K_2 e^{\alpha t} \cdot e^{-j\beta t}$$

$$\Rightarrow i = e^{\alpha t}\left[K_1 e^{j\beta t} + K_2 e^{-j\beta t}\right] \text{..................} \ (5)$$

Equation (5) represents damped transient oscillatory current as shown in Fig. 4.13.

Case - 3 : $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ *i.e* critical damping.

In this case β is zero. Hence the roots P_1 and P_2 are real and equal.

$$P_1 = \alpha + 0 = \alpha \quad \text{and} \quad P_2 = \alpha - 0 = \alpha$$

For the case of real, repeated roots, not only $i = K_1 e^{\alpha t}$ satisfies the differential equation, equation (1), but also $i = K_2 e^{\alpha t}$ can be seen to satisfy equation (1). By superposition, the form of the transient solution will be seen to be $i = K_1 e^{\alpha t} + K_2 t e^{\alpha t}$......(6) which contains two constants, as it should for the solution of a second order differential equation.

It is a case of critical damping because current is reduced to almost zero in the shortest possible time as shown in fig. 4.14.

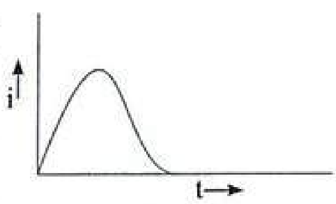

Fig. 4.14

Application of Thevenin's Theorem

Solution of simple circuits is possible with classical method as discussed above. When delt with series parallel circuits classical method become more complex for getting the solution. Hence Laplace transformation is used to solve such circuits. However, Thevenin's theorem can be used for first order differential equation with series / parallel configuration.

Procedure to solve a R-L circuit with Thevenin's theorem :

It is already derived that the charging current in a inductor is given by $i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$. Where L is the inductance across which the current in measured and R is the Thevenin's equivalent resistance of the circuit seen across inductor terminals, V is the Thevenin's equivalent voltage of the circuit seen across the inductor terminals. It is explained in detail in the following example.

Example 4.1 : The switch of the circuit shown in fig. 4.15 has been open for a very long time. At t = 0 the switch closes and again t = 50ms the switch opens. Determine the capacitor voltage as a function of time using Thevenin's theorem.

$R_1 = 1000\Omega, \quad R_2 = 1000\Omega,$

$R_3 = 500\Omega, \quad C = 25\mu F$

Solution : Given the switch has been open for a very long time. So $V_C(t) = 0$ at $t = 0^-$.

At $t = 0^+$ (i.e. switch has been closed for a long time) steady - state condition occurs since the voltage across the capacitor can not change instantaneously, this voltage $[V_C(t) = 0]$ will also be the capacitor voltage immediately after the switch is closed.

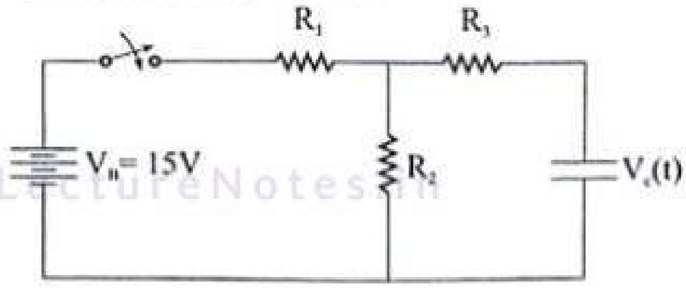

Fig. 4.15

$$\therefore \quad V_C(0^+) = V_C(0^-) = 0$$

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Example 4.7 : A constant voltage is applied to R-L series circuit at t = 0 by closing a switch. The voltage across L is 25 V at t =0 and drops to 5V at t = 0.025 Sec. If L = 2 H. find

i. the applied voltage and (ii) the value of R

Solution :

i. At t =0, the entire voltage is dropped across inductor and no voltage is dropped across resistor. So applied voltage V = 25 volt.

ii. At any instant voltage across inductor is 5V. (i.e. $V_L = 5$ volt)

$$V_L = Ve^{-\frac{t}{\tau}}$$

$$\Rightarrow 5 = 25e^{\frac{-0.025}{\tau}}$$

$$\Rightarrow \tau = 0 \cdot 01553$$

$$\Rightarrow \frac{L}{R} = 0 \cdot 01553$$

$$\Rightarrow R = \frac{L}{0 \cdot 01553} = \frac{2}{0 \cdot 01553} = 128 \cdot 78\Omega$$

Example 4.8 : The winding of an electromagnet has an inductance of 3H and resistance of 15 Ω, when it is connected to 120 V dc supply, Calculate

i. The steady state value of current flowing in the winding.

ii. The time constant of the circuit

iii. The value of induced emf after 0.1 Sec.

iv. The time for the current to rise to 85% of its final value.

v. The value of current after 0.3 Sec.

Solution : Given R = 15Ω, L = 3H, and V = 120V.

i. Steady state current, $I_0 = \frac{V}{R} = \frac{120}{15} = 8A$

ii. The time constant of the circuit

$$\tau = \frac{L}{R} = \frac{3}{15} = 0 \cdot 2 \text{ sec}$$

iii. The value of induced emf after 0.1 sec is,

$$V_L = Ve^{-\frac{t}{\tau}} = 120\left(e^{-\frac{0.1}{0.2}}\right) = 72.783 \text{ volt}$$

iv. Current at any instant during rise is,

Example 4.18 : Write the differential equation for t >0 for the circuit of Fig. 4.38

$V_{s1} = 13V, \quad V_{s2} = 13V$

$L = 170\,mH, \quad R_1 = 2\cdot 7\Omega$

$R_2 = 4\cdot 3K\Omega, \quad R_3 = 29K\Omega$

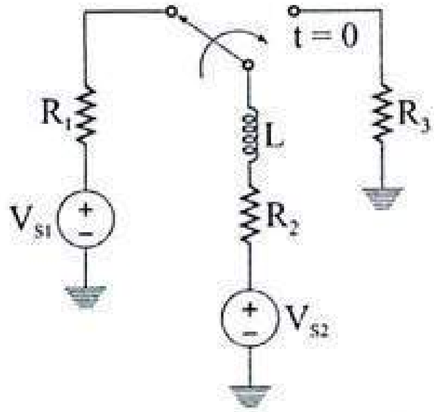

Fig 4.38

Solution : The differential equation for t > 0 (switch open) for the circuit as shown in Fig. 4.39

Applying KVL, we get

$$V_{s2} + i_L R_2 + V_L + i_L R_3 = 0$$

$$\Rightarrow (R_2 + R_3)\, i_L + V_L + V_{s2} = 0$$

$$\Rightarrow (R_2 + R_3)\, i_L + L\cdot \frac{di_L}{dt} + V_{s2} = 0$$

$$\Rightarrow \frac{di_L}{dt} + \left(\frac{R_2 + R_3}{L}\right) i_L + \frac{V_{s2}}{L} = 0$$

Substituting the numerical values, we get

$$\frac{di_L}{dt} + 1\cdot 96\times 10^5\, i_L + 76\cdot 5 = 0$$

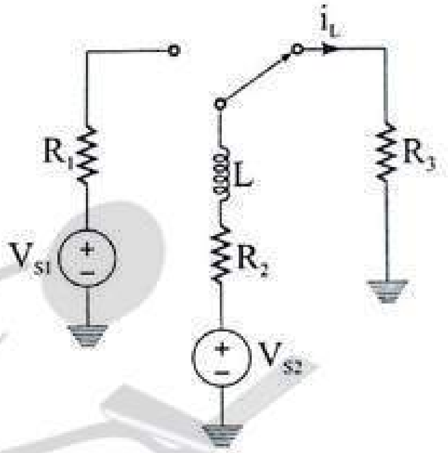

Fig 4.39

Example : 4.19 Write the differential equation for t >0 for the circuit of Fig. 4.40

$I_0 = 17mA, \quad C = 055\mu F$

$R_1 = 7K\Omega$ and $R_2 = 3\cdot 3K\Omega$

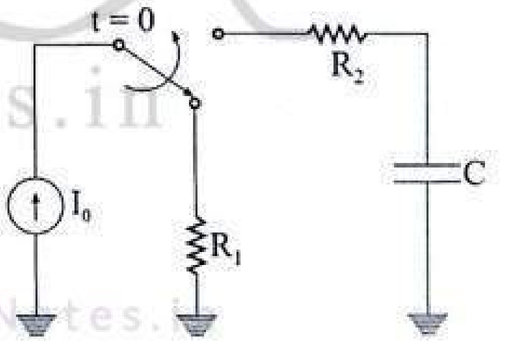

Fig 4.40

Solution : The differential equation for t>0 is shown in fig. 4.41

From figure, capacitor current is,

$$I_0 = C\cdot \frac{dv_c}{dt}$$

$$\Rightarrow \frac{dv_c}{dt} = \frac{I_0}{C}$$

Example 4.27 : Determine the current through the capacitor just before and just after the switch is closed in figure 4.51 Assume steady - state conditions for t < 0.

$V_1 = 12V, \qquad C = 0\cdot5\mu F,$

$R_1 = 0\cdot68\,k\Omega, \quad R_2 = 1\cdot8\,k\Omega,$

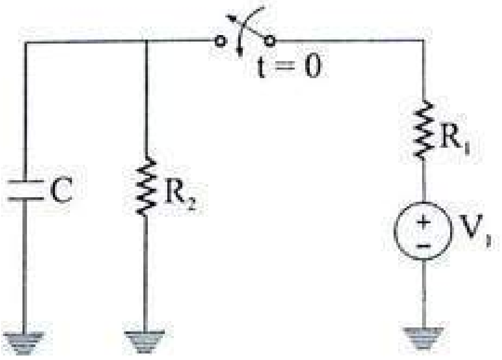

Fig 4.51

Solution : At $t = 0^-$, assume steady state conditions exist. If charge is stored on the plates of the capacitor, then there will be energy stored in the electric field of the capacitor and a voltage across the capacitor. This will cause a current flow through R_2 which will dissipate energy until no energy is stored in the capacitor at which time the current ceases and the voltage across the capacitor is zero.

These are steady state conditions.

$i_C(0^-) = 0, \qquad V_c(0^-) = 0$

At $t = 0^+$, the switch is closed and the transient starts. Since the voltage across a capacitor can not change instantaneously, this voltage will be the capacitor voltage immediately after the switch thrown.

$\therefore \quad V_C(0^+) = V_C(0^-) = 0$

At this instant capacitor behaves as a short circuit. Therefore, all of the voltage V_1 is across the resistor R_1.

Let $i_c(0^+)$ = current through capacitor.

Apply KCL at top node,

$$i_c(0^+) + \frac{V_c(0^+) - 0}{R_2} + \frac{V_c(0^+) - V_1}{R_1} = 0$$

$$\Rightarrow i_C(0^+) + 0 + \frac{0 - V_1}{R_1} = 0$$

$$\Rightarrow i_C(0^+) = \frac{V_1}{R_1} = \frac{12}{0\cdot68\times10^3} = 17\cdot65\,mA$$

Example 4.28 : Just before the switch is opened at $t = 0$ in fig. 4.52 the current through the inductor is 1.70 mA in the direction shown. Determine the voltage across R_3 just after the switch is opened.

L = 0.9 mH, $\qquad V_s = 12V$

At steady state, the capacitor behaves as an open circuit and we can calculate the equivalent open circuit voltage (Thevenin's voltage V_{th}) and equivalent resistance R_{th}.

From fig. 4.16 the Thevenin's voltage V_{Th} = Voltage across R_2.

According to voltage division rule, $V_{Th} = V_B \cdot \dfrac{R_2}{R_1 + R_2} = 15 \times \dfrac{1000}{1000+1000} = 7 \cdot 5V$

To calculate R_{Th} short the voltage source V_B as shown in fig. 4.17

$$\therefore \quad R_{Th} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 500 + \frac{1000 \times 1000}{1000+1000} = 1000\,\Omega$$

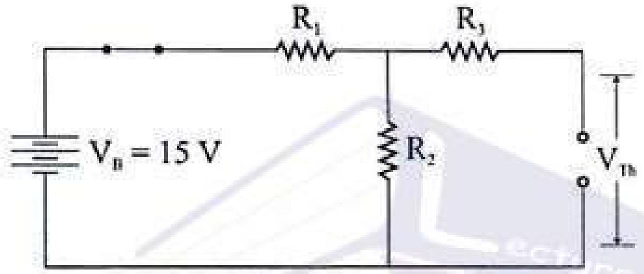

Fig. 4.16

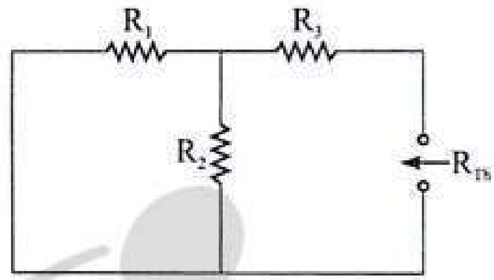

Fig. 4.17

To write the differential equation we use Thevenin's equivalent circuit as shown in fig. 4.18.

for $t \geq 0$

Let $i_C(t)$ = Current through capacitor C.

Apply KVL,

$$V_{Th} - R_{Th} i_c(t) - V_c(t) = 0$$

$$\Rightarrow V_{Th} - R_{Th} C \frac{dv_c(t)}{dt} - V_c(t) = 0$$

$$\Rightarrow R_{Th} C \frac{dv_c(t)}{dt} + V_c(t) = V_{Th}$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{R_{Th} C} V_c(t) = \frac{V_{Th}}{R_{Th} C}$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{1}{1000 \times 25 \times 10^{-6}} V_c(t) = \frac{7 \cdot 5}{1000 \times 25 \times 10^{-6}}$$

$$\Rightarrow \frac{dv_c(t)}{dt} + 40 V_c(t) = 300 \text{ .... (1)}$$

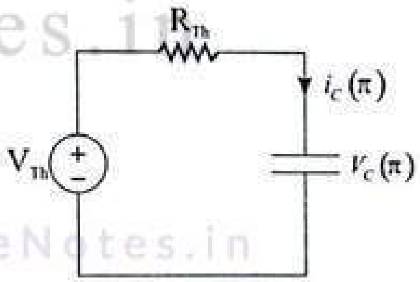

Fig. 4.18

$$i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \frac{85}{100} I_0 = I_0\left(1 - e^{\frac{-t}{0.2}}\right)$$

$$\Rightarrow t = 0 \cdot 379 \text{sec.}$$

v. After 0.3 sec. the current $i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$

$$= 8\left(1 - e^{\frac{-0.3}{0.2}}\right) = 6 \cdot 21 A$$

Example 4.9 : The time constant of a coil was found to be 2.5 ms. With a resistance of 100 Ω added in series, a new time constant of 0.5 ms was obtained. Find R and L of the coil.

Solution : Given time constant $\tau = \frac{L}{R} = 2 \cdot 5 \times 10^{-3}$ sec.

When resistance (R) added then time constant

$$\tau^1 = \frac{L}{R + 100} = 0 \cdot 5$$

$$\therefore \quad \frac{\tau}{\tau^1} = \frac{2 \cdot 5}{0 \cdot 5}$$

$$\Rightarrow \frac{\frac{L}{R}}{\frac{L}{R} + 100} = 5$$

$$\Rightarrow \frac{R + 100}{R} = 5$$

$$\Rightarrow R = 25\Omega$$

But given $\frac{L}{R} = 2 \cdot 5 \times 10^{-3}$.

So, $L = 2 \cdot 5 \times 10^{-3} R = 2 \cdot 5 \times 10^{-3} \times 25 = 62 \cdot 5 \times 10^{-3} H$

Example : 4.10 An 8μF capacitor is connected in series with 0.5 MΩ resistance across 200V supply. Calculate (i) initial charging current (ii) the current and p.d across capacitor 4 seconds after it is connected to the supply.

Solution : Given $C = 8 \times 10^{-6} F$, $R = 0 \cdot 5 \times 10^6 \Omega$,

V = 200 Volt.

(i) Initial charging current $= \frac{V}{R} = \frac{200}{0 \cdot 5 \times 10^6} = 4 \times 10^{-4} A$

Substituting the numerical values we get,

$$\frac{dv_c}{dt} = 30909$$

$$\Rightarrow \frac{dv_c}{dt} - 30909 = 0$$

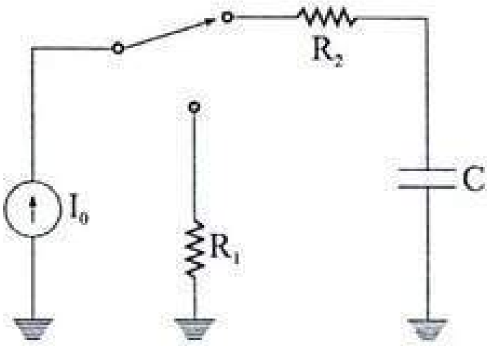

Fig 4.41

Example : 4.20 Write the differential equation for t >0 for the circuit shown in Fig. 4.42

$V_{s1} = V_{s2} = 11V$, $C = 70nF$,

$R_1 = 14K\Omega$ $R_2 = 13K\Omega$

and $R_3 = 14K\Omega$

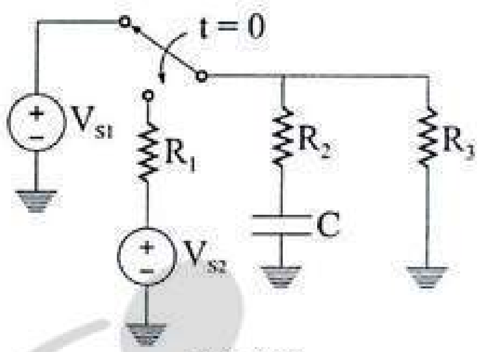

Fig 4.42

Solution : The differential equation for t > 0 (switch closed) as shown in fig. 4.43

Let V_1 = Voltage at node 1.

$\therefore \quad V_C + i_C R_2 = V_1$

Apply KCL at node 1

$$\frac{V_{s2} - V_1}{R_1} - i_C + \frac{0 - V_1}{R_3} = 0$$

$$\Rightarrow \frac{V_1 - V_{s2}}{R_1} + i_C + \frac{V_1}{R_3} = 0$$

$$\Rightarrow \frac{V_C + i_C R_2 - V_{s2}}{R_1} + i_C + \frac{V_C + i_C R_2}{R_3} = 0$$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_3}\right) V_C + \left(\frac{R_2}{R_1} + 1 + \frac{R_2}{R_3}\right) i_C - \frac{V_{s2}}{R_1} = 0$$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_3}\right) V_C + \left(\frac{R_2}{R_1} + 1 + \frac{R_2}{R_3}\right) C \cdot \frac{dv_c}{dt} - \frac{V_{s2}}{R_1} = 0$$

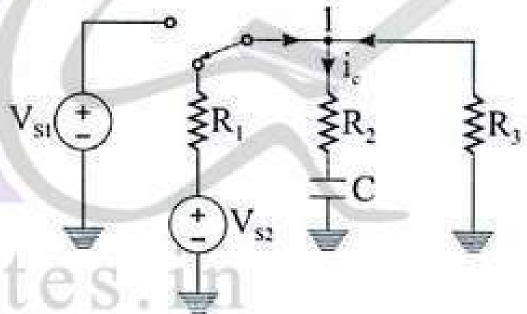

Fig 4.43

Substituting the numerical values we get,

$$\frac{dV_c}{dt} + 714 \cdot 3V_c - 3929 = 0$$

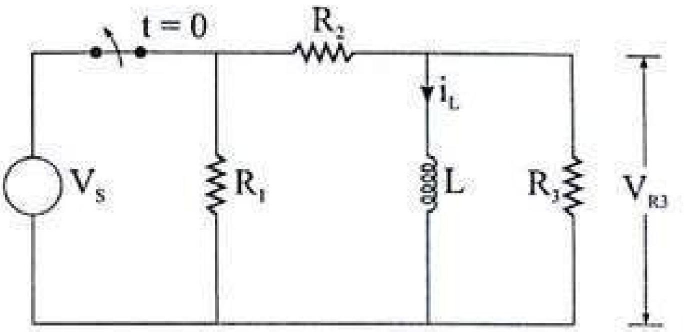

Fig 4.52

$R_1 = 6k\Omega$, $R_2 = 6k\Omega$

$R_3 = 3k\Omega$

Solution : Given $i_L = 1.70\,mA$ before the switch is opened. When the switch is opened the voltage source is disconnected from the circuit. Since the current through the inductor can not change instantaneously the current through the inductor at $t = 0^+$ is 1.70 mA. At this instant the inductor behaves as a DC current source.

Let $V_{R3} =$ Voltage across R_3.

$$i_L(0^+) = i_L(0^-) = 1.7\,mA$$

Apply KCL at top node,

$$\frac{V_{R3}(0^+)}{R_3} + \frac{V_{R3}(0^+)}{R_1 + R_2} = i_L(0^+)$$

$$\Rightarrow V_{R3}(0^+) = \frac{i_L(0^+)}{\frac{1}{R_3} + \frac{1}{R_1 + R_2}} = \frac{1.7 \times 10^{-3}}{\frac{1}{3\times10^3} + \frac{1}{12\times10^3}} = 4 \cdot 080V$$

Example 4.29 : Determine the voltage across the inductor just before and just after the switch is changed in figure 4.53 Assume steady state conditions exist for t < 0.

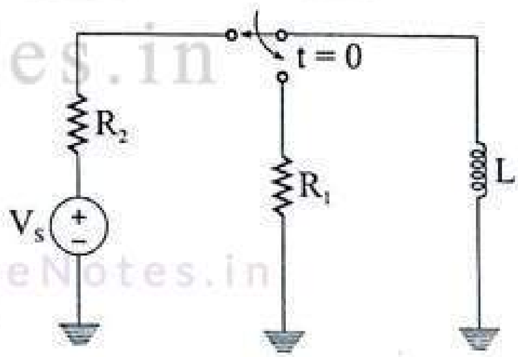

Fig 4.53

$R_1 = 22\,k\Omega$ $R_2 = 0 \cdot 7\Omega$

$V_s = 12V$, L = 100 mH

Solution : (i) At $t = 0^-$ (switch is connected to R_2)

Steady state condition exist and the inductor behaves as short circuit.

$$\therefore \; V_L(0^-) = 0$$

Apply KVL, $V_s - i_L(0^-)R_2 - V_L(0^-) = 0$

$$\Rightarrow i_L(0^-) = \frac{V_s}{R_2} = \frac{12}{0 \cdot 7} = 17 \cdot 14\,A$$

Equation (1) is a non-homogeneous equation. Its solution is given by,

$$V_c(t) = e^{-40t}\int e^{40t}\cdot 300\cdot dt + Ke^{-40t}$$

$$\Rightarrow V_c(t) = e^{-40t}\times\frac{300}{40}\times e^{40t} + Ke^{-40t}$$

$$\Rightarrow V_c(t) = 7\cdot 5 + Ke^{-40t} \text{ .... (2)}$$

Where K is a constant whose value can be calculated from initial conditions.

We know $\frac{dy}{dx}+ay=Q$

$\therefore y=e^{-ax}\int e^{ax}Q\,dx+Ke^{-ax}$

Here $V_C(t)=y$

$t=x$

$40=a$

$300=Q$

At $t=0^+$, $V_c(t)=0$

$\therefore \quad 0 = 7\cdot 5 + Ke^{-40\times 0}$

$\Rightarrow K = -7\cdot 5$

Putting this value of K in equation (2) we get,

$$V_c(t) = 7\cdot 5 - 7\cdot 5e^{-40t}$$

$$\Rightarrow V_c(t) = 7\cdot 5\left(1-e^{-40t}\right)$$

Example 4.2 : At $t = 0$, the switch of the circuit is closed. With no initial current through the inductance, determine the instantaneous voltage across the 6 Ω resistance as shown in fig 4.19.

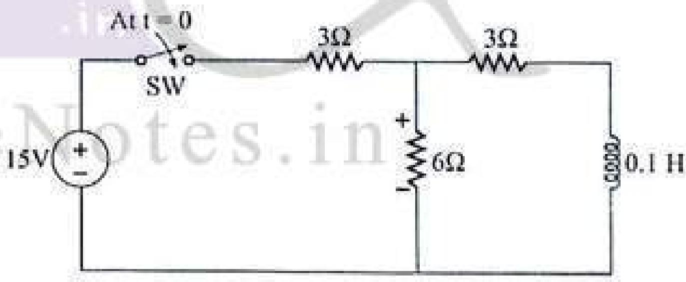

Fig 4.19

Solution : At $t = \bar{0}$ (before closing)

Voltage across 6 Ω resistor = 0.

At $t = \overset{+}{0}$ (after closing the switch), the circuit is shown in fig 4.20.

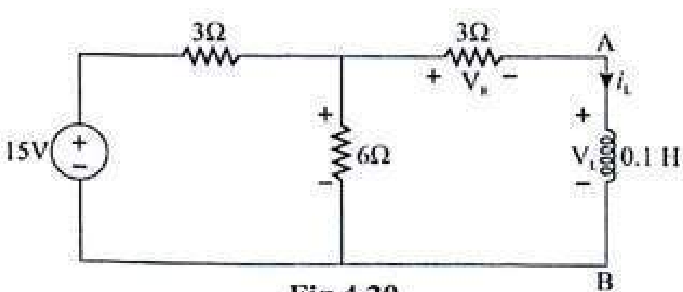

Fig 4.20

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(ii) Time constant $\tau = RC = 0\cdot5\times10^{6}\times8\times10^{-6} = 4$ Sec.

The current across capacitor is 4 seconds after it is connected to the supply is,

$$i = \frac{V}{R}e^{-\frac{t}{\tau}}$$

$$\Rightarrow i = 4\times10^{-4}e^{\frac{-4}{4}} = 1\cdot47\times10^{-4}A$$

The p.d. across capacitor in 4 seconds after it is connected to the supply is $V_C = V\left(1-e^{-\frac{t}{RC}}\right)$

$$\Rightarrow V_C = 200\left(1-e^{\frac{-4}{4}}\right) = 126\cdot44 \text{ volt.}$$

Example 4.11 : A capacitor of 1μF and resistance 82 KΩ are connected in series with an emf of 100V. Calculate the magnitude of energy and the time in which energy stored in the capacitor will reach half of its equilibrium value.

Solution : Given C = 1μF = 10^{-6}F, $R = 82\times10^{3}\,\Omega$ and V = 100 Volt.

Time constant = RC = 0.082 sec.

Equilibrium value of energy = $\frac{1}{2}CV^2$

Let V_1 = potential of capacitor when energy stored through it is $\frac{1}{4}CV^2$ (i.e. half of equilibrium value).

t = time taken by potential V_1 across capacitor.

Energy stored in capacitor in time $t = \frac{1}{2}CV_1^2$

Given $\frac{1}{2}CV_1^2 = \frac{1}{4}CV^2$

$$\Rightarrow V_1 = \frac{V}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70\cdot721\, volt$$

Also, $V_1 = V\left(1-e^{-\frac{t}{\tau}}\right)$

$$\Rightarrow 70\cdot721 = 100\left(1-e^{\frac{-t}{0\cdot082}}\right)$$

$$\Rightarrow t = 0\cdot1 \text{ sec.}$$

Energy stored in capacitor = half of equilibrium energy =
$\frac{1}{4}CV^2 = \frac{1}{4}\times10^{-6}\times100^2 = 2\cdot5\times10^{-3}$ Joule.

Example 4.21 : Write the differential equation for $t > 0$ for the circuit shown in fig. 4.44

$R_1 = 5\Omega$, $R_2 = 4\Omega$,

$R_3 = 3\Omega$, $R_4 = 6\Omega$

$C_1 = C_2 = 4F$

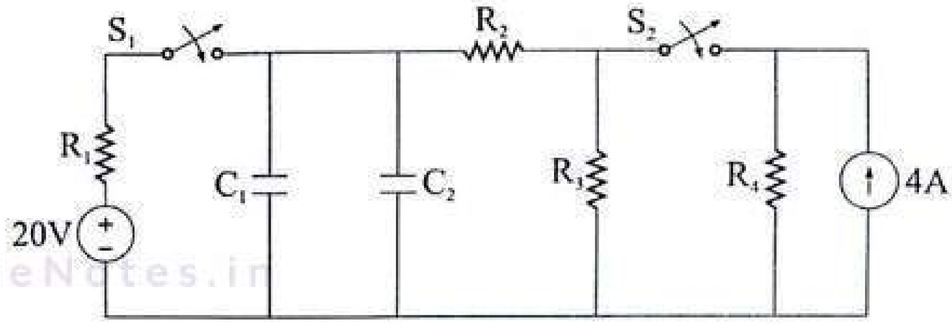

Fig 4.44

Solution : The differential equation for t > 0 (switch closed) as shown in fig. 4.45

Assume that switch S_1 is always open and switch S_2 closes at t = 0

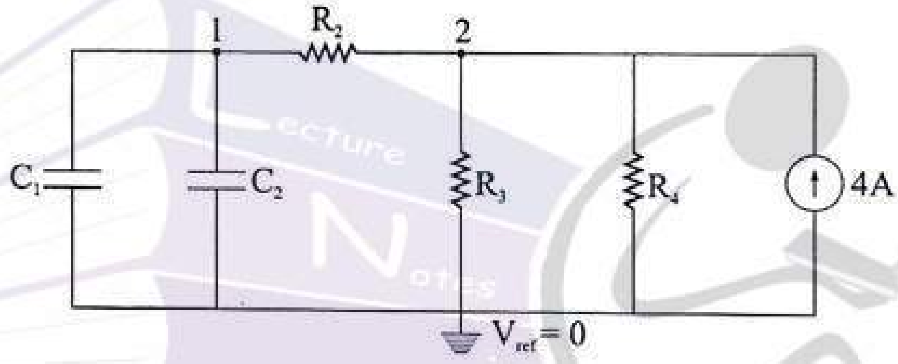

Fig 4.45

Node 1 voltage is equal to the two capacitor voltages (i.e. $V_{c1} = V_{c2} = V_c$)

Apply KCL to node 1,

$$i_{c1} + i_{c2} + \frac{V_2 - V_c}{4} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1) \qquad \{\because V_1 = V_c\}$$

Apply KCL to node 2,

$$\frac{V_c - V_2}{4} - \frac{V_2}{3} - \frac{V_2}{6} + 4 = 0$$

$$\Rightarrow V_2 = \frac{V_c}{3} + \frac{16}{3}$$

Putting this value of V_2 in equation (1) we get,

(ii) At $t = 0^+$ (switch is connected to R_1) transient occurs. Since current through inductor can not change instantaneously so $i_L(0^+) = i_L(0^-) = 17 \cdot 14A$

Let $V_L(0^+)$ = Voltage across inductor.

Apply KVL, $-V_L(0^+) - i_L(0^+)R_1 = 0$

$$\Rightarrow V_L(0^+) = -i_L(0^+)R_1 = -17 \cdot 14 \times 22 \times 10^3 = -337 \cdot 1KV$$

Example 4.30 : At $t < 0$, the circuit shown in Fig. 4.54 is at steady state. The switch is changed as shown t = 0

$V_{s1} = 13V, \quad V_{s2} = 13V$

$L = 170\,mH, \quad R_1 = 2 \cdot 7\Omega$

$R_2 = 4 \cdot 3\,k\Omega, \quad R_3 = 29\,k\Omega$

Determine the time constant of the circuit for t > 0.

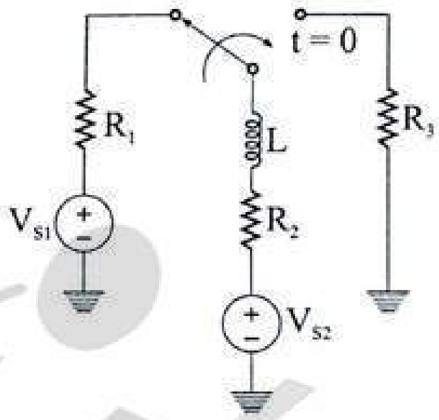

Fig 4.54

Solution : Given the circuit is in steady state condition for t < 0. For t > 0 (switch is connected to R_3) transient starts.

Time constant

$$(\tau) = \frac{L}{R_2 + R_3} = \frac{170 \times 10^{-3}}{4 \cdot 3 \times 10^3 + 29 \times 10^3} = \frac{170 \times 10^{-3}}{(4 \cdot 3 + 29)10^3} = 5 \cdot 105 \times 10^{-6} \text{ second.}$$

Example 4.31 : Determine $V_c(t)$ for t > 0. The voltage across the capacitor in fig. 4.55 just before the switch is changed is given below.

$V_C(0^-) = -7V, \quad I_0 = 17\,mA$

$C = 0 \cdot 55\mu F, \quad R_1 = 7\,k\Omega$

$R_2 = 3 \cdot 3\,k\Omega$

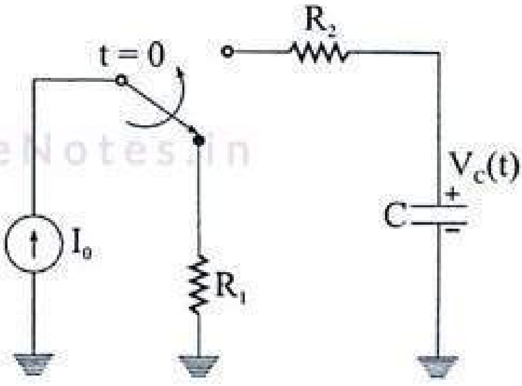

Fig 4.55

Solution : When switch is connected to R_1, the voltage across capacitor is - 7V (given). When switch is connected to R_2 the current source (I_0) and capacitor are connected in series. So current to capacitor is constant at I_0.

Now let us find the Thevnins equivalent circuit across AB as shown in fig 4.21 and hence find out the current i_L.

$$\therefore V_{th} = \frac{15\,(6)}{9} = 10V$$

$$R_{th} = (3 \,||\, 6) + 3 = 5\Omega$$

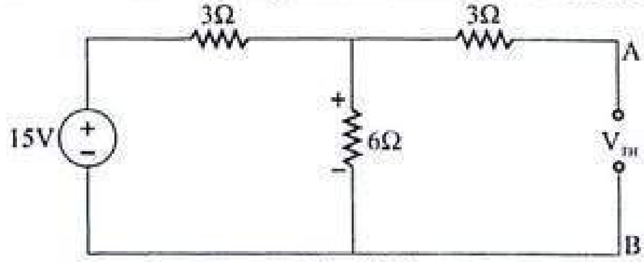

Fig 4.21

Referring to the Thevenins equivalent circuit shown in fig 4.22 the charging current in the inductor is given by

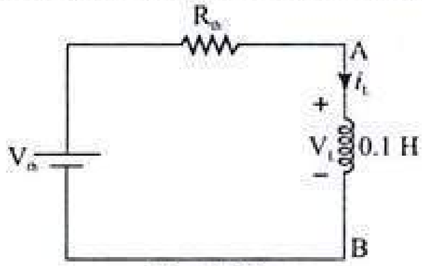

Fig 4.22

$$i_L = \frac{V_{th}}{R_{th}}\left(1 - e^{\frac{-R_{th}t}{L}}\right)$$

$$\Rightarrow i_L = \frac{10}{5}\left(1 - e^{\frac{-5t}{0.1}}\right) = 2\left(1 - e^{-50t}\right)A$$

Voltage across the inductor is

$$V_L = L \cdot \frac{di}{dt} = 0.1\,\frac{d}{dt}\left[2\left(1 - e^{-50t}\right)\right]$$

$$\Rightarrow V_L = 10e^{-50t} \text{ volts}$$

Voltage across $3\,\Omega$ resistor is $V_R = 3\,i_L$

$$\Rightarrow V_R = 3 \times 2\left(1 - e^{-50t}\right) = 6 - 6\,e^{-50t} \text{ volts.}$$

$\therefore$ Voltage across $6\,\Omega$ resistor is $= V_R + V_L = 6 - 6e^{-50t} + 10e^{-50t} = 6 + 4e^{-50t}$ volts.

Example 4.3: With no initial energy stored in the inductance of the circuit shown in fig 4.23 the switch is closed at t = 0. Evaluate

(*i*) the voltage across the inductance.
(*ii*) the source current.

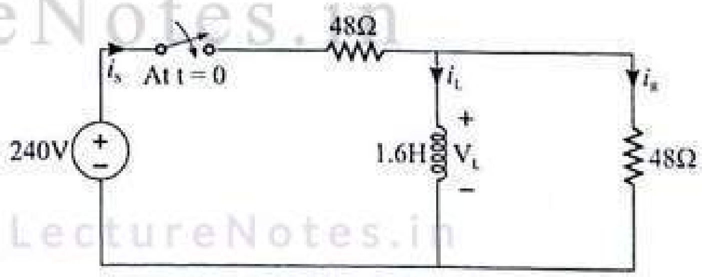

Fig 4.23

Solution : Let us find Thevnins equivalent circuit across the inductor and hence the charging current i_L through it. From fig 4.24,

$$V_{th} = 240\left(\frac{48}{48 + 48}\right) = 120 \text{ volts.}$$

$$R_{th} = 48 \,||\, 48 = 24 \; ohms.$$

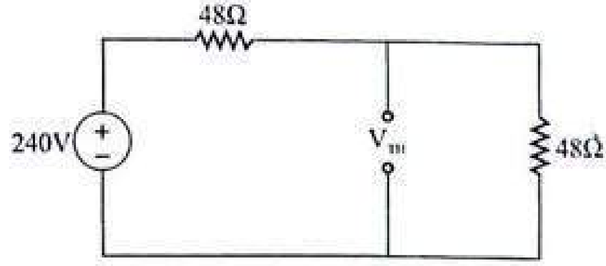

Fig 4.24

Example 4.12: Write the differential equation for $t > 0$ for the circuit of fig. 4.32.

$R_1 = R_2 = 6K\Omega$

$R_3 = 3K\Omega, \quad L = 0\cdot 9\, mH$

$V_s = 12$ volt.

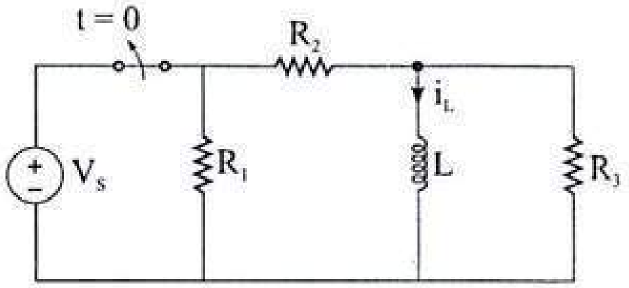

Fig 4.32

Solution : Consider a reference node whose potential is zero. The differential equation for t > 0 (switch open) for the circuit as shown in fig. 4.33.

Let the direction of currents through R_2 and R_3 are towards the node

The top node voltage is equal to inductor voltage V_L. Apply KCL to top node,

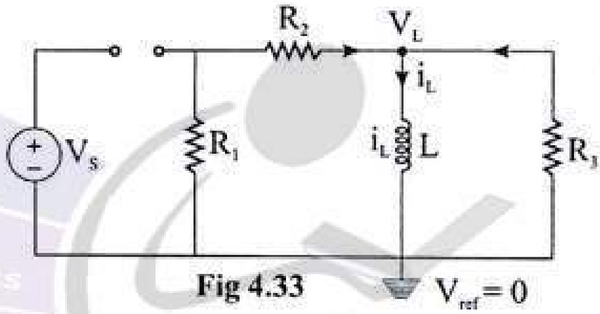

Fig 4.33

$$\frac{0 - V_L}{R_1 + R_2} + \frac{0 - V_L}{R_3} = i_L$$

$$\Rightarrow \frac{V_L}{R_1 + R_2} + \frac{V_L}{R_3} + i_L = 0$$

$$\Rightarrow \frac{L}{R_1 + R_2} \cdot \frac{di_L}{dt} + \frac{L}{R_3}\frac{di_L}{dt} + i_L = 0$$

$$\Rightarrow L \cdot \frac{di_L}{dt}\left[\frac{R_1 + R_2 + R_3}{(R_1 + R_2)R_3}\right] + i_L = 0 \qquad \left[\because V_L = L \cdot \frac{di_L}{dt}\right]$$

$$\Rightarrow \frac{L \cdot \frac{di_L}{dt}(R_1 + R_2 + R_3) + (R_1 + R_2)R_3 \cdot i_L}{(R_1 + R_2)R_3} = 0$$

$$\Rightarrow \frac{di_L}{dt} + \frac{(R_1 + R_2)R_3}{L(R_1 + R_2 + R_3)} i_L = 0$$

Putting the numerical values we get,

$$\frac{di_L}{dt} + 2\cdot 67 \times 10^6\, i_L = 0$$

$$i_{c1} + i_{c2} + \frac{\frac{V_c}{3} + \frac{16}{3} - V_c}{4} = 0$$

$$\Rightarrow i_{c1} + i_{c2} + \frac{8}{6} - \frac{V_c}{6} = 0$$

$$\Rightarrow C_1 \cdot \frac{dV_c}{dt} + C_2 \cdot \frac{dV_c}{dt} + \frac{8}{6} - \frac{V_c}{6} = 0$$

$$\Rightarrow (C_1 + C_2)\frac{dV_c}{dt} + \frac{8}{6} - \frac{V_c}{6} = 0$$

$$\Rightarrow (4+4)\frac{dV_c}{dt} + \frac{8}{6} - \frac{V_c}{6} = 0$$

$$\Rightarrow \frac{dV_c}{dt} - \frac{1}{48}V_c + \frac{1}{6} = 0$$

Example 4.22 : Determine the initial and final conditions for the circuit shown in fig. 4.46

$L = 0.9$ mH, $V_s = 12V$

$R_1 = R_2 = 6K\Omega$

$R_3 = 3K\Omega$

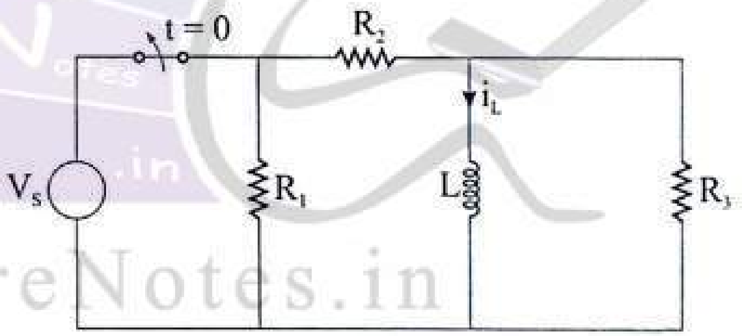

Fig 4.46

Solution : In fig. 4.46 the switch has been closed for a long a time. Thus we have steady state condition and inductor behaves as a short circuit. The voltage across R_3 is zero. The current flows through the resistor R_2 is

$$\therefore \quad i_L(0) = \frac{V_s}{R_2} = \frac{12}{6\times10^3} = 2\,mA$$

After the switch has been opened for a long time, again we have a steady - state condition and the inductor behaves as short circuit. When the switch is open V_s is not connected to the circuit.

Thus $i_L(\infty) = 0A$

Therefore the rate at which charge accumulates on the capacitor is also constant and the voltage across the capacitor rises at a constant rate.

$$V_C(0^+) = V_C(0^-) = -7V$$

$$i_C(t) = I_0 = 17\,mA$$

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)\cdot dt = \frac{1}{C}\left[\int_{-\infty}^{0} i_C(t)\cdot dt + \int_{0}^{t} I_0 dt\right]$$

$$= V_C(0^+) + \frac{I_0}{C}\int_0^t dt = V_C(0^+) + \frac{I_0}{C}t$$

$$= -7 + \frac{17\times10^{-3}}{0\cdot55\times10^{-6}}t = -7 + 30\cdot91\times10^3 t$$

Example 4.32 : The circuit of fig. 4.56 is a simple model of an automotive ignition system. The switch models the points that switch electric power to the cylinder when the fuel - air mixture is compressed and R is the resistance between the electrodes (i.e. the gap) of the spark plug.

$V_G = 12V$, $R_G = 0\cdot37\Omega$ $R = 1\cdot7k\Omega$

Determine the value of L and R_1 so that the voltage across the spark plug gap just after the switch is changed is 23KV and so that this voltage will change exponentially with a time constant $\tau = 13\,ms$.

Solution : Given the voltage across the spark plug gap just after the switch is changed is $V_R = 23\,KV$.

i. At $t = 0^-$ (Switch connected to R_G) steady state condition exist and inductor behaves as short circuit.

$\therefore\ V_L(0^-) = 0$

The current through inductor $i_L(0^-) = \dfrac{V_G}{R_G + R_1}$

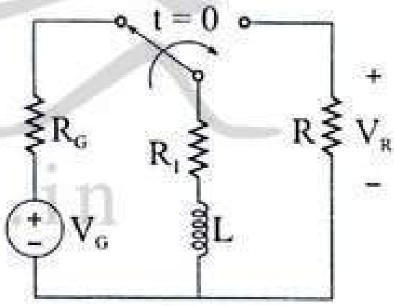

Fig 4.56

ii. At $t = 0^+$ (Switch connected to R) transient starts. Since the current through the inductor can not change instantaneously the current through inductor at $t = 0^+$ is also $V_G / R_G + R_1$.

$$V_R(0^+) = -i_L(0^+)R = -\frac{V_G R}{R_G + R_1}$$

[-ve sign indicate that the current is discharging current]

$$\Rightarrow R_1 = \frac{12\times1\cdot7\times10^3}{-23\times10^3} = -0\cdot37 = 0\cdot5170\Omega$$

From the Thevnins equivalant circuit shown in fig 4.25

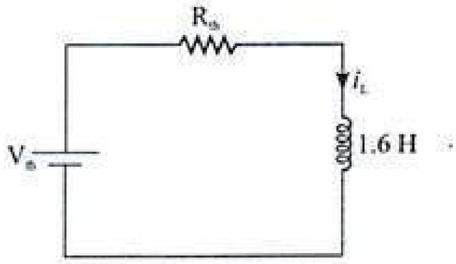

Fig 4.25

$$i_L = \frac{V_{th}}{R_{th}}\left(1-e^{\frac{-R_{th}t}{L}}\right) = \frac{120}{24}\left(1-e^{\frac{-24t}{1\cdot 6}}\right)$$

$$\Rightarrow i_L = 5\left(1-e^{-15t}\right) A$$

(i) $\therefore V_L = L\cdot\frac{di_L}{dt} = 1\cdot 6\frac{d}{dt}\left[5\left(1-e^{-15t}\right)\right] = 120\, e^{-15t}$ volts.

$$i_R = \frac{V_L}{48} = \frac{120}{48}e^{-15t} = 2\cdot 5e^{-15t}\ A$$

(ii) Source current $(i_S) = i_L + i_R$

$$\Rightarrow i_S = 5 - 5e^{-15t} + 2\cdot 5e^{-15t} = 5 - 2\cdot 5e^{-15t} A$$

Example 4.4 : The capacitor of the circuit shown in fig 4.26 has no charge when the switch is closed at t = 0. Calcutate (i) the current through and (ii) the voltage across the capacitor.

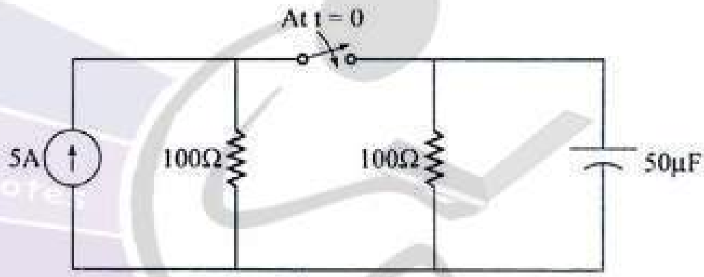

Fig 4.26

Solution : The Thevnins equivalent circuit across the capacitor is shown in fig 4.27.

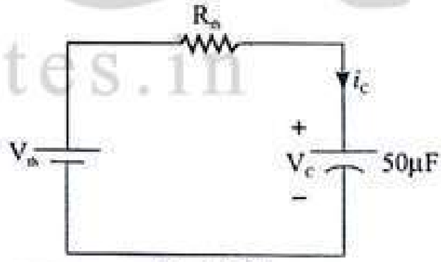

Fig 4.27

$$V_{th} = 5\left(\frac{100\times 100}{100+100}\right) = 250 \text{ volts.}$$

$$R_{th} = \frac{100\times 100}{100+100} = 50\Omega.$$

(i) Charging current through the capacitor is given by,

$$i_c = \frac{V_{th}}{R_{th}}\left(e^{\frac{-t}{R_{th}C}}\right)\ 5\ A$$

$$\Rightarrow i_C = \frac{250}{50}\left(e^{\frac{-t}{50\times 50\times 10^{-6}}}\right)$$

$$\Rightarrow i_C = 5e^{-400t} A$$

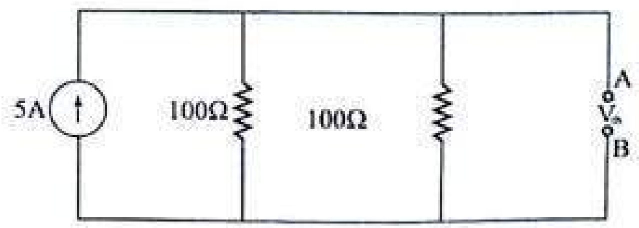

Fig 4.28

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Example 4.13: Write the differential equation for t > 0 for the circuit of fig. 4.34.

$R_1 = 0 \cdot 68K\Omega \quad R_2 = 1 \cdot 8K\Omega$

$V_1 = 12V, \quad C = 0 \cdot 5\mu F$

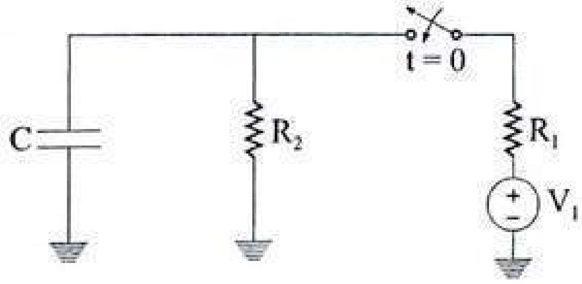

Fig 4.34

Solution : Consider a reference node whose potential is zero. the differential equation for t >0 (switch closed) for the circuit as shown in fig. 4.35.

The top node voltage is equal to capacitor voltage V_c. Apply KCL to top node.

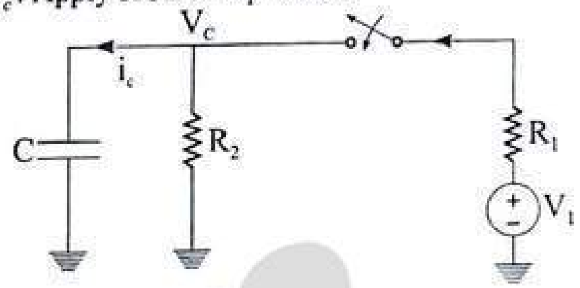

Fig 4.35

$$\frac{V_1 - V_C}{R_1} = \frac{V_C}{R_2} + i_C$$

$$\Rightarrow i_C + \frac{V_C}{R_2} + \frac{V_C - V_1}{R_1} = 0$$

$$\Rightarrow C \cdot \frac{dV_C}{dt} + \frac{V_C}{R_2} + \frac{V_C}{R_1} - \frac{V_1}{R_1} = 0$$

$$\Rightarrow C \cdot \frac{dV_C}{dt} + \left(\frac{R_1 + R_2}{R_1 R_2}\right) V_C = \frac{V_1}{R_1} \qquad \left[\because \ i_C = c \cdot \frac{dv_c}{dt}\right]$$

$$\Rightarrow \frac{dV_c}{dt} + \left(\frac{R_1 + R_2}{R_1 R_2}\right)\frac{V_C}{C} = \frac{V_1}{R_1 C}$$

Substituting numerical values we get,

$$\frac{dV_C}{dt} + 4052V_C - 35292 = 0$$

Example 4.14 : Write the differential equation for t > 0 for the circuit of fig 4.36.

$V_1 = 12V,$

$R_1 = 0 \cdot 68K\Omega$

$R_2 = 2 \cdot 2K\Omega$

$R_3 = 1 \cdot 8K\Omega$

$C = 0.47\,\mu F$

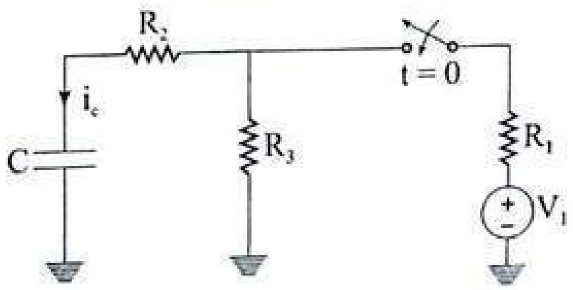

Fig 4.36

Example : 4.23 Determine the initial and final conditions for the circuit of fig. 4.47

$V_1 = 12V, \quad C = 0.5\,\mu F$

$R_1 = 0.68\,K\Omega \quad R_2 = 1.8\,K\Omega$

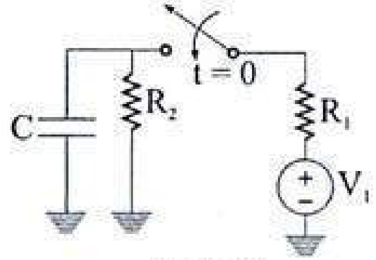

Fig 4.47

Solution : From fig. 4.47 the switch has been opened for a long time. The voltage source is not connected to the circuit. Thus $V_C(0) = 0$ Volt.

After the switch has been closed for a long time, we have a steady-state condition and the capacitor behaves as open circuit. The voltage across the capacitor is equal to voltage across R_2 . According to voltage division rule,

$$V_c(\infty) = \frac{R_2 V_1}{R_1 + R_2} = \frac{1800 \times 12}{1800 + 680} = 8.71 \text{ volt.}$$

Example 4.24 : Determine the initial and final conditions for the circuit of Fig. 4.48

$V_{s1} = V_{s2} = 13V, \quad R_1 = 2\cdot7\Omega$

$L = 170\text{ mH}, \quad R_2 = 4\cdot3\,K\Omega$

$R_3 = 29\,K\Omega$

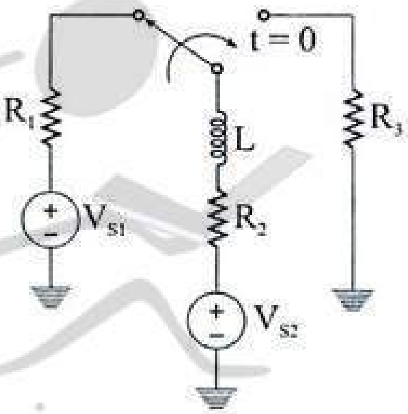

Fig 4.48

Solution : From fig. 4.48 the switch is connected to R_1 and voltage source V_{S1}. In steady state condition the inductor behaves as short circuit. Applying KVL we get, $V_{s1} - i_L(0)R_1 - i_L(0)R_2 - V_{s2} = 0$

$$\Rightarrow \quad i_L(0) = \frac{V_{s1} - V_{s2}}{R_1 + R_2} = \frac{13 - 13}{2.7 + 43 \times 10^3} = 0$$

When switch is connected to R_3 then again steady state condition will attained and inductor behaves short circuit. Applying KVL we get,

$$V_{s2} - i_L(\infty)R_2 - i_L(\infty)R_3 = 0$$

$$\Rightarrow i_L(\infty) = \frac{V_{s2}}{R_2 + R_3} = \frac{13}{(4\cdot3 + 29)10^3} = 0\cdot39\,mA$$

Example 4.25 : Determine the initial and final conditions for the circuit of Fig. 4.49

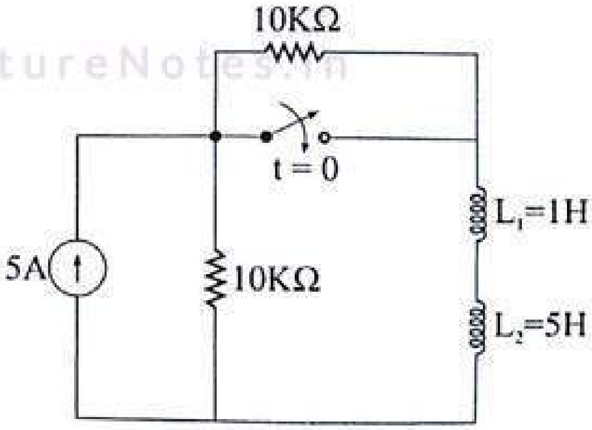

Fig 4.49

Given time constant $\tau = 13 \times 10^{-3}$ sec

$$\Rightarrow \frac{L}{R_1 + R} = 13 \times 10^{-3}$$

$$\Rightarrow L = 22 \cdot 11 \text{Henry.}$$

Example 4.33 : At t = 0, the switch in the circuit of fig. 4.57 closes. Assume $i_L(0) = 0A$. For $t \geq 0$, find (a) $i_L(t)$ (b) $V_{L1}(t)$.

$L_1 = 1H, L_2 = 5H$

Solution : When switch is closed the inductors behave short circuits in steady - state condition. All of the current (5A) from the source will travel through the inductors.

$$\therefore \quad i_L(\infty) = 5A$$

In this case time constant,

$$\tau = \frac{L_1 + L_2}{10 \times 10^3} = \frac{1+5}{10 \times 10^3} = 0 \cdot 6\, ms$$

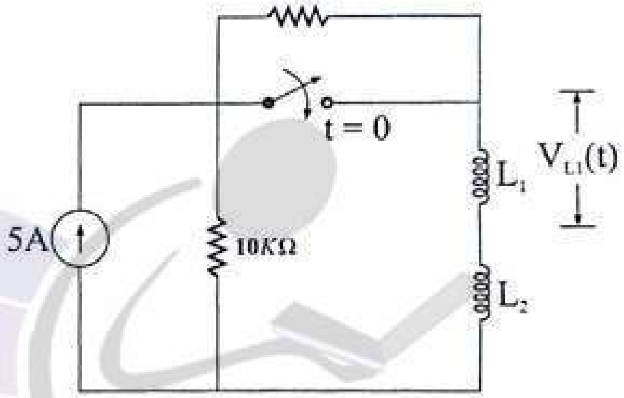

Fig 4.57

$\therefore$ (a) $i_L(t) = 5\left(1 - e^{-t/\tau}\right) = 5\left(1 - e^{-t/0 \cdot 6 \times 10^{-3}}\right)$ Ampere.

(b) $V_{L1}(t) = L_1 \cdot \frac{di_L(t)}{dt} = L_1 \cdot \frac{d}{dt}\left[5\left(1 - e^{-t/0 \cdot 6 \times 10^{-3}}\right)\right]$

$$= 8 \cdot 333\, e^{-t/0 \cdot 6 \times 10^{-3}}\, KV$$

(ii) Voltage across the capacitor is,

$$V_C = V_{th}\left(1-e^{-t/R_{th}C}\right) = 250\ \left(1-e^{-400t}\right)\text{ volt.}$$

Example 4.5 : Switch S of the network shown in fig 4.29 has been closed for a long time. At t = 0, the switch is suddenly opened. Evaluate the voltage across the 200 Ω resistor.

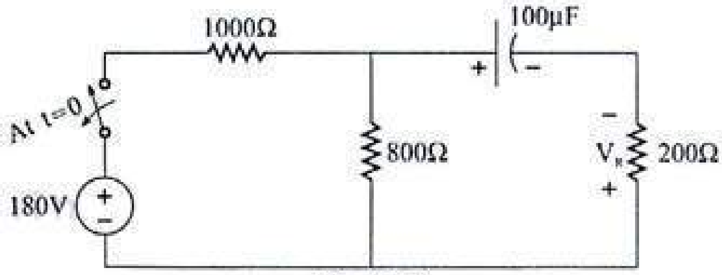

Fig 4.29

Solution : At $t=0^+_-$ circuit is a shown in fig 4.30.

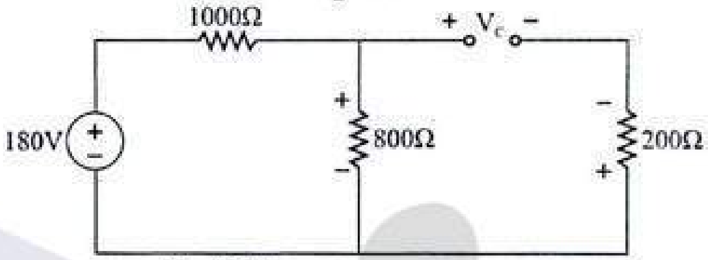

Fig 4.30

At $t=\bar{0}$, $V_C = 180\left(\frac{800}{1800}\right) = 80$ volts

At $t = 0^+$, circuit is as shown in fig 4.31.

$V_c(0) = V_C(\bar{0}) = 80$ volts.

∴ The discharging current in the capacitor $i_C = \frac{V_C(0)}{800+200}\ e^{-\frac{t}{RC}}$

$$\Rightarrow i_C = \frac{80}{1000}\ e^{\frac{-t}{1000\times100\times10^{-6}}} \Rightarrow i_C = 0{\cdot}08\ e^{-10t}$$

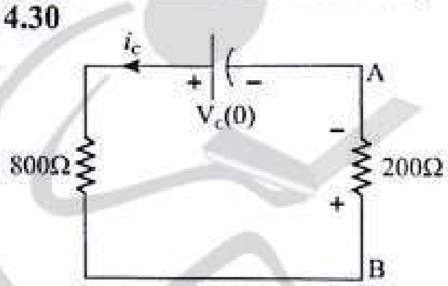

Fig 4.31

∴ Voltage across the resistor $V_R = i_C\ (200)$

$\Rightarrow\ V_R = 0{\cdot}08\ e^{-10t}\ (200) = 16\ e^{-10t}$ volts.

Example 4.6 : A coil having L=10 H and R = 15Ω is connected to a 75V supply. Find (i) the rate of change of current at the instant of closing the switch. (ii) final steady value (iii) time constant (iv) the time taken for the current to reach a value of 4A.

Solution : Given R = 15 Ω, L =10H and V = 75V

i. Initial rate of change of current at the instant of closing the switch i.e. at $t=0$, is $\frac{di}{dt} = \frac{V}{L} = \frac{75}{10} = 75\frac{A}{\text{sec}}$

ii. Final steady current = Maximum current in circuit $= I_0 = \frac{V}{R} = \frac{75}{15} = 5A$

iii. Time constant $\tau = \frac{L}{R} = \frac{10}{15} = 0.67$ sec.

iv. Current at any instant $i = I_0\left(1-e^{-\frac{t}{\tau}}\right)$

Given $i = 4A$ ∴ $4 = 5\left(1-e^{\frac{-t}{0.67}}\right)$ $\Rightarrow t = 1{\cdot}078$ sec.

Solution : Consider a reference node whose potential is zero. The differential equation for t >0 (switch closed) for the circuit as shown in fig. 4.37

Node 1 voltage is the capacitor voltage V_C and node 2 voltage is V_2.

Apply KCL to node 1

$$\frac{V_2 - V_C}{R_2} - i_C = 0$$

$$\Rightarrow \frac{V_C - V_2}{R_2} + i_C = 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

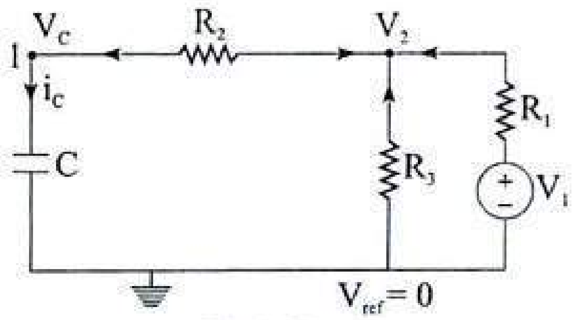

Fig 4.37

Apply KCL to node 2,

$$\frac{V_C - V_2}{R_2} + \frac{0 - V_2}{R_3} + \frac{V_1 - V_2}{R_1} = 0$$

$$\Rightarrow \frac{V_2 - V_C}{R_2} + \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_1} = 0 \ldots\ldots\ldots\ldots (2)$$

Solving eqn (1) and (2) we get,

$$V_C = \frac{R_3}{R_1 + R_3} V_1 - \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_3} i_C$$

$$\Rightarrow V_C + \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_3} \cdot C \frac{dV_c}{dt} = \frac{R_3}{R_1 + R_3} V_1$$

$$\Rightarrow \frac{dV_c}{dt} + \frac{R_1 + R_3}{C(R_1 R_2 + R_1 R_3 + R_2 R_3)} \cdot V_C = \frac{R_3}{(R_1 R_2 + R_1 R_3 + R_2 R_3)C} V_1$$

Substituting the numerical values, we get

$$\frac{dV_c}{dt} + 790 V_C - 6876 = 0$$

Solution : From fig. 4.49 the switch has been opened for a long time. Thus we have a steady state condition, and inductors behave as short circuits. The values of the two resistors are equal. So the current flowing through the inductor is $i_L(0)=\frac{5}{2}=2\cdot 5A$

After the switch has been closed for a long time, we have again a steady state condition and again inductors behave short circuits. In this case resistors are short circuited.

So all current is flowing through inductors $\therefore \quad i_L(\infty)=5A$.

Example 4.26: At $t<0$, the circuit shown in figure 4.50 is at steady state. The switch is changed as shown at t = 0

$V_{s1}=35V, \quad V_{s2}=130V, \quad C=11\mu F \quad R_1=17K\Omega$

$R_2=7K\Omega \quad R_3=23K\Omega$

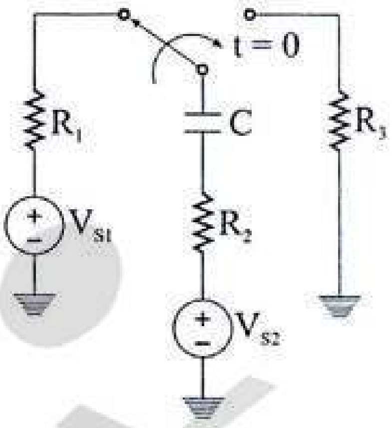

Fig 4.50

Determine at $t=0^+$ the initial current through R_3 just after the switch is changed.

Solution : (i) At $t=0^-$ (i.e. the switch is connected to R_1)

Let $V_C(0^-)$ = steady state voltage across capacitor before the switch is thrown.

At steady state, the capacitor is modeled as an open circuit. So voltage drops across R_1 and R_2 are zero. Apply KVL,

$$V_{s1}+0-V_c(0^-)+0-V_{s2}=0$$

$$\Rightarrow V_C(0^-)=V_{s1}-V_{s2}=35-130=-95 \text{ volt.}$$

(ii) At $t=0^+$ (i.e. the switch is connected to R_3)

Since voltage across capacitor can not change instantaneously, this voltage will be the capacitor voltage immediately after the switch is thrown.

$$\therefore \quad V_c(0^+)=V_C(0^-)=-95V$$

Let $i(0^+)$ = Current flows through R_2 and R_3

Apply KVL, $V_{s2}-i(0^+)R_2+V_C(0^+)-i(0^+)R_3=0$

$$\Rightarrow i(0^+)=\frac{V_{s2}+V_c(0^+)}{R_2+R_3}=\frac{130-95}{7\times10^3+23\times10^3}=1\cdot167\,mA$$

Transient in R-L circuit

GROWTH

1. At t = 0, entire voltage is dropped across inductor and no voltage is dropped across resistor. i.e. $V_R = 0, V_L = V$

2. At t = ∞, entire voltage is dropped across resistor and no voltage is dropped across inductor i.e. $V_R = V, V_L = 0$.

3. Instantaneous growth of current, $I = \frac{V}{R}\left(1 - e^{-Rt/L}\right) = I_0\left(1 - e^{-t/\tau}\right)$

4. Rate of growth of current is, $\frac{dI}{dt} = \frac{V}{L}e^{-t/\tau} = \frac{V}{L}e^{-Rt/L}$

5. Initial rate of growth of current (i.e. at t = 0), $\frac{dI}{dt} = \frac{V_L}{L}e^0 = \frac{V_L}{L}$ (where at t = 0, $V = V_L$)

6. Maximum Current, $I_O = \frac{V}{R}$

7. Time constant, $\tau = L/R$

8. Voltage drop across inductor is $V_L = Ve^{-t/\tau}$

9. Voltage drop across resistor is $V_R = V\left(1 - e^{-t/\tau}\right)$

DECAY

10. Instantaneous decay of current, $I = I_0 e^{-t/\tau}$

11. Voltage drop across inductor is, $V_L = -Ve^{-t/\tau}$

12. Voltage drop across resistor $V_R = -Ve^{-t/\tau}$

Transient in R-C circuit

CHARGING

1. At t = 0, i.e instant of switching the charging current is maximum and decreases gradually as the voltage across the capacitor increases.

2. When capacitor is fully charged to applied voltage (v) then charging current becomes zero.

3. Charging Current at any instant, $I = \frac{V}{R}e^{-t/\tau} = I_0 e^{-t/\tau}$

4. Initial charging current i.e. at t = 0, $I = \frac{V}{R}e^0 = \frac{V}{R}$ = Maximum Current

5. Charge on capacitor at any instant, $q = q_0\left(1 - e^{-t/RC}\right) = q_0\left(1 - e^{-t/\tau}\right)$ where $q_0 = CV$ = full charge of capacitor

6. Final charging current = 0

7. Time constant, $\tau = RC$

8. Instantaneous voltage across capacitor, $V_C = V\left(1 - e^{-t/\tau}\right) = V\left(1 - e^{-t/RC}\right)$

9. Rate of rise of voltage across capacitor, $\frac{dv_C}{dt} = \frac{V}{RC}e^{-t/RC}$

10. Initial rate of rise of voltage across capacitor (i.e. at t = 0), $\frac{dv_C}{dt} = \frac{V}{RC}$

11. Capacitor is almost fully charged in a time equal to 5RC i.e. 5τ

DISCHARGING

12. Charge at any instant, $q = q_0 e^{-t/\tau}$

13. Voltage at any instant, $V_C = Ve^{-t/\tau}$

14. Discharging current at any instant, $I = -I_0 e^{-t/\tau}$

BPUT PREVIOUS YEAR QUESTIONS SOLVED

1. The resistance and inductance of a series circuit are 5Ω and 20 H respectively. At the instant of closing the d.c. supply switch, the current increases at the rate of 4 $A/_{sec}$. Calculate

(i) the applied voltage.

(ii) the rate of growth of current when 5A flows in the circuit.

(iii) the stored energy under steady state conditon.

(1st semester 2003)

solution : $R = 5\Omega$, L = 20 H, $\frac{di}{dt} = 4\frac{A}{\sec}$

(i) At the instant of closing the d.c. supply switch, the inductance behaves as open circuit, so no current flows through the circuit. The voltage drop across resistance is zero and the total voltage applied is droped across the inductance.

∴ Voltage drop across inductance $V_L = L \cdot \frac{di}{dt} = 20 \times 4 = 80$ volts.

(ii) $V = V_R + L \cdot \frac{di}{dt}$

$$\Rightarrow \frac{di}{dt} = \frac{V - V_R}{L} = \frac{80 - (5 \times 5)}{20} = 2 \cdot 75 \frac{A}{\sec}$$

(iii) Steady Current $I = \frac{V}{R} = \frac{80}{5} = 16A$

Stored energy under steady state condition is $= \frac{1}{2}LI^2 = \frac{1}{2}(20)(16)^2 = 2560\,Joule$

2. What is the time required for the capacitor voltage in a RC circuit with $R = 2\Omega$ and $C = 4F$ to reach 63.2% of its steady state value ? ***(1st semester 2004)***

Solution : $R = 2\Omega, C = 4F$

Time constant $\tau = RC = 2 \times 4 = 8$ sec.

$$q = Q\left(1 - e^{-t/RC}\right)$$

$$\Rightarrow \frac{63 \cdot 2}{100}Q = Q\left(1 - e^{-t/8}\right)$$

BURN IGNORANCE

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17. A capacitor is charged from a dc supply of 100 V through a resistor in series having resistance 100 Ω. If the time constant for the given setup is 15 milli seconds, calculate the value of the capacitance. Also calculate the time requirement for the capacitor to acquire 90% of steady state charge assuming zero initial charge in the capacitor.

(1st semester 2009)

Solution : V = 100 volts, $R = 100\Omega$

Time constant $= \tau = RC = 15\times10^{-3}$ seconds.

Capacitance $= C = \dfrac{15\times10^{-3}}{100} = 1\cdot5\times10^{-4}\ F$

$q = Q\left(1 - e^{-t/\tau}\right)$

$\Rightarrow 0\cdot9Q = Q\left(1 - e^{-t/15\times10^{-3}}\right)$

$\Rightarrow$ $t = 0.034$ second.

18. A direct voltage of 200 V is applied to a coil of resistance 20 Ω and inductance of 2000 mH. Find the time taken for the current through the coil to reach one half of its final value.

(2nd semester 2009)

Solution : V = 200 volts, $R = 20\Omega$ $L = 2000\times10^{-3}\ H,$

Final current $= \dfrac{V}{R} = 10A$

Time constant $= \tau = L/R = 0\cdot1$ second

$i = \dfrac{V}{R}\left(1 - e^{-t/\tau}\right)$

$\Rightarrow 5 = 10\left(1 - e^{-t/0\cdot1}\right)$

$\Rightarrow t = 0\cdot0693$ second.

$\Rightarrow 0\cdot 632 = 1 - e^{-t/8}$

$\Rightarrow \frac{-t}{8} = \log_e 0\cdot 368$

$\Rightarrow t = 7\cdot 997$ sec.

3. A resistance R and 3.5 μF capacitor are connected in series across a 230 V dc source through a switch. A voltmeter is connected across the capacitor.

(i) Find R, so that the voltmeter reads 165 V at 5.65 sec after the switch is closed.

(ii) the initial charging current. ***(1st semester 2004)***

(iii) the ultimate energy stored in the capacitor.

(iv) the current at t = 5.65 sec.

Solution : $R = ?$ $C = 3\cdot 5\times 10^{-6} F$, V = 230 volts.

(i) $V_c = 165$ volts $t = 5\cdot 65$ sec.

$V_C = V\left(1 - e^{-t/RC}\right)$

$\Rightarrow 165 = 230\left(1 - e^{-5\cdot 65/RC}\right)$

$\Rightarrow e^{-5\cdot 65/RC} = 0\cdot 2826$

$\Rightarrow \frac{-5\cdot 65}{RC} = \log_e 0\cdot 2826$

$\Rightarrow RC = 4\cdot 47$

$\Rightarrow R = 1\cdot 277\times 10^6\,\Omega$

(ii) Initial charging current $I_m = \frac{230}{1\cdot 277\times 10^6} = 180\cdot 1\mu A$

(iii) Energy stored in capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}\left(3\cdot 5\times 10^{-6}\right)(230)^2 = 0\cdot 0925$ Joule.

(iv) The current at t = 5.65 sec is,

$i = \frac{V}{R}e^{-t/RC} = \frac{230}{1\cdot 277\times 10^6}e^{\frac{-5\cdot 65}{1\cdot 277\times 10^6\times 3\cdot 5\times 10^{-6}}}$

$\Rightarrow i = 180\cdot 1\times 10^{-6}\, e^{-1\cdot 264} = 50\cdot 88\mu A$

4. A series R-C circuit is excited by dc voltage E through a switch. Find the value of initial current. ***(2nd semester 2004)***

Solution : Initial current $= E/R$ Ampere.

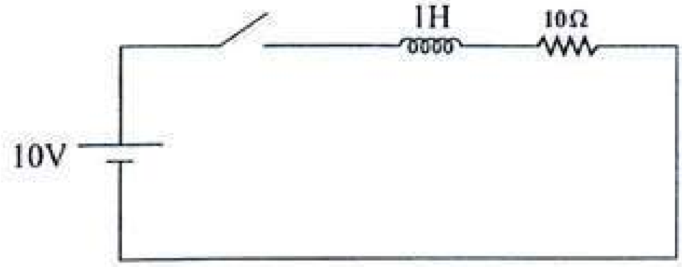
5. Find the value of final current in a series R - L circuit impressed by dc voltage V.

(2nd semester 2004)

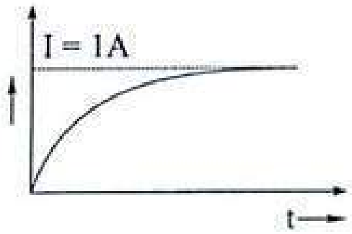
Solution : Final Current $= \frac{V}{R}$ ampere.

6. Switch is closed at t = 0

Draw i - t graph. What is steady state Current ?



(Supplementary Exam 2004)



Solution : Steady Current $= \frac{10}{10} = 1A$

The i - t graph is shown.

7. A 100 μF capacitor in series with an 800 Ω resistor is switched on to a 100 V d.c. supply. Calculate (i) time constant (ii) initial charging current.

(1st semester 2005)

Solution : $C = 100 \times 10^{-6} F$ $R = 800\Omega,$ $V = 100$ volts.

(i) Time constant $\tau = RC = 800 \times 100 \times 10^{-6} = 0 \cdot 08 \text{sec}$

(ii) Initial charging current $= \frac{100}{800} = 0 \cdot 125 A$

8. A coil having R = 1 ohm and L = 500 mH is switched on to a voltage source of 3v d.c.

Find (i) initial and final value of current

(ii) time constant of the circuit.

(iii) current at t = 500 ms. ***(1st semester 2005)***

Solution : R = 1 ohm, $L = 500 \times 10^{-3} H,$ V = 3 volts.

(i) Initial value of current = 0

Final value of current $= \frac{V}{R} = \frac{3}{1} = 3A$

(ii) Time constant $= \frac{L}{R} = \frac{500 \times 10^{-3}}{1} = 0 \cdot 5 \text{sec.}$

(iii) $i=\frac{V}{R}\left(1-e^{\frac{-Rt}{L}}\right)=3\left(1-e^{-500\times10^{-3}/500\times10^{-3}}\right)=1\cdot896A$

9. A coil having a resistance of 2 ohms and an inductance of 1 henry is switched on to a 10 volt d.c. supply. Write down the expression of current $i(t)$ in the coil as a function of time.

(1st semester 2006)

Solution : $R=2$ ohms, $L=1H$, V = 10 volts.

Time constant $=\frac{L}{R}=\frac{1}{2}=0\cdot5$ sec.

Steady current $=\frac{V}{R}=\frac{10}{2}=5A$

Expression of current $i(t)=\frac{V}{R}\left(1-e^{-t/\tau}\right)$

$=5\left(1-e^{-t/0.5}\right)A$

10. A coil with a self inductance of 4 H and a resistance of 10 ohm is suddenly switched across a 20 V dc supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.5 sec. and the time taken for the current to reach 80% of its final value. What is the final steady value of the current ? *(1st semester 2006)*

Solution : L = 4 H, $R=10\Omega$, V = 20 volts.

(i) Time constant $\tau=\frac{L}{R}=\frac{4}{10}=0\cdot4$ sec.

(ii) Final steady value of current $=\frac{V}{R}=\frac{20}{10}=2A$

(iii) Instantaneous value of the current after 0.5 second is

$i=\frac{V}{R}\left(1-e^{-t/\tau}\right)=2\left(1-e^{-0.5/0.4}\right)=1\cdot427$ Ampere

(iv) $i=\frac{V}{R}\left(1-e^{-t/\tau}\right)$

$\Rightarrow 0\cdot8(2)=2\left(1-e^{-t/0.4}\right)$

$\Rightarrow t=0\cdot6436$ sec.

11. An 8 micro farad capacitor is connected in series with a 500 kilo ohm resistor and the RC combination is connected across 200 V dc supply, Calculate

(i) the time constant

(ii) the initial charging current

(iii) the time taken for the capacitor voltage to reach 160 volts.

(iv) the current and p.d. across the capacitor 4 seconds after it is connected to supply.

(2nd semester 2006)

Solution : $C = 8\times10^{-6} F$, $R = 500\times10^{3}\,\Omega, V = 200$ volts

(i) Time constant $\tau = RC = 500\times10^{3}\times8\times10^{-6} = 4$ sec.

(ii) The initial charging current $= \frac{V}{R} = \frac{200}{500\times10^{3}} = 4\times10^{-4} A$

(iii) $V_C = V\left(1-e^{-t/\tau}\right)$

$\Rightarrow 160 = 200\left(1-e^{-t/4}\right)$

$\Rightarrow t = 6\cdot437$ sec.

(iv) The current through capacitor after 4 second is,

$i = \frac{V}{R}e^{-t/RC} = \frac{200}{500\times10^{3}}e^{-4/4} = 0\cdot14\times10^{-3} A$

P.d across capacitor is $V_C = V\left(1-e^{-t/\tau}\right)$

$= 200\left(1-e^{-4/4}\right)$

$= 126\cdot42$ volts.

12. A direct voltage of 150 V is applied to a coil of resistance 15 ohm and inductance 15 H. What is the value of the current 0.1 second after switching on ?

(1st semester 2007)

Solution : V = 150 volts, $R = 15\Omega$, $L = 15H$

Time constant $= \tau = L/R = 1$ sec.

The current, 0.1 second after switching on is,

$i = \frac{V}{R}\left(1-e^{-t/\tau}\right)$

$= \frac{150}{15}\left(1-e^{-0\cdot1/1}\right) = 0\cdot951 A$

13. A 10 microfarad capacitor is connected to a constant voltage source through a resistance of 2.5 mega ohm. Calculate the time taken for the capacitor to lose 50% of its charge when the voltage source is short circuited ?

(1st semester 2007)

Solution : $C = 10\times10^{-6} F$ $R = 2\cdot5\times10^{6}\,\Omega$

Time constant $= \tau = RC = 25$ second.

$q = Qe^{-t/\tau}$

$$\Rightarrow \frac{Q}{2} = Qe^{-t/25}$$

$\Rightarrow t = 17 \cdot 32$ second.

14. A coil has a resistance of 10 Ω and inductance of 1 henry. What will be the value of current after 0.1 second of switching this coil to a 100 V dc supply ?

(2nd semester 2007)

Solution : $R = 10\Omega$, L=1H, V = 100 volts.

Time constant $= \tau = \frac{L}{R} = \frac{1}{10} = 0 \cdot 1$ second.

$$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right) = \frac{100}{10}\left(1 - e^{-0\cdot1/0\cdot1}\right) = 6 \cdot 321A$$

15. A condenser of $8\mu F$ capacitance is connected to a dc source through a resistance of one mega ohm. Calculate the time taken for the condenser to receive 95% of its final charge.

(2nd semester 2007)

Solution $C = 8 \times 10^{-6} F$ $R = 10^6\ \Omega$

Time constant $= \tau = RC = 8$ second.

$$q = Q\left(I - e^{-t/\tau}\right)$$

$$\Rightarrow 0 \cdot 95Q = Q\left(I - e^{-t/8}\right)$$

$\Rightarrow t = 23 \cdot 965$ second.

16. A 15 μF capacitor in series with a $15 \times 10^3\ \Omega$ resistance is connected across a constant dc voltage source of 250 volts. The fully charged capacitor is disconnected from the supply and is discharged by connecting a 1000Ω resistance across its terminals. Compute the initial value of the charging current and initial value of discharging current. *(1st semester 2008).*

Solution : $C = 15 \times 10^{-6}\ F$, $R = 15 \times 10^3\ \Omega$, V = 250 volts.

Initial value of charging current $= \frac{V}{R} = \frac{250}{15 \times 10^3} = 0 \cdot 016A$

Initial value of discharging current $= -\frac{250}{1000} = -0 \cdot 25A$

BPUT PREVIOUS YEAR QUESTIONS SOLVED

1. The resistance and inductance of a series circuit are 5Ω and 20 H respectively. At the instant of closing the d.c. supply switch, the current increases at the rate of $4\ ^{A}/_{sec}$. Calculate

(i) the applied voltage.

(ii) the rate of growth of current when 5A flows in the circuit.

(iii) the stored energy under steady state conditon.

(1st semester 2003)

solution : $R = 5\Omega$, L = 20 H, $\frac{di}{dt} = 4\frac{A}{\sec}$

(i) At the instant of closing the d.c. supply switch, the inductance behaves as open circuit, so no current flows through the circuit. The voltage drop across resistance is zero and the total voltage applied is droped across the inductance.

∴ Voltage drop across inductance $V_L = L\cdot\frac{di}{dt} = 20\times 4 = 80$ volts.

(ii) $V = V_R + L\cdot\frac{di}{dt}$

$$\Rightarrow \frac{di}{dt} = \frac{V - V_R}{L} = \frac{80-(5\times 5)}{20} = 2\cdot 75\frac{A}{\sec}$$

(iii) Steady Current $I = \frac{V}{R} = \frac{80}{5} = 16A$

Stored energy under steady state condition is $= \frac{1}{2}LI^2 = \frac{1}{2}(20)(16)^2 = 2560\,Joule$

2. What is the time required for the capacitor voltage in a RC circuit with $R = 2\Omega$ and $C = 4F$ to reach 63.2% of its steady state value ? ***(1st semester 2004)***

Solution : $R = 2\Omega, C = 4F$

Time constant $\tau = RC = 2\times 4 = 8$ sec.

$$q = Q\left(1 - e^{-t/RC}\right)$$

$$\Rightarrow \frac{63\cdot 2}{100}Q = Q\left(1 - e^{-t/8}\right)$$

17. A capacitor is charged from a dc supply of 100 V through a resistor in series having resistance 100 Ω. If the time constant for the given setup is 15 milli seconds, calculate the value of the capacitance. Also calculate the time requirement for the capacitor to acquire 90% of steady state charge assuming zero initial charge in the capacitor.

(1st semester 2009)

Solution : V = 100 volts, $R = 100\Omega$

Time constant $= \tau = RC = 15\times10^{-3}$ seconds.

Capacitance $= C = \dfrac{15\times10^{-3}}{100} = 1\cdot5\times10^{-4}\ F$

$q = Q\left(1 - e^{-t/\tau}\right)$

$\Rightarrow 0\cdot9Q = Q\left(1 - e^{-t/15\times10^{-3}}\right)$

$\Rightarrow t = 0.034$ second.

18. A direct voltage of 200 V is applied to a coil of resistance 20 Ω and inductance of 2000 mH. Find the time taken for the current through the coil to reach one half of its final value.

(2nd semester 2009)

Solution : V = 200 volts, $R = 20\Omega$ $L = 2000\times10^{-3}\,H$,

Final current $= \dfrac{V}{R} = 10A$

Time constant $= \tau = L/R = 0\cdot1$ second

$i = \dfrac{V}{R}\left(1 - e^{-t/\tau}\right)$

$\Rightarrow 5 = 10\left(1 - e^{-t/0\cdot1}\right)$

$\Rightarrow t = 0\cdot0693$ second.

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$\Rightarrow 0\cdot632 = 1 - e^{-t/8}$

$\Rightarrow \frac{-t}{8} = \log_e 0\cdot368$

$\Rightarrow t = 7\cdot997$ sec.

3. A resistance R and 3.5 μF capacitor are connected in series across a 230 V dc source through a switch. A voltmeter is connected across the capacitor.

(i) Find R, so that the voltmeter reads 165 V at 5.65 sec after the switch is closed.

(ii) the initial charging current. ***(1st semester 2004)***

(iii) the ultimate energy stored in the capacitor.

(iv) the current at t = 5.65 sec.

Solution : $R = ?$ $C = 3\cdot5\times10^{-6} F,$ V = 230 volts.

(i) $V_c = 165$ volts $t = 5\cdot65$ sec.

$V_C = V\left(1 - e^{-t/RC}\right)$

$\Rightarrow 165 = 230\left(1 - e^{-5\cdot65/RC}\right)$

$\Rightarrow e^{-5\cdot65/RC} = 0\cdot2826$

$\Rightarrow \frac{-5\cdot65}{RC} = \log_e 0\cdot2826$

$\Rightarrow RC = 4\cdot47$

$\Rightarrow R = 1\cdot277\times10^6\,\Omega$

(ii) Initial charging current $I_m = \frac{230}{1\cdot277\times10^6} = 180\cdot1\mu A$

(iii) Energy stored in capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}\left(3\cdot5\times10^{-6}\right)(230)^2 = 0\cdot0925$ Joule.

(iv) The current at t = 5.65 sec is,

$i = \frac{V}{R}e^{-t/RC} = \frac{230}{1\cdot277\times10^6}e^{\frac{-5\cdot65}{1\cdot277\times10^6\times3\cdot5\times10^{-6}}}$

$\Rightarrow i = 180\cdot1\times10^{-6}\,e^{-1\cdot264} = 50\cdot88\mu A$

4. A series R-C circuit is excited by dc voltage E through a switch. Find the value of initial current. ***(2nd semester 2004)***

Solution : Initial current $= E/R$ Ampere.

5. Find the value of final current in a series R - L circuit impressed by dc voltage V.

(2nd semester 2004)

Do Your Self

T4.1 A coil has an inductance of 1.2H and a resistance of 40Ω and is connected to a 200 V, d.c. supply. Either by drawing the current/time characteristic or by calculation determine the value of the current flowing 60 ms after connecting the coil to the supply. **[4.32 A]**

T4.2 A 25 V d.c. supply is connected to a coil of inductance 1H and resistance 5Ω. Determine the value of the current flowing 100 ms after being connected to the supply. **[1.97 A]**

T4.3 An inductor has a resistance of 20Ω and an inductance of 4H. It is connected to a 50V d.c. supply. Calculate (a) the value of current flowing after 0.1s and (b) the time for the current to grow to 1.5A. **[(a) 0.984A (b) 0.183s]**

T4.4 The field winding of a 200 V d.c. machine has a resistance of 20Ω and an inductance of 500 mH. Calculate (a) the time constant of the field winding, (b) the value of current flow one time constant after being connected to the supply, and (c) the current flowing 50 ms after the supply has been switched on. **[(a) 25 ms (b) 6.32A (c) 8.65A]**

T4.5 A circuit comprises an inductor of 9H of negligible resistance connected in series with a 60Ω resistor and a 240V d.c. source. Calculate (a) the time constant, (b) the current after 1 time constant, (c) the time to develop maximum current, (d) the time for the current to reach 2.5A, and (e) the initial rate of change of current.

[(a) 0.15s (b) 2.528A (c) 0.75s (d) 0.147s (e) 26.67A/s]

T4.6 In the inductive circuit shown in Figure T4.1, the switch is moved from position A to position B until maximum current is flowing. Calculate (a) the time taken for the voltage across the resistance to reach 8 volts, (b) the time taken for maximum current to flow in the circuit, (c) the energy stored in the inductor when maximum current is flowing, and (d) the time for current to drop to 750mA after switching to position C.

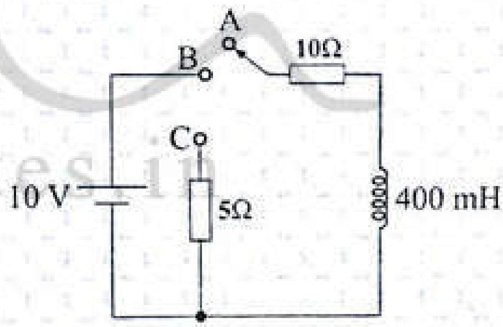


Fig. T4.1

[(a) 64.38 ms (b) 0.20 s (c) 0.20 J (d) 7.67 ms]

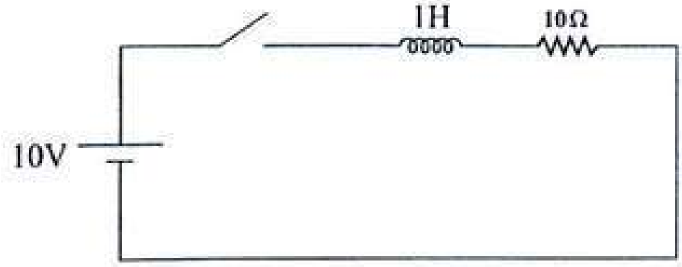
T4.7 An uncharged capacitor of 0.2μF is connected to a 100V, d.c. supply through a resistor of 100 kΩ. Determine, either graphically or by calculation the capacitor voltage 10 ms after the voltage has been applied. **[39.35V]**

T4.8 A circuit consists of an uncharged capacitor connected in series with a 50 kΩ resistor and has a time constant of 15 ms. Determine either graphically or by calculation (a) the capacitance of the capacitor and (b) the voltage drop across the resistor 5 ms after connecting the circuit to a 20V, d.c. supply. **[(a) 0.3μF (b) 14.33V]**

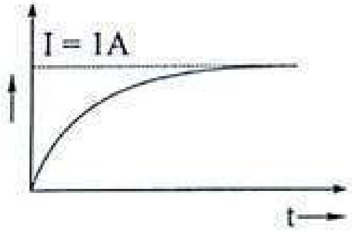
Solution : Final Current $= \frac{V}{R}$ ampere.

6. Switch is closed at t = 0

Draw i - t graph. What is steady state Current ?



(Supplementary Exam 2004)



Solution : Steady Current $= \frac{10}{10} = 1A$

The i - t graph is shown.

7. A 100 μF capacitor in series with an 800 Ω resistor is switched on to a 100 V d.c. supply. Calculate (i) time constant (ii) initial charging current.

(1st semester 2005)

Solution : $C = 100 \times 10^{-6} F$ $R = 800\Omega$, $V = 100$ volts.

(i) Time constant $\tau = RC = 800 \times 100 \times 10^{-6} = 0 \cdot 08\,\text{sec}$

(ii) Initial charging current $= \frac{100}{800} = 0 \cdot 125A$

8. A coil having R = 1 ohm and L = 500 mH is switched on to a voltage source of 3v d.c.

Find (i) initial and final value of current

(ii) time constant of the circuit.

(iii) current at t = 500 ms.

(1st semester 2005)

Solution : R = 1 ohm, $L = 500 \times 10^{-3} H$, V = 3 volts.

(i) Initial value of current = 0

Final value of current $= \frac{V}{R} = \frac{3}{1} = 3A$

(ii) Time constant $= \frac{L}{R} = \frac{500 \times 10^{-3}}{1} = 0 \cdot 5\,\text{sec.}$

T4.9 A 10μF capacitor is charged to 120V and then discharged through a 1.5MΩ resistor. Determine either graphically or by calculation the capacitor voltage 2s after discharging has commenced. Also find how long it takes for the voltage to fall to 25V. **[105.0V, 23.53s]**

T4.10 A capacitor is connected in series with a voltmeter of resistance 750 kΩ and a battery. When the voltmeter reading is steady the battery is replaced with a shorting link. If it takes 17s for the voltmeter reading to fall to two-thirds of its original value, determine the capacitance of the capacitor. **[55.9μF]**

T4.11 When a 3μF charged capacitor is connected to a resistor, the voltage falls by 70% in 3.9s. Determine the value of the resistor. **[1.08 MΩ]**

T4.12 A 50μF uncharged capacitor is connected in seris with a 1 kΩ resistor and the circuit is switched to 100 V, d.c. supply. Determine (a) the initial current flowing in the circuit, (b) the time constant, (c) the value of current when t is 50 ms and (d) the voltage across the resistor 60 ms after closing the switch. **[(a) 0.1A (b) 50 ms (c) 36.8mA (d) 30.1 V]**

T4.13 An uncharged 5μF capacitor is connected in series with a 30 kΩ resistor across a 110V, d.c. supply. Determine the time constant of the circuit and the initial charging current. Determine the current flowing 120 ms after connecting to the supply. **[150 ms, 3.67 mA, 1.65 mA]**

T4.14 An uncharged 80μF capacitor is connected in series with a 1 kΩ resistor and is switched across a 110V supply. Determine the time constant of the circuit and the initial value of current flowing. Determine the value of current flowing after (a) 40 ms and (b) 80 ms.

[80 ms, 0.11A (a) 66.7mA (b) 40.5 mA]

T4.15 A 60 μF capacitor is connected in series with a 10kΩ resistor and connected to a 120V d.c. supply. Calculate (a) the timke constant, (b) the initial rate of voltage rise, (c) the initial charging current, and (d) the time for the capacitor voltage to reach 50V.

[(a) 0.60 s (b) 200 V/s (c) 12mA (d) 0.323s]

T4.16 A 200V d.c. supply is connected to a 2.5MΩ resistor and a 2μF capacitor in series. Calculate (a) the current flowing 4s after connecting, (b) the voltage across the resistor after 4s, and (c) the energy stored in the capacitor after 4s. **[(a) 35.95μA (b) 89.87V (c) 12.13 mJ]**

T4.17 (a) In the circuit shown in Figure T4.2 with the switch in position 1, the capacitor is uncharged. If the switch is moved to position 2 at time t = 0s, calculate (i) the initial current through the 0.5 MΩ, (ii) the voltage across the capacitor when t = 1.5s and (iii) the time taken for the voltage across the capacitor to reach 12V.

(b) If at the time t = 1.5s, the switch is moved to position 3, calculate (i) the initial current through the 1MΩ resistor, (ii) the energy stored in the capacitor 3.5 s later (i.e when t = 5s). (c) Sketch a graph of the voltage across the capacitor against time from t = 0 to t = 5s, showing the main points.

[(a) (i) 80 μA (ii) 18.05 V (iii) 0.892 s (b) (i) 40 μA (ii) 48.30μJ]

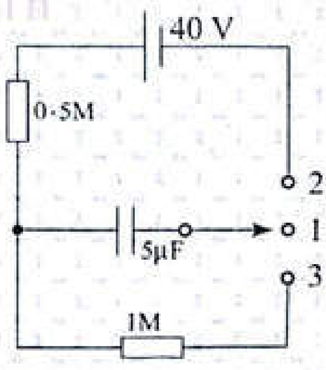


Fig. T4.2

(iii) $i = \frac{V}{R}\left(1 - e^{\frac{-Rt}{L}}\right) = 3\left(1 - e^{-500\times10^{-3}/500\times10^{-3}}\right) = 1\cdot896A$

9. A coil having a resistance of 2 ohms and an inductance of 1 henry is switched on to a 10 volt d.c. supply. Write down the expression of current $i(t)$ in the coil as a function of time.

(1st semester 2006)

Solution : $R = 2$ ohms, $L = 1H$, V = 10 volts.

Time constant $= \frac{L}{R} = \frac{1}{2} = 0\cdot5$ sec.

Steady current $= \frac{V}{R} = \frac{10}{2} = 5A$

Expression of current $i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$

$= 5\left(1 - e^{-t/0.5}\right)A$

10. A coil with a self inductance of 4 H and a resistance of 10 ohm is suddenly switched across a 20 V dc supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.5 sec. and the time taken for the current to reach 80% of its final value. What is the final steady value of the current ? *(1st semester 2006)*

Solution : L = 4 H, $R = 10\Omega$, V = 20 volts.

(i) Time constant $\tau = \frac{L}{R} = \frac{4}{10} = 0\cdot4$ sec.

(ii) Final steady value of current $= \frac{V}{R} = \frac{20}{10} = 2A$

(iii) Instantaneous value of the current after 0.5 second is

$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right) = 2\left(1 - e^{-0.5/0.4}\right) = 1\cdot427$ Ampere

(iv) $i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$

$\Rightarrow 0\cdot8(2) = 2\left(1 - e^{-t/0.4}\right)$

$\Rightarrow t = 0\cdot6436$ sec.

11. An 8 micro farad capacitor is connected in series with a 500 kilo ohm resistor and the RC combination is connected across 200 V dc supply. Calculate

(i) the time constant

(ii) the initial charging current

(iii) the time taken for the capacitor voltage to reach 160 volts.

T4.18 For the circuit of Fig. T4.3 obtain the complete solution for the current $i_L(t)$ through the 5-H inductor and the voltage $V_x(t)$ across the 6Ω resistor.

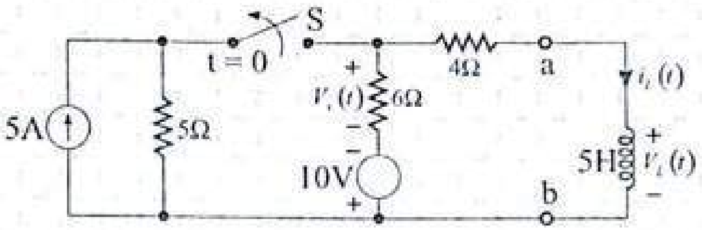


Fig. T4.3

$$\left[i_L(t)=\left(\frac{50}{37}+1\right)e^{-2t}-1=\left(\frac{87}{37}e^{-2t}-1\right)A,\ \text{for } t>0\ vx(t)=-(522/37)e^{-2t}+6\right]$$

T4.19 Consider the circuit of Figure T4.4 and obtain the complete solution for the voltage $V_c(t)$ across the 5-F capacitor and the voltage $V_x(t)$ across the 5Ω resistor.

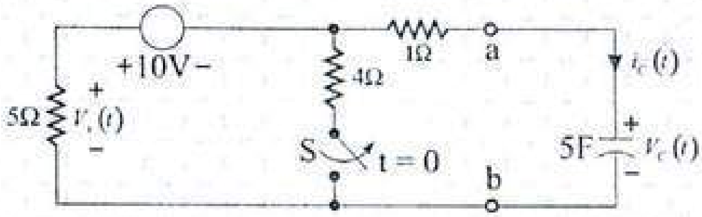


Fig. T4.4

$$i_C(t)=C\frac{dv_c(t)}{dt}=(5)\left(-\frac{50}{9}\cdot\frac{1}{30}\right)e^{-t/30}=-\frac{25}{27}e^{-t/30}A$$

$$V_x(t)=-5i_C(t)=(125/27)e^{-t/30}A$$

T4.20 The swich in the circuit shown in Fig. T4.5 has been in position 1 for a long time. At $t=0$, the switch is suddenly brought to position 2. Determine the current $i(t)$ through the capacitance for $t>0$.

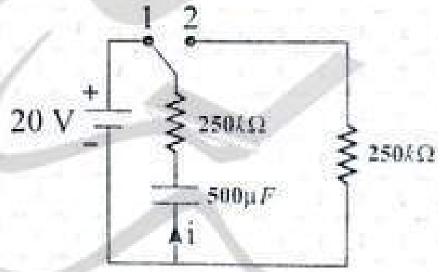


Fig. T4.5

$$[-40e^{-(t/250)}(\mu A)]$$

T4.21 In the circuit of Fig. T.4.6 swich S represents a current - operated relay, the contacts of which close when $i_L=0.9A$ and open when $i_L=0\cdot25A$. Determine the time period for one cycle of the relay operation. **[4.11 ms.]**

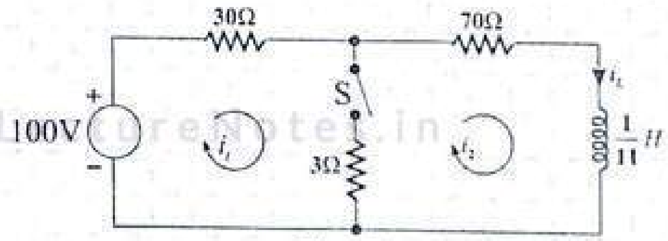


Fig T4.6

T4.22 Switch S of the network shown in Fig. T4.7 has been closed for a long time. At $t=0$, the switch is suddenly opened. Evaluate the voltage across the 200 Ω resistor.

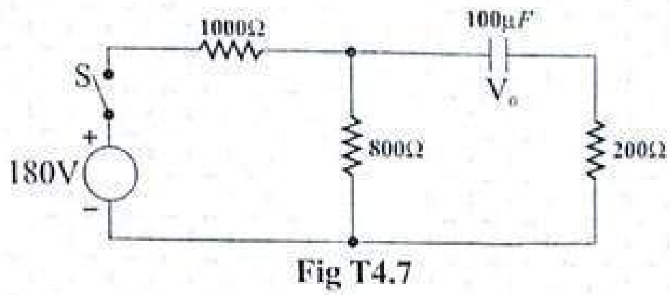


Fig T4.7

(iv) the current and p.d. across the capacitor 4 seconds after it is connected to supply.

(2nd semester 2006)

Solution : $C = 8\times10^{-6}\,F$, $R = 500\times10^{3}\,\Omega, V = 200$ volts

(i) Time constant $\tau = RC = 500\times10^{3}\times8\times10^{-6} = 4$ sec.

(ii) The initial charging current $= \frac{V}{R} = \frac{200}{500\times10^{3}} = 4\times10^{-4}\,A$

(iii) $V_C = V\left(1-e^{-t/\tau}\right)$

$\Rightarrow 160 = 200\left(1-e^{-t/4}\right)$

$\Rightarrow t = 6\cdot437$ sec.

(iv) The current through capacitor after 4 second is,

$i = \frac{V}{R}e^{-t/RC} = \frac{200}{500\times10^{3}}e^{-4/4} = 0\cdot14\times10^{-3}\,A$

P.d across capacitor is $V_C = V\left(1-e^{-t/\tau}\right)$

$= 200\left(1-e^{-4/4}\right)$

$= 126\cdot42$ volts.

12. A direct voltage of 150 V is applied to a coil of resistance 15 ohm and inductance 15 H. What is the value of the current 0.1 second after switching on ?

(1st semester 2007)

Solution : V = 150 volts, $R = 15\Omega$, $L = 15H$

Time constant $= \tau = L/R = 1$ sec.

The current, 0.1 second after switching on is,

$i = \frac{V}{R}\left(1-e^{-t/\tau}\right)$

$= \frac{150}{15}\left(1-e^{-0\cdot1/1}\right) = 0\cdot951A$

13. A 10 microfarad capacitor is connected to a constant voltage source through a resistance of 2.5 mega ohm. Calculate the time taken for the capacitor to lose 50% of its charge when the voltage source is short circuited ?

(1st semester 2007)

Solution : $C = 10\times10^{-6}\,F$ $R = 2\cdot5\times10^{6}\,\Omega$

Time constant $= \tau = RC = 25$ second.

$q = Qe^{-t/\tau}$

T4.23 The switch of the network of Fig. T4.8 is suddenly opened at $t = 0$; prior to this, the network was under steady state. Determine the voltage across the 6Ω resistance for $t > 0$.

$[12e^{-90t}\,V]$

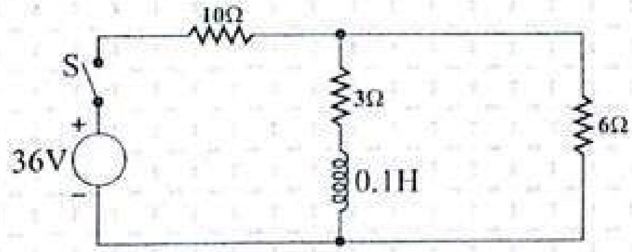


Fig T4.8

T4.24. The capacitor of the circuit shown in Fig. T4.9 has no charge when the switch is closed at $t = 0$. Calculate (a) the current through and (b) the voltage across, the capacitor.

$[(a)\ 5e^{-400t}\ (A);\ (b)\ 250\left(1 - e^{-400t}\right)$

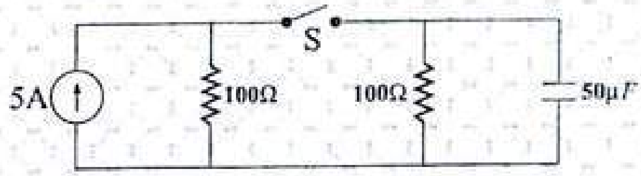


Fig T4.9

T4.25. With no initial energy stored in the inductance of the circuit shown in Fig. T4.10 the switch is closed at $t = 0$. Evaluate (a) the source current, and (b) the voltage across the inductance.

$[(a)\ 2.5\left(2 - e^{-15t}\right)(A);\ (b)\ 120e^{-15t}\ (V)$

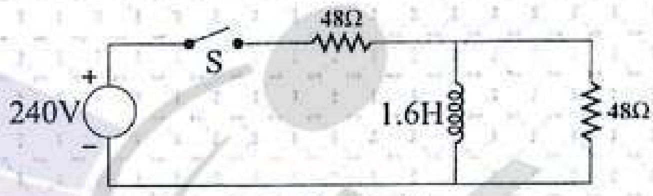


Fig T4.10

T4.26 The switch in the network of Fig. T4.11 has been closed for a long time; at $t = 0$ it is suddenly opened. Solve for the voltage v for $t > 0$.

$[4e^{-0.1t}\,V]$

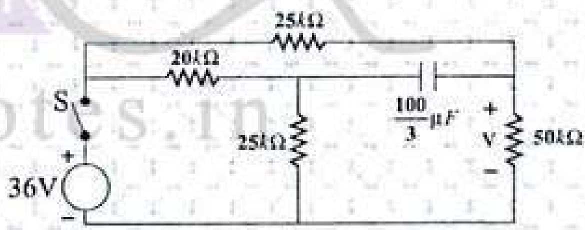


Fig T4.11

T4.27 At $t = 0$ the switch of the circuit of Fig. T4.12 is closed. With no initial current through the inductance, determine the instantaneous voltage across the 6Ω resistance. $[6 + 4e^{-50t}\,V]$

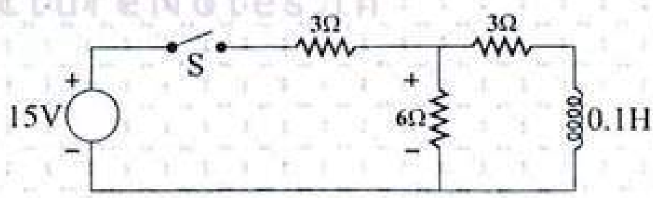


Fig T4.12

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$$\Rightarrow \frac{Q}{2} = Qe^{-t/25}$$

$\Rightarrow t = 17 \cdot 32$ second.

14. A coil has a resistance of 10 Ω and inductance of 1 henry. What will be the value of current after 0.1 second of switching this coil to a 100 V dc supply ?

(2nd semester 2007)

Solution : $R = 10\Omega$, L=1H, V = 100 volts.

Time constant $= \tau = \frac{L}{R} = \frac{1}{10} = 0 \cdot 1$ second.

$$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right) = \frac{100}{10}\left(1 - e^{-0.1/0.1}\right) = 6 \cdot 321A$$

15. A condenser of $8\mu F$ capacitance is connected to a dc source through a resistance of one mega ohm. Calculate the time taken for the condenser to receive 95% of its final charge.

(2nd semester 2007)

Solution $C = 8 \times 10^{-6} F$ $R = 10^6\ \Omega$

Time constant $= \tau = RC = 8$ second.

$$q = Q\left(I - e^{-t/\tau}\right)$$

$$\Rightarrow 0 \cdot 95Q = Q\left(I - e^{-t/8}\right)$$

$\Rightarrow t = 23 \cdot 965$ second.

16. A 15 μF capacitor in series with a $15 \times 10^3\ \Omega$ resistance is connected across a constant dc voltage source of 250 volts. The fully charged capacitor is disconnected from the supply and is discharged by connecting a 1000Ω resistance across its terminals. Compute the initial value of the charging current and initial value of discharging current. *(1st semester 2008).*

Solution : $C = 15 \times 10^{-6}\ F$, $R = 15 \times 10^3\ \Omega$, V = 250 volts.

Initial value of charging current $= \frac{V}{R} = \frac{250}{15 \times 10^3} = 0 \cdot 016A$

Initial value of discharging current $= -\frac{250}{1000} = -0 \cdot 25A$

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Basic Electrical Engineering

Topic:

Resistive Network Analysis

Contributed By:

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Silicon Institute Of Technology

Chapter - 4

Transient Analysis

4.1 Introduction

When an electric circuit containing an ohmic resistance only is switched on, the electric current acquires its maximum value almost in zero time. Similarly when such a circuit is switched off the current reduces to zero almost in zero time. When electric circuit contains an inductor or a capacitor or both, the growth as well as decay of current are opposed by emf induced. Therefore electric current takes some finite time to reach its maximum value, when the circuit is switched on. Similarly when the circuit is switched off, the current takes some finite time to decay to zero value. Electric currents which vary for a small finite time, while growing from zero to maximum value or while decaying from maximum value to zero value are called ***transient currents***

Transients are produced whenever :

(i) a circuit is shorted

(ii) there is a sudden change in applied voltage

(iii) a circuit is suddenly connected to or disconnected from the supply.

Transient currents are associated with the changes in stored energy in inductors and capacitors. There are no transients in pure resistive circuit. It is because resistors are not stored energy.

4.2 Differential equations for circuits containing inductors and capacitors

Inductor

When a wire of finite length twisted into a coil then it represents an inductor. Inductance is a property which opposes any change of magnitude or direction of electric current passing through the conductor. When a steady current (D.C) is allowed to flow in a inductor, it behaves like an inert element (i.e. presence is not felt). However, when the current changes with respect to time then inductor shows its presence and opposes to the change in current. As a result a voltage is induced across the inductor whose magnitude is given by $V_L = L\frac{di}{dt}$ and direction is opposite to the flow of current (by Lenz's law). If current through the inductor is constant then change in current is zero (i.e. di = 0). So voltage across inductor in zero (i.e. V_L=0). This means that an inductor behaves as a short circuited coil in steady state. For a minute change in current within zero time (dt = 0) the voltage across the inductor is infinite (i.e. $V_L = \infty$). This means that an inductor behaves as open circuited just after switching across d.c voltage.

Power absorbed by inductor is given by $P = Vi = L\cdot\frac{di}{dt}\cdot i = Li\cdot\frac{di}{dt}$

Energy absorbed by inductor is given by $W = \int P\cdot dt = \int Li\cdot\frac{di}{dt}\cdot dt = \frac{1}{2}Li^2$

It may be noted that the inductor can store finite amount of energy, even the voltage across it may be nil.

Capacitor

A capacitor is formed by any two conducting surfaces separated by a dielectric medium. Capacitance of the capacitor is defined as the ability to store energy in its electrostatic field. However, a capacitor has a typical characteristics which does not allow any change in voltage across it. If a change in voltage with respect to time is imposed across it then it allows a current through its dielectric medium (ie. *displacement current*). The magnitude of this current is given by

$$i = C\frac{dv}{dt} \quad \text{.....................................} (1)$$

Where C in the capacitance in farad and $\frac{dv}{dt}$ is the rate of change of voltage in volt/sec.

$$(1) \qquad dv = \frac{1}{C}i\cdot dt$$

$$dt \Rightarrow V = \frac{1}{C}\int_{-\infty}^{t} i\cdot dt$$

$$\Rightarrow V = \frac{1}{C}\int_{-\infty}^{0} i \cdot dt + \frac{1}{C}\int_{0}^{t} i \cdot dt$$

$$\Rightarrow V = V_0 + \frac{1}{C}\int_{0}^{t} i \cdot dt \text{ .... (2)} \qquad \left[\because V_0 = \frac{1}{C}\int_{-\infty}^{0} i \cdot dt\right]$$

Where V_0 is the initial voltage (if any) across the capacitor at the instant of inserting in the circuit.

From equation (2) $\quad V = V_0 + \frac{1}{C} q(t) \qquad \left[\begin{array}{l} As\, i = \frac{dq}{dt} \\ \Rightarrow q(t) = \int i \cdot dt \end{array}\right]$

Power absorbed by capacitor is, $P = Vi = V \cdot C\frac{dv}{dt}$

Energy sorted by capacitor is, $W = \int_0^t p \cdot dt = \int_0^t V \cdot C\frac{dv}{dt} \cdot dt = \frac{1}{2}CV^2$

The capacitor, on application of d.c. voltage and with no initial charge first acts as short circuit but as soon as the full charge it retains, the capacitor behaves as open circuit. It may be noted that the capacitor can store finite amount of energy even the current across it may be nil.

4.2.1 Solution of Differential Equation

Differential equations are used in the treatment of transient i.e. single energy transients (involved in R-L and R-C circuits) and double -energy transients (involved in R-L-C circuit). We will consider both first - order and second - order differential equations.

1. First order Equations :

(i) Let $\frac{dy}{dx} + ay = 0$, Where a is a constant. The solution of this homogeneous differential equation is $y = Ke^{-ax}$, where K is the constant of integration whose value can be found from the boundary conditions.

(ii) Let $\frac{dy}{dx} + ay = Q$, where Q is either a function of independent variable or a constant.

The solution of this non homogeneous differential equation is

$y = e^{-ax}\int e^{ax} \cdot Q \cdot dx + Ke^{-ax}$

y = Particular integral + Complementary function

The particular integral part $\left(e^{-ax}\int e^{ax}\cdot Q\cdot dx\right)$ does not contain the arbitrary constant K and complementary function part $\left(Ke^{-ax}\right)$ does not depend on function Q.

2. Second Order Equations :

Let $A\frac{d^2y}{dx^2}+B\frac{dy}{dx}+Cy=0$, where A,B and C are constants.

The solution of this homogenous differential equation is $y=K_1e^{P_1x}+K_2e^{P_2x}$

Where K_1 and K_2 are the constants and P_1 and P_2 are the roots of the characteristic equation.

The characteristic equation is $AP^2+BP+C=0$

The roots of this equation $P_1,P_2=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$

$$=\frac{-B+\sqrt{B^2-4AC}}{2A},\ \frac{-B-\sqrt{B^2-4AC}}{2A}$$

The values of K_1 and K_2 can be evaluated by using initial conditions for the original equations and derivative of the original equations, which is time. To summerise the behaviour of the basic elements at t = 0 and t = ∞ discussed elaborately in the examples.

4.2.2 Initial and steady state conditions

Every circuit when excited from a energy source the behaviour of the circuit is characterised by a differential equation, while solving the equation we get constants of integration. To evaluate these equations it is necessary to know the behaviour of the circuit (of each element) at various instant, particularly at initial (t = 0) and final (t = ∞) the following table is given.

Table

Sl.No.	Name of the element with symbol	At t = 0	At t = ∞
1	Resistance R	R	R
2	Inductance L	O.C	S.C
3	Capacitance C	S.C	O.C
4	Inductance with initial current Io (Io→)	Io	Io S.C
5	Capacitance with initial voltage Vo (C, − +, V_o)	− + V_o SC	+ − V_o O.C

4.3. Transient response of 1st order circuit

The first order circuit contains resistance and one energy storing element i.e. one inductor or capacitor. This first order circuit, during its transient state of operation, is governed by first order linear differential equation.

4.3.1 D.C steady state solutions of R-L Circuit

Consider a R-L circuit connected in series with a battery of voltage V and a switch S. The R-L combination gets connected to the battery when switch S is connected to terminal 1 and is short circuited when switch S is connected to terminal 2.

Growth of current in RL circuit

When switch S moves to terminal 1, the battery is connected and current grows in R-L circuit. Due to self induction an induced emf is set up across inductance L. By lenz's law this induced emf opposes the growth of current. The current rises gradually and it takes a definite time to reach its steady value (i.e. maximum value). This time is called transient time.

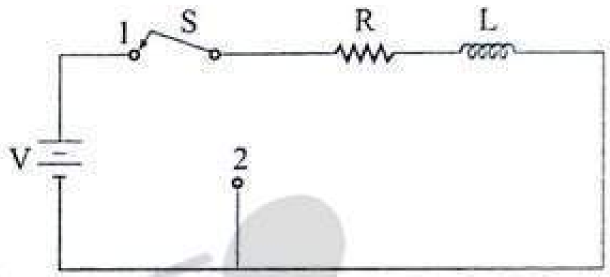

Fig. 4.1

Consider some instant t seconds after the voltage is applied,

i = Current flowing through the circuit at instant t seconds.

$\frac{di}{dt}$ = rate of growth of current at this instant.

$V_R = iR$ = Voltage across R

$V_L = L \cdot \frac{di}{dt}$ = Voltage across L

Applying Kirchhoff's Voltage law (KVL),

$$V = Ri + L \cdot \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{V}{L} \quad \text{............................ (1)}$$

Equation (1) is a non-homogeneous differential equation.

Solution of this equation is,

$$i = e^{-\frac{R}{L}t} \int e^{\frac{R}{L}t} \cdot \frac{V}{L} dt + Ke^{-\frac{R}{L}t} \quad \text{.............} (2)$$

$$\Rightarrow i = i_p + i_c$$

we know $\frac{dy}{dx} + ay = Q$

$$\therefore y = e^{-ax} \int e^{ax} . Q . dx + Ke^{-ax}$$

$i = y$

$t = x$

here $\frac{R}{L} = a$

$\frac{V}{L} = Q$

Where i_p is the particular solution that provides the steady state response and i_c is the complementary function that always goes to zero value in a short time (transient period).

Solution of equation (2) is,

$$i = \frac{V}{R} + Ke^{-\frac{R}{L}t} \quad \text{......................} (3)$$

An inductance has a property called as *"electrical inertia"* which does not allow sudden change of current through it following the laws of electromagnetic induction.

So current flowing through inductor just before switching is equal to current just after switching. But before switching there was no current through the inductor. Therefore, just after switching (i.e. at time $t = 0^+$) the current through the inductor will be zero.

At $t = 0^+$ (just after switching) i.e. with initial condition, equation (3) becomes

$$0 = \frac{V}{R} + Ke^{-\frac{R}{L}\times 0} = \frac{V}{R} + K$$

$$\Rightarrow K = -\frac{V}{R}$$

Putting this value of K in equation (3) we get

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{.........................} (4)$$

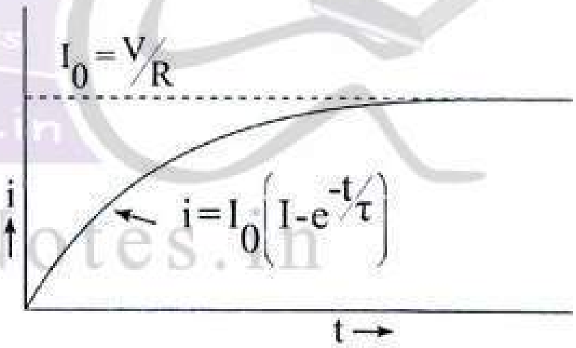

Fig. 4.2

Where $I_0 = \frac{V}{R}$ = maximum current or steady state current.

$\tau = \frac{L}{R}$ = time constant (or inductive time constant)

Equation (4) is called *Helmholtz equation* for growth of current in R-L circuit. This equation clearly shows the exponential rise of current i charging the inductor. The graph between i and t is shown in fig 4.2.

If we put $t = \tau = \frac{L}{R}$ in equation (4) we get $i = I_0\left(1 - e^{-1}\right) = 0 \cdot 632 I_0 = 63 \cdot 2\%$ of I_0.

Thus time constant is defined as the time during which current in an inductor rises to 63.2 percent of its maximum value.

If we put $t = \infty$ in equation (4) then we get $i = I_0\left(1 - e^{-\infty/\tau}\right) = I_0$. Thus current in R-L circuit would attain maximum value (I_0) only after infinite time. But practically current reaches its maximum value after a time which is five times the time constant.

Voltage drop across inductor is, $V_L = L \cdot \frac{di}{dt}$

$$\Rightarrow V_L = L \cdot \frac{d}{dt}\left[I_0\left(1 - e^{-\frac{t}{\tau}}\right)\right] = L \cdot \frac{d}{dt}\left[I_0 - I_0 e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_L = L\left[\frac{d}{dt}I_0 - \frac{d}{dt}I_0 e^{-\frac{t}{\tau}}\right] = L\left[0 + \frac{1}{\tau}I_0 e^{-\frac{t}{\tau}}\right]$$

$$\Rightarrow V_L = L\left[\frac{1}{L/R}I_0 e^{-\frac{t}{\tau}}\right] = I_0 R e^{-\frac{t}{\tau}}$$

$$\Rightarrow V_L = V e^{-\frac{t}{\tau}} \qquad (\because where\ V = I_0 R)$$

Voltage drop across resistor is

$$V_R = iR = I_0\left(1 - e^{-\frac{t}{\tau}}\right)R = V\left(I - e^{-\frac{t}{\tau}}\right)$$

In transient period voltage across resistor exponentially rising and voltage across inductance exponentially decaying. Once the transient dies out within a short time then steady current $\left(I_0 = \frac{V}{R}\right)$ remains in the circuit.

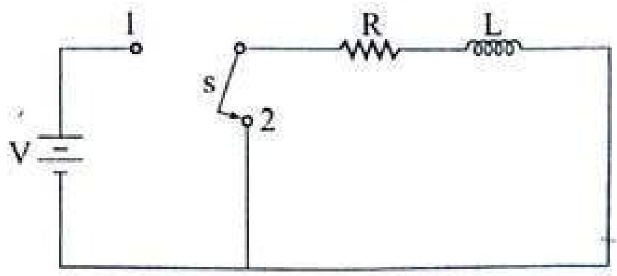

Fig. 4.3

Decay of current in R-L circuit :

When switch S suddenly moves to terminal 2 then battery gets disconnected from the circuit and current in R-L circuit decays.

Let i = decaying current at any instant.

Applying Kirchhoff's law (KVL),

$$Ri + L\cdot\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0 \quad \text{.............................. (5)}$$

Equation (5) is a homogeneous differential equation. Solution of this equation is

we know $\frac{dy}{dx} + ay = 0$

$\therefore y = Ke^{-ax}$

$i = y$

here $t = x$

$\frac{R}{L} = a$

$$i = Ke^{-\frac{R}{L}t} \quad \text{.................................. (6)}$$

Where K is a constant whose value can be calculated from initial conditions.

With the initial conditions, at $t = 0, i = I_0 = \frac{V}{R}$

$\therefore$ equation(6) becomes.

$$\frac{V}{R} = Ke^{-\frac{R}{L}\times 0}$$

$$\Rightarrow K = \frac{V}{R}$$

Putting this value of K in equation (6) we get.

$$i = \frac{V}{R}e^{-\frac{R}{L}t} = I_0 e^{-\frac{t}{\tau}} \quad \text{.......... (7)}$$

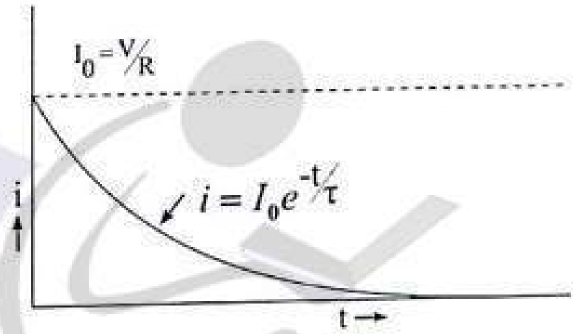

Fig. 4.4

where $\frac{V}{R} = I_0$ and $\frac{L}{R} = \tau$ = time constant.

Equation (7) is called Helmholtz equation for decay of current in R-L circuit. The graph between i and t is shown in fig.4.4.

If we put $t = \tau = \frac{L}{R}$ in equation (7) we get $i = I_0 e^{-1} = 0\cdot 37\, I_0 = 37\%$ of I_0.

Thus time constant is defined as the time during which current in an inductor falls to 37% of its maximum value.

Voltage drop across inductor, $V_L = L\frac{di}{dt}$

$$\Rightarrow V_L = L\frac{d}{dt}I_0 e^{-\frac{t}{\tau}} = L\cdot I_0\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} = L\cdot I_0\left(-\frac{1}{\frac{L}{R}}\right)e^{-\frac{t}{\tau}}$$

$$\Rightarrow V_L = -I_0 R\, e^{-\frac{t}{\tau}} = -Ve^{-\frac{t}{\tau}}$$

Voltage drop across resistance, $V_R = iR = I_0 Re^{-\frac{t}{\tau}} = Ve^{-\frac{t}{\tau}}$

4.3.2 D.C Steady state solutions of R-C circuit

Consider a R-C circuit connected in series with a battery of voltage V and a switch S. The R-C combination gets connected to the battery when switch S is connected to terminal 1 and is short circuited when switch S is connected to terminal 2.

Charging of R.C. Circuit

When switch S moves to terminal 1 then battery is connected to the circuit. Initially capacitor is uncharged and voltage across it is zero. (i.e. $V_C = 0$). The whole of supply voltage V appears across resistance (i.e. $V_R = V$). As V_C is zero so capacitor behaves as a short circuit. According to Ohm's law the initial current in the circuit is $I_0 = \frac{V}{R}$.

As current flows the capacitor starts charging and capacitor voltage V_C increases. As a result voltage across resistance (V_R) decreases and charging current also decreases.

When capacitor becomes fully charged (i.e. $V_C = V$) then V_R becomes zero and charging current also becomes zero. As charging current is zero, the capacitor behaves as open circuit. This current takes a definite time to reach its zero value. This time is called transient time.

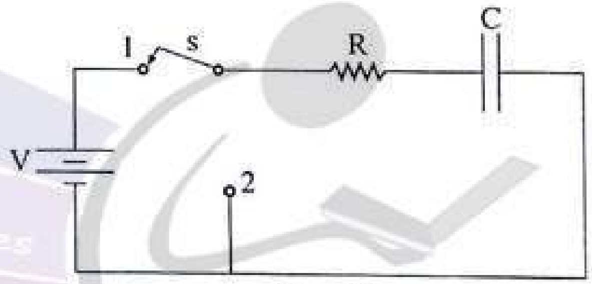

Fig. 4.5

Consider some instant t seconds after the voltage is applied.

i = current flowing through the circuit at instant t seconds.

$V_R = Ri$ = Voltage drop across R.

$V_C = \frac{1}{C}\int i \cdot dt$ = Voltage across C.

Applying Kirchhoff's voltage law (KVL)

$$Ri + \frac{1}{C}\int i \cdot dt = V$$

Differentiating both sides w.r.t time 't' we get,

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{1}{RC} i = 0 \text{ .... (1)}$$

we know $\frac{dy}{dx} + ay = 0$

$\therefore y = Ke^{-ax}$

here $i = y$

$t = x$

$\frac{1}{RC} = a$

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2.3.1 Delta to star Conversion

Consider three resistors R_{AB}, R_{BC} and R_{CA} connected in delta to three terminals A,B, and C as shown in fig. 2.15.

It is desired to replace these three delta connected resistors by three resistors R_A, R_B and R_c connected in star. The expressions of R_A, R_B and R_c are given below.

$$R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{BC}\, R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{CA}\, R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

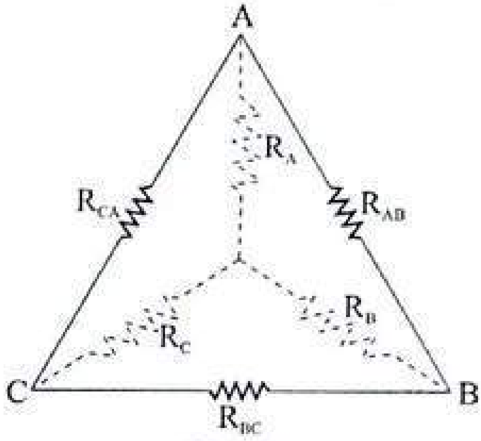


Fig 2.15

How to remember question? There is an easy way to remember it. Referring to star connected resistance $R_A, R_B, \& R_C$ are electrically equivalent to delta connected resistance $R_{AB}, R_{BC} \& R_{CA}$. We have seen above that : $R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$

i.e. Any arm of star connection $= \frac{\text{Product of two adjacent arms of } \Delta}{\text{Sum of arms of } \Delta}$

2.3.2 Star to delta conversion

Consider three resistors R_A, R_B and R_c connected in star to three terminals A,B and C as shown in fig. 2.16

It is desired to replace these three star connected resistors by three resistors $R_{AB}, R_{BC} \& R_{CA}$ Connected in delta. The expressions of $R_{AB}, R_{BC} \& R_{CA}$ are given below.

$$R_{AB} = R_A + R_B + \frac{R_A\, R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B\, R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C\, R_A}{R_B}$$

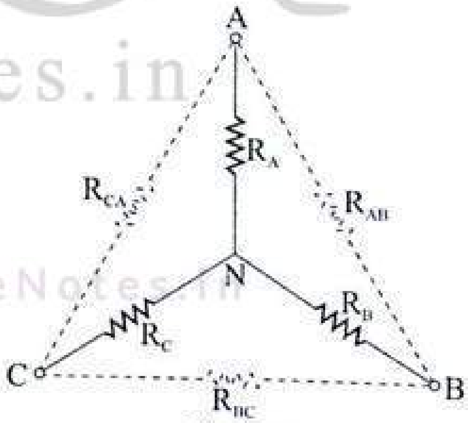


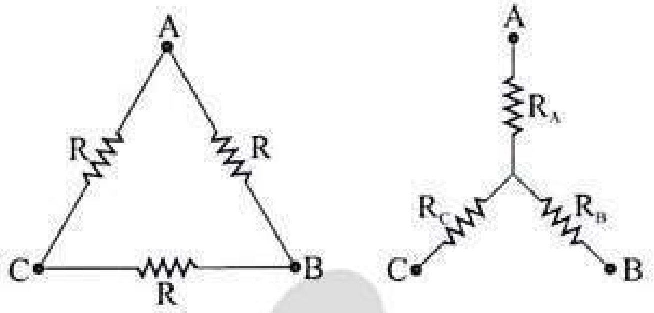
Fig 2.16

How to remember ? There is an easy way to remember it. Referring to fig 2.16 star connected resistance R_A, R_B and R_c are electrically equivalent to delta connected resistance R_{AB}, R_{BC} & R_{CA}.

We have seen above that : $R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$ i.e. Resistance between two terminals of delta = sum of star resistance connected to those terminals + product of the same to resistance divided by the third resistance.

Example 2.7 : A delta connection contains three equal resistance R. Find the resistance of the equivalent star connection.

Solution :



Given, $R_{AB} = R_{BC} = R_{CA} = R$ (in delta)

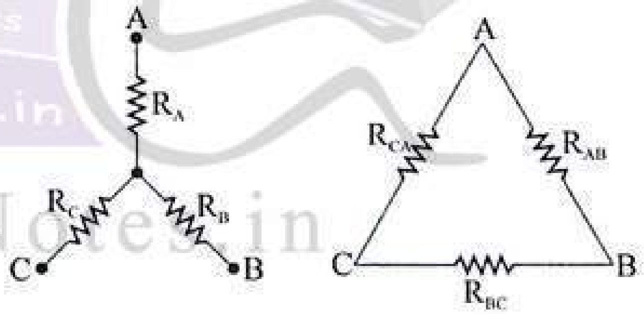
Let R_A, R_B *and* R_c are resistance in star connection.

$$\therefore \; R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{R.R}{R+R+R} = \frac{R}{3}$$

Similarly $R_B = R_C = \frac{R}{3}$

Example 2.8 : A star connection contains three equal resistance R. Find the resistance of the equivalent delta connection.

Solution :



Given $R_A = R_B = R_C = R$ (in star)

Let R_{AB}, R_{BC} & R_{CA} are resistance in delta connection.

$$R_{AB} = R_A + R_B \frac{R_A\, R_B}{R_C} = \; R + R + \frac{RR}{R} = 3R$$

Similarly $R_{BC} = R_{CA} = 3R$.

Example 2.9 : Find the resistance between the terminals a-b of the bridge circuit as shown in fig 2.17 by using delta- star transformation.

Given $R_1 = 4\Omega, R_2 = 6\Omega,\; R_3 = 14\Omega, R_4 = 10\Omega$ and R= 2m.

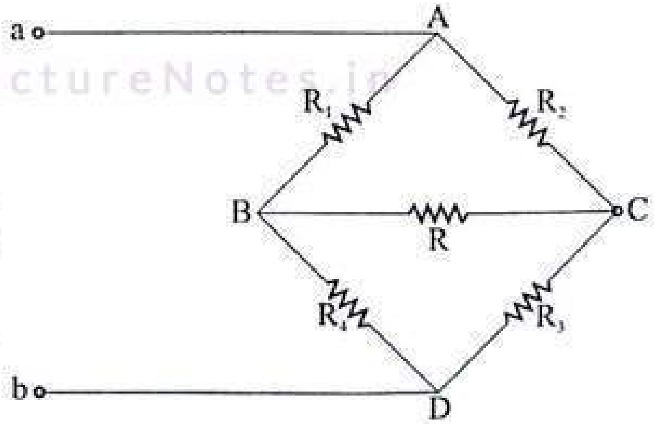


Fig 2.17

Solution : We shall transform the delta network between terminals A,B,C to star network In fig. 2.18 (a)

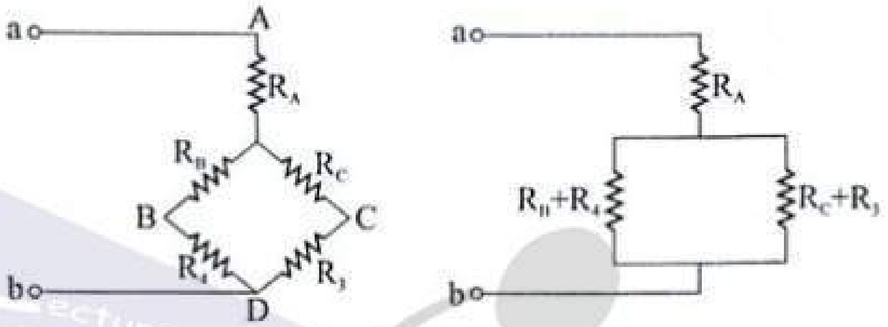


Fig 2.18

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R} = \frac{4 \times 6}{4+6+2} = 2\Omega$$

$$R_B = \frac{R R_2}{R_1 + R_2 + R} = \frac{2 \times 6}{4+6+2} = 1\Omega$$

$$R_c = \frac{R_1 R}{R_1 + R_2 + R} = \frac{4 \times 2}{4+6+2} = 0.667\Omega$$

In fig 2.18 (b)

$$R_{ab} = R_A + \left[(R_B + R_4) \| (R_C + R_3) \right]$$

$$\Rightarrow R_{ab} = 2 + \left[(1+10) \| (0.667 + 14) \right] = 8285\Omega$$

2.4 Network Theorems

To solve electric network we use two basic laws viz, ohm's law and kirchhoff's laws, occasions arise when these laws applied to certain networks do not yield quick and easy solution. to overcome this difficulty, network theorems have been developed which are very useful in analysing both simple and complex electrical circuits. In this chapter we shall discuss the following network theorems :

(i) Superposition theorem.

(ii) Thevenin's theorem

(iii) Norton's theorem

(iv) Maximum power transfer theorem.

2.4.1 Superposition theorem

This theorem finds use in solving a network where two or more sources are present and connected not in series or in parallel.

The superposition theorem states ***that in a liner bilateral network containing N sources, each branch current (or branch voltage) is the algebraic sum of N currents (or branch voltages), each of which is determined by considering one source at a time and removing all other sources. In removing the sources, voltage sources are short-circuited (or replaced by resistance equal to their internal resistance for non-ideal sources), while the ideal current sources are open circuited.***

These are the following steps for solving a network using superposition theorem.

Step - 1 : Take only one independent source of voltage/current and deactivate the other independent voltage/current sources. (for voltage sources, remove the source and short circuit the respective circuit terminals and for current sources, just delete the source keeping the respective circuit terminals open). Obtain branch currents.

Step 2 : Repeat the above step for each of the independent sources.

Step - 3 : To determine the net branch current, just add the currents obtained in step-1 and step-2 for each branch. If the currents obtained in step-1 and step-2 are in same direction, just add them. If the currents are directed opposite in each step, subtract them.

Example 2.10 : Determine the current through 2Ω resistor in circuit shown in fig. 2.19 by superposition theorem.

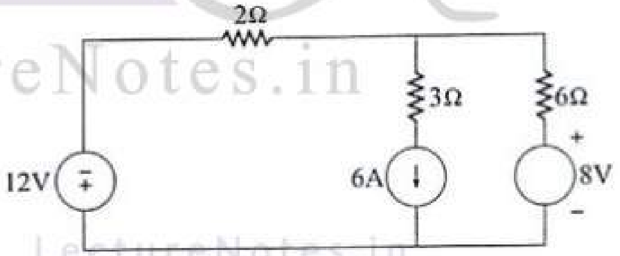


Fig 2.19

Solution : There are three sources in the given circuit. We shall determine the current due to each source acting alone.

Step - 1: Only 12V source in the circuit.

The 6A current source is open circuited and the 8V voltage source is short circuited .The resulting circuit is shown in fig 2.20 .

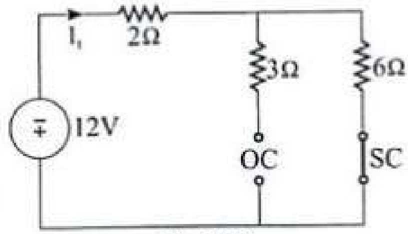


Fig 2.20

There is no current through the open circuit. Hence the current through the 3Ω resistor is zero.

$$\therefore \; I_1 = \frac{-12}{6+2} = -1.5A$$

Step -2 :Only 6A current source in the circuit.

The 12V voltage source and 8V voltage source are short circuited as shown in fig 2.21. The 6A current source is supplying current to 2Ω and 6Ω resistors. By current division rule.

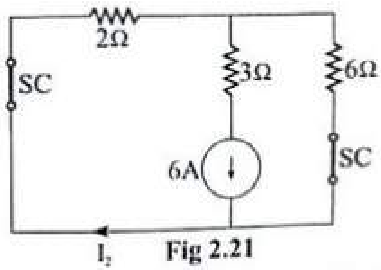


Fig 2.21

$$\therefore \; I_2 = 6 \times \frac{6}{2+6} = 4 \cdot 5A$$

Example 2.14 : In the circuit shown in fig 2.34 the source and node voltage are $V_{s1} = V_{s2} = 110V, V_A = 103V$ *and* $V_B = -107V$ Determine the voltage across each of the five resistor.

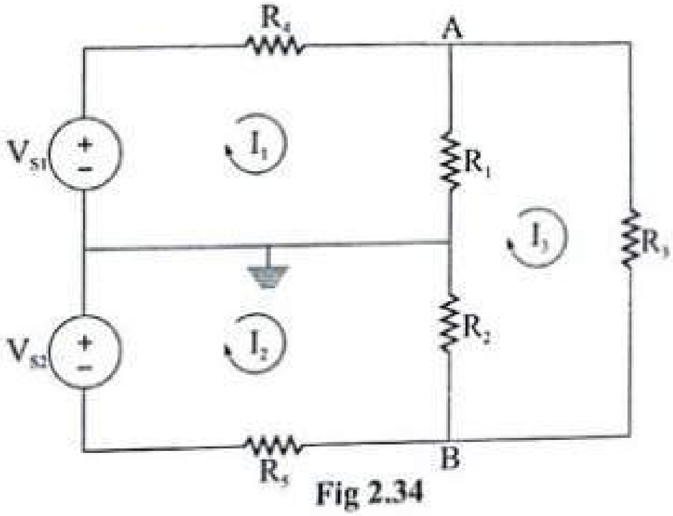


Fig 2.34

Solution : Assume a polarity for the voltages across R_1 *and* R_2 (i.e. from ground to node A, and from node B to ground) R_1 is connected between node A and ground. Therefore the voltage across R_1 is equal this node voltage.

So $V_{R1} = V_A = 103\ V$

R_2 is connected between node B and ground. Therefore the voltage across R_2 is equal to the negative of this voltage.

So $V_{R2} = -V_B = -(-107) = 107\,volt.$

Assume a polarity for the voltage across R_3 i.e. from node B to node A. Then apply KVL we get,
$V_{R3} = V_A - V_B = 103-(-107) = 210V.$

Assume polarities for voltage across R_4 *and* R_5 (i.e. from node A to ground and from ground to node B).

Apply KVL, $V_{s1} - V_{R4} - V_A = 0$

$\Rightarrow V_{R4} = V_{S1} - V_A = 110 - 103 = 7V$

Apply KVL, $-V_{S2} - V_{R5} - V_B = 0$

$\Rightarrow V_{R5} = -V_{S2} - V_B = -110-(-107) = -3V$

Example : 2.15 Using node voltage analysis in the circuit of fig 2.35 find the three indicated node voltages.

Solution : From figure 2.36 $i = \dfrac{V_1 - V_2}{75}$

Apply KCL to node -1

$$2 + \frac{0 - V_1}{200} + \frac{V_2 - V_1}{75} = 0$$

$$\Rightarrow 7V_1 - 4V_2 = 1200 \quad \text{..........} \quad (1)$$

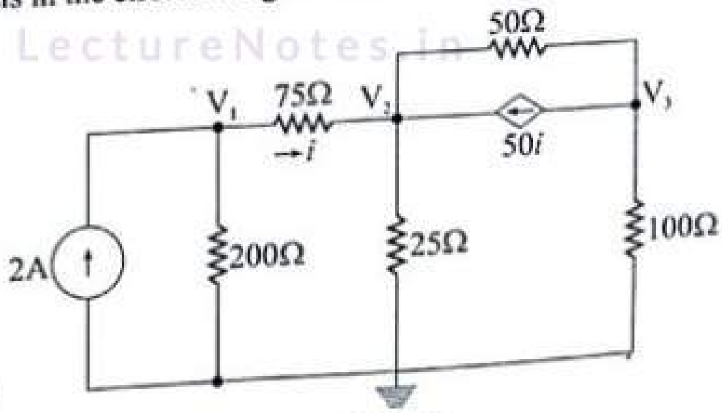


Fig 2.35

Example 2.25 Determine, Using superposition, the voltage across R in the circuit shown in Fig.2.51

$I_B = 12A, \quad R_B = 1\Omega$

$V_G = 12V, \; R_G = 0.3\Omega$

$R = 0.23\Omega$

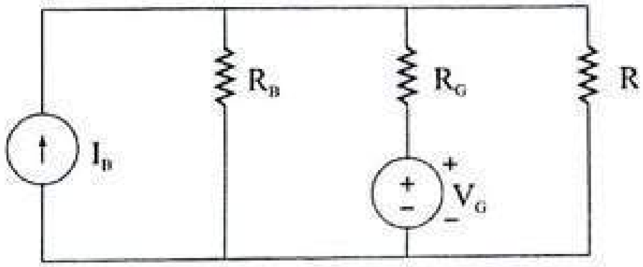


Fig 2.51

Solution : Only current source in the circuit, suppress the voltage source by replacing it with a short circuit as shown in fig 2.52.

Let V_1= Voltage across 'R'.

Apply KCL, $\quad I_B - \frac{V_1}{R_B} - \frac{V_1}{R_G} - \frac{V_1}{R} = 0$

$$\Rightarrow V_1 = \frac{I_B}{\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}} = \frac{12}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}} = 1.38V$$

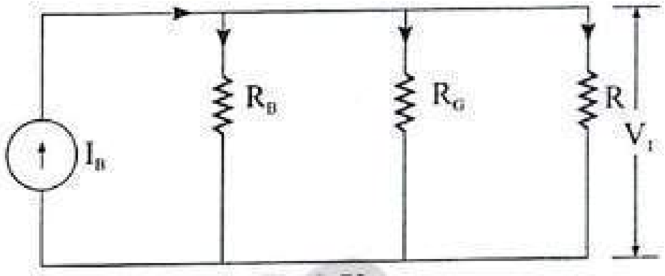


Fig 2.52

Only voltage source in the circuit, suppress the current source by replacing it with an open circuit.

Let V_2 = Voltage across R.

Apply KCL, $-\frac{V_2}{R_B} - \frac{V_2 - V_G}{R_G} - \frac{V_2}{R} = 0$

$$\Rightarrow \frac{V_2}{R_B} + \frac{V_2}{R_G} - \frac{V_G}{R_G} + \frac{V_2}{R} = 0$$

$$\Rightarrow V_2\left[\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}\right] = \frac{V_G}{R_G}$$

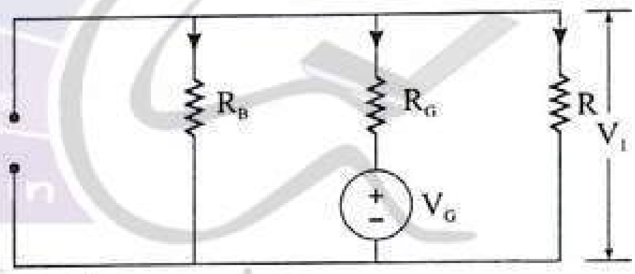


Fig 2.53

$$\Rightarrow V_2 = \frac{\frac{V_G}{R_G}}{\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}} = \frac{\frac{12}{0.3}}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}} = 4.61V$$

$$\therefore \; V_R = V_1 + V_2 = 1.38 + 4.61 = 5.99V$$

Example 2.26 Using superposition, determine the voltage across R_2 in the circuit of fig 2.54

$V_{S1} = V_{S2} = 12V, \quad R_1 = R_2 = R_3 = 1000\Omega$

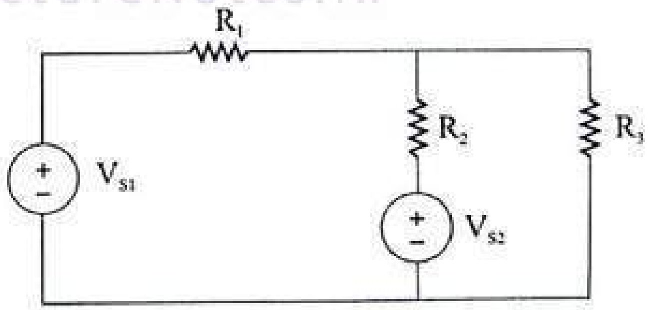


Fig 2.54

For node - 1 :

$$I + \frac{0 - V_1}{R_1} + \frac{V_2 - V_1}{R_2} = 0$$

$$\Rightarrow 0.2 - \frac{V_1}{200} + \frac{V_2 - V_1}{75} = 0$$

$$\Rightarrow 11V_1 - 8V_2 = 120 \quad ....(1)$$

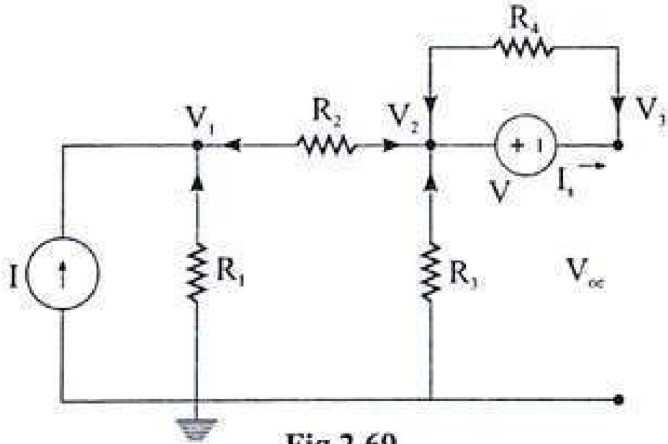


Fig 2.69

For node - 2 :

$$\frac{V_1 - V_2}{R_2} + \frac{0 - V_2}{R_3} + \frac{V_3 - V_2}{R_4} - I_s = 0 \quad (\because I_s = \text{Current through voltage source})$$

$$\Rightarrow \frac{V_1 - V_2}{75} - \frac{V_2}{25} + \frac{V_3 - V_2}{50} - I_s = 0$$

$$\Rightarrow 2V_1 - 11V_2 + 3V_3 - 150I_s = 0 \quad .... (2)$$

For node - 3 :

$$\frac{V_2 - V_3}{R_4} + I_s = 0$$

$$\frac{V_2 - V_3}{50} + I_s = 0$$

$$\Rightarrow V_2 - V_3 + 50I_s = 0 \quad ......(3)$$

For voltage source, $V_3 + 10 = V_2$ ....(4)

Solving these four equations we get,

$V_1 = 13.33$ volt, $V_2 = 3.33$ Volt, $V_3 = -6.67$ volt.

Therefore $V_{oc} = V_3 = -6.67$ volt.

(3) Replace the load with a short circuit as shown in fig 2.70

The current source I and resistance R_1 are connected in parallel. Convert it into a voltage source of voltage IR_1 connected in series with resistance R_1.

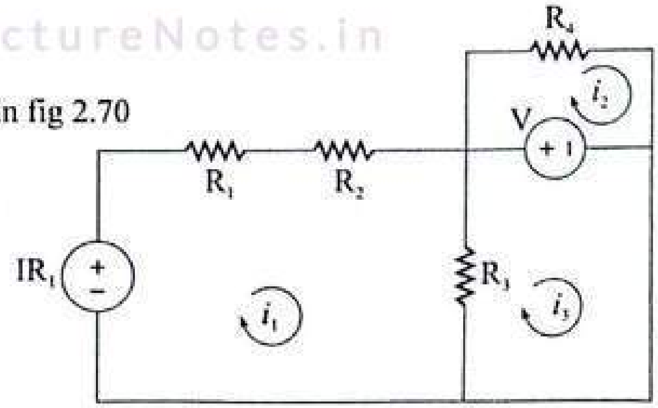


Fig 2.70

For mesh-1,

$$IR_1 - i_1R_1 - i_1R_2 - (i_1 - i_3)R_3 = 0$$

Apply KVL to the loop as shown in fig 2.88

$100-10I-15I=0$

$\Rightarrow \quad I=\frac{100}{10+15}=4A$

$\therefore \quad V_{th}=15\times 4=60$ volts,

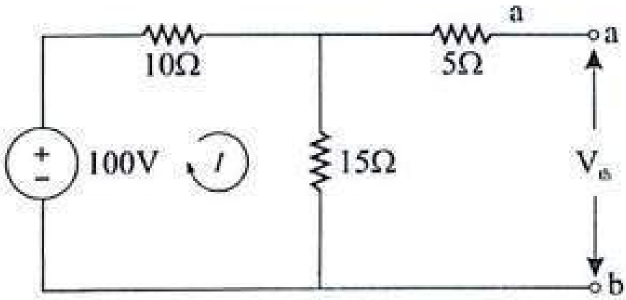


Fig 2.88

From fig 2.89

$R_{th}=(10\parallel 15)+5=11\Omega$

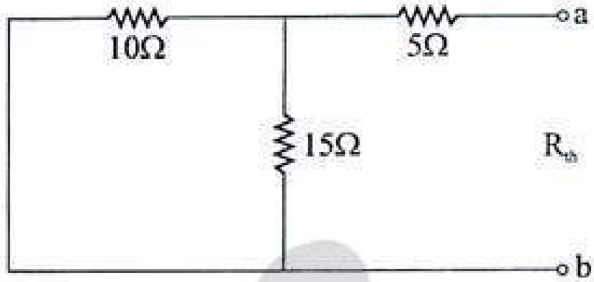


Fig 2.89

Thevenins equivalent circuit is shown in fig 2.90

Power = Maximum = P_{max},

when $R=R_{th}=11\Omega$

$\therefore \quad I=\frac{V_{th}}{R_{th}+R}=\frac{60}{11+11}=2.727\,A$

$P_{max}=I^2R=(2\cdot 727)(11)=81\cdot 81$ watts

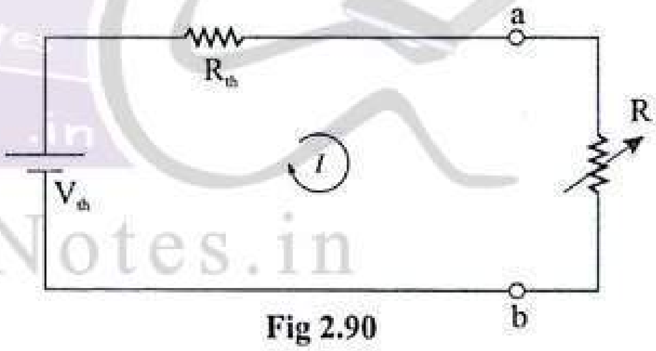


Fig 2.90

Example 2.37 Find the maximum power that the active network to the left of terminals 'ab' can deliver to the adjustable resistor R as shown in fig 2.91.

Solution : Let us find Thevenin's equivalent circuit across 'ab'

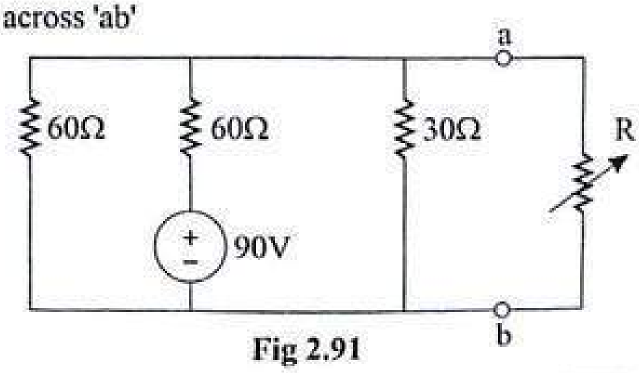


Fig 2.91

2.4.2. Thevenin's Theorem

Thevenin's theorem states that ***any pair of terminals AB of a linear active network may be replaced by an equivalent voltage source, in series with an equivalent resistance R_{Th}. The value of V_{Th} (called the thevenin voltage) is equal to p.d between the terminals AB when they are open-circuited, and R_{Th} is the equivalent resistance looking into the network at AB with the independent active sources set to zero i.e. with all the independent voltage sources short-circuited and all the independent current sources open-circuited.***

These are the following steps for solving a network by thevenin's theorem.

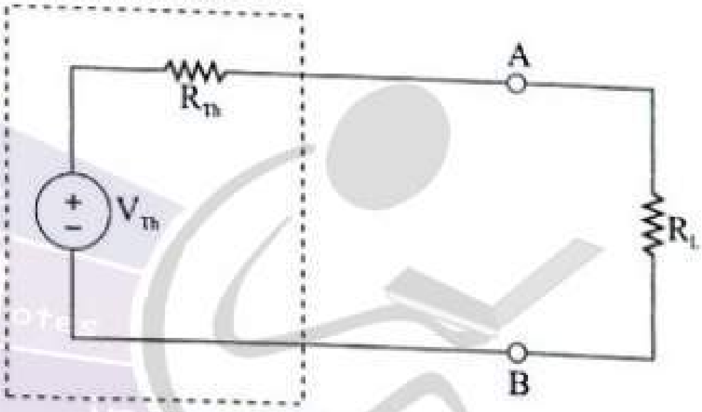
Step-1 : Mark the pair of terminals AB of the network. Remove the load resistance R_L and find the open circuit voltage (V_{Th}) across the open circuited load terminals AB.

Step-2 : Deactive the constant sources (for voltage sources, remove it by internal resistance and for current source delete the source by open circuit). Find the internal resistance R_{Th} of source side looking through the open circuited load terminals Ab.

Step-3 : Obtain Thevenin's equivalent circuit by placing R_{Th} in series with V_{Th}.

Step -4 : Reconnect R_L across terminals AB and calculate the current following through it.

Equivalent source



Thevenin's Equivalent network

$$\therefore \text{Current through } R_L \text{ is, } I = \frac{V_{Th}}{R_{Th} + R_L}$$

Step 3 : Only 8V source in the circuit.

The 6A current source is open-circuit and the 12V voltage source is short-circuited as shown in fig 2.22

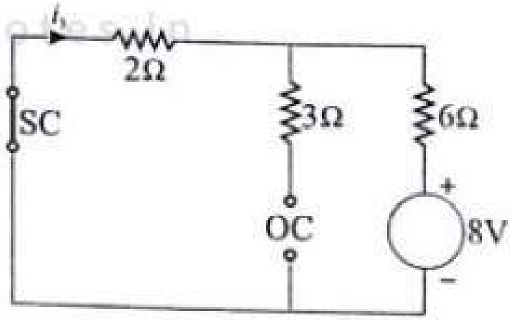


Fig 2.22

$$I_3 = \frac{-8}{6+2} = -1A$$

Step 4 : Total current through 2 Ω resistor is,

$$I = I_1 + I_2 + I_3$$

$$\Rightarrow I = -1.5 + 4.5 - 1 = 2A$$

Apply KCL to node -2

$$\frac{V_1 - V_2}{75} + \frac{0 - V_2}{25} + 50i + \frac{V_3 - V_2}{50} = 0$$

$$\Rightarrow \frac{V_1 - V_2}{75} - \frac{V_2}{25} + 50\left(\frac{V_1 - V_2}{75}\right) + \frac{V_3 - V_2}{50} = 0$$

$$\Rightarrow 102V_1 - 111V_2 + 3V_3 = 0 \text{.................} \quad (2)$$

Apply KCL to node - 3

$$\frac{V_2 - V_3}{50} + \frac{0 - V_3}{100} - 50i = 0$$

$$\Rightarrow -200V_1 + 206V_2 - 7V_3 = 0 \text{..............} \quad (3)$$

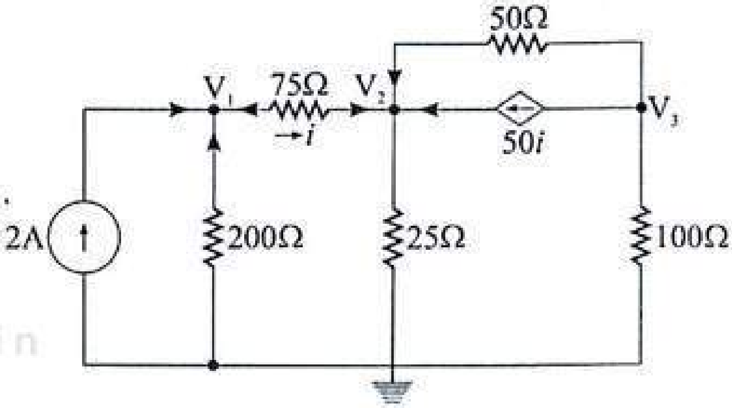


Fig 2.36

Solving these equations we get, $V_1 = 290 \cdot 41$ volts, $V_2 = 208 \cdot 219$ volts and $V_3 = -2169 \cdot 86$ volts.

Example 2.16 : Using node voltage analysis in the circuit of fig 2.37 find the currents I_1 and I_2. Given $R_1 = 3\,\Omega$, $R_2 = 1\Omega$ and $R_3 = 6\,\Omega$.

Solution : In fig 2.38,

Apply KCL to node-1,

$$1 - I_1 + V_2 - V_1 = 0$$

$$\Rightarrow 1 - \frac{V_1}{3} + V_2 - V_1 = 0$$

$$(\therefore\ I_1 = V_1 / 3)$$

$$\Rightarrow 4V_1 - 3V_2 = 3 \text{..................} \quad (1)$$

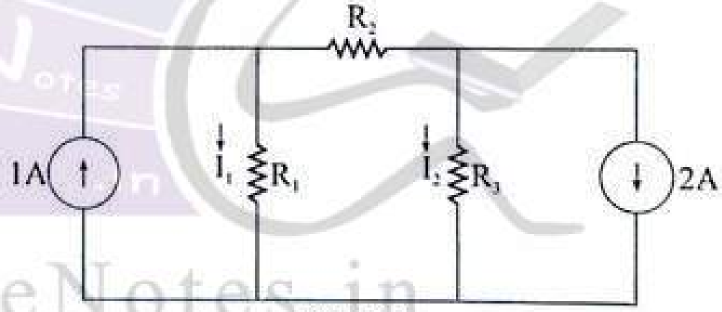


Fig 2.37

Apply to KCL to node -2

$$V_1 - V_2 - I_2 - 2 = 0$$

$$\Rightarrow V_1 - V_2 - \frac{V_2}{6} - 2 = 0$$

$$(\therefore I_2 = V_2 / 6)$$

$$\Rightarrow 6V_1 - 7V_2 = 12 \text{..............} \quad (2)$$

Solving eqn(1) and (2) we get, $V_1 = -1.5$ volt and $V_2 = -3$ volt.

So $I_1 = \frac{V_1}{3} = -0.5A$ and $I_2 = \frac{V_2}{6} = -0.5A$

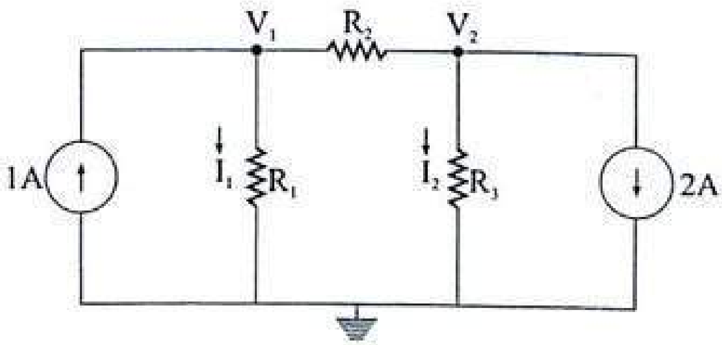


Fig 2.38

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Solution : Only voltage source V_{S2} in the circuit, suppress the voltage source V_{S1} by replacing it with a short circuit as shown in fig 2.55

$R_{eq}=R_1 \parallel R_3=500\Omega$

Let V_2 = Voltage across R_2

Applying voltage division rule,

$$V_2 = V_{S2}\frac{R_2}{R_2+R_{eq}} = 12\times\frac{1000}{1000+500} = 8V$$

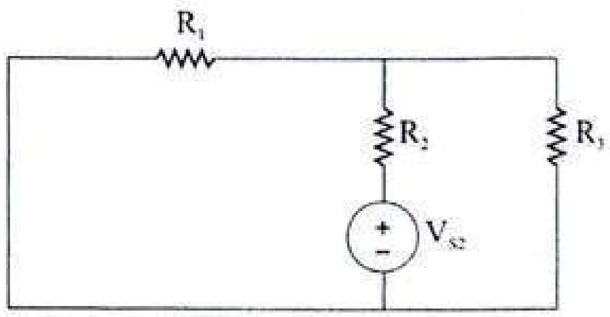

Fig 2.55

Only voltage source V_{S1} in the circuit, suppress the voltage source V_{S2} by replacing it with a short circuit as shown in fig 2.56

$R_{eq}=R_2 \parallel R_3=500\Omega$

Let V_1 = voltage across R_1

Applying voltage division rule

$$V_1 = -V_{S1}\frac{R_{eq}}{R_1+R_{eq}} = -12\times\frac{500}{1000+500} = -4V$$

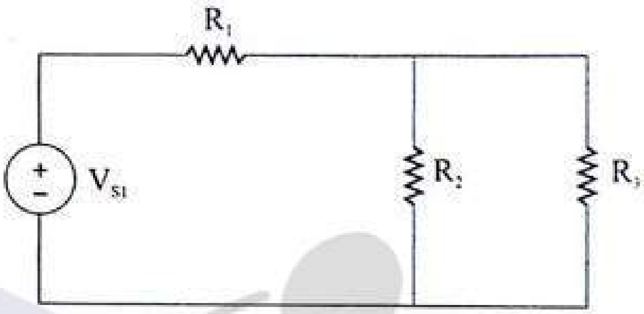

Fig 2.56

$\therefore$ voltage across R_2 is, $V = V_1 + V_2 = -4+8 = 4V$.

Example 2.27 The circuit shown in fig 2.57 is a simplified DC version of an AC three phase electrical distribution system.

$V_{S1} = V_{S2} = V_{S3} = 170V$

$R_{w1} = R_{w2} = R_{w3} = 0.7\Omega$

$R_1 = 1.9\Omega, R_2 = 2.3\Omega, \quad R_3 = 11\Omega$

find current through R_1, using superposition.

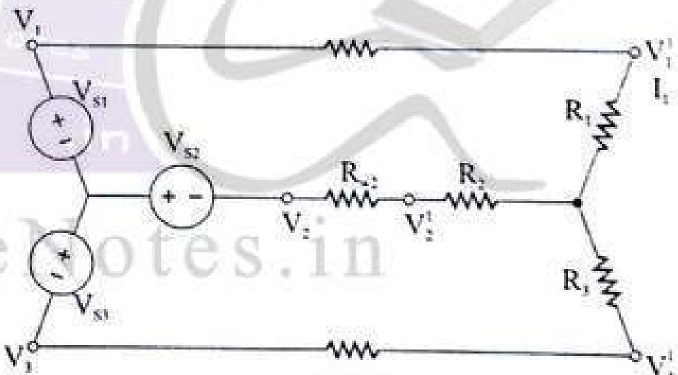

Fig 2.57

Solution : Only voltage source V_{S1} in the circuit, suppress V_{S2} and V_{S3} by replacing short circuits as shown in fig 2.58

$R_{eq1} = R_{w1} + R_1 = 2.6\Omega$

$R_{eq2} = R_{w2} + R_2 = 3\Omega$

$R_{eq3} = R_{w3} + R_3 = 11.7\Omega$

$$\therefore R_{eq} = R_{eq1} + \frac{R_{eq2}R_{eq3}}{R_{eq2}+R_{eq3}} = 4.99\Omega$$

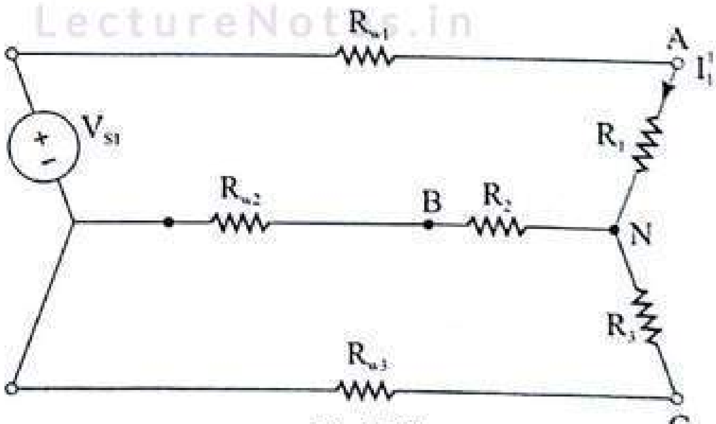

Fig 2.58

$$\Rightarrow\ 0.2\times 200-200i_1-75i_1-25i_1+25i_3=0$$

$$\Rightarrow -300i_1+25i_3=-40 \quad ....(5)$$

For mesh -2 $i_2 R_4 = 10$

$$\Rightarrow\ 50\, i_2 = 10$$

$$i_2=\frac{10}{50}=0.2\,A \quad ....(6)$$

For mesh - 3, $-(i_3-i_1)R_3-10=0$

$$\Rightarrow\ -(i_3-i_1)25=10$$

$$\Rightarrow\ 25i_1-25i_3=10 \quad ....(7)$$

Solving equations 5,6, and 7 we get,

$$i_1=0.109\ A,\quad i_2=0.2\,A\quad and\quad i_3=-0.291\,A$$

Therefore short- circuit current $I_{sc}=i_3=-0.291\,A$

Example 2.31 Find the Thevenin equivalent resistance seen by resistor R_5, as shown in figure 2.71

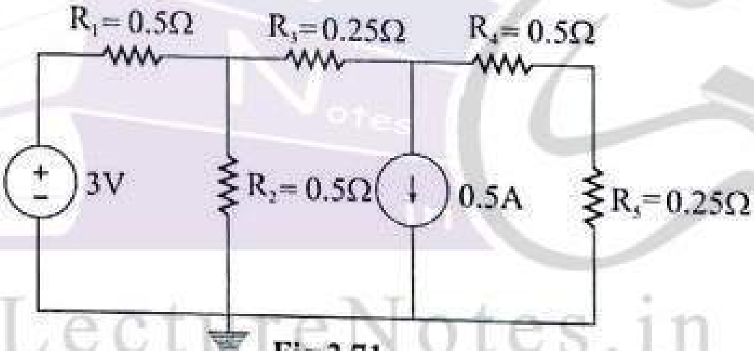

Fig 2.71

Solution : (i) Remove the load, leaving the load terminals open circuited, short the voltage source and open the current source.

$R_{th} = (0.5\ ||\ 0.5)+0.25+0.5=1\Omega$

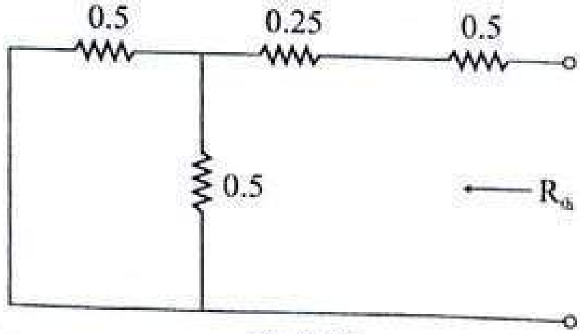

Fig 2.72

In fig 2.92 apply KCL to point 'a'

$$\frac{0-V}{60}+\frac{90-V}{60}+\frac{0-V}{30}=0$$

$\Rightarrow \quad V=22\cdot 5$ volts.

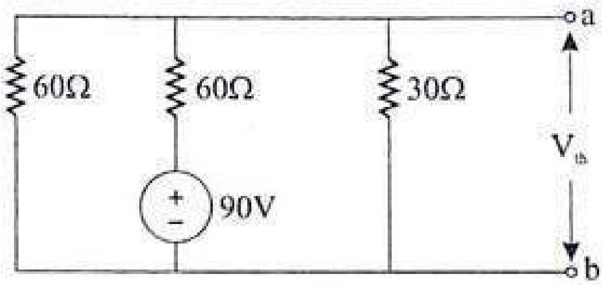

Fig 2.92

In fig 2.93

$$\frac{1}{R_{th}}=\frac{1}{60}+\frac{1}{60}+\frac{1}{30}=\frac{1}{15}$$

$\Rightarrow \quad R_{th}=15\Omega$

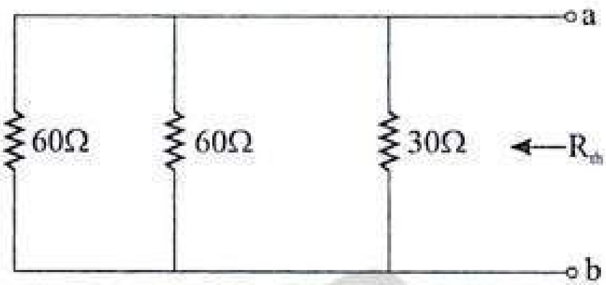

Fig 2.93

The Thevenin's equivalent circuit

is shown in fig 2.94

Powert = maximum = P

when $R_{th}=R=15\Omega$

$$\therefore \quad I=\frac{V_{th}}{R_{th}+R}=\frac{22\cdot 5}{15+15}=0\cdot 75A$$

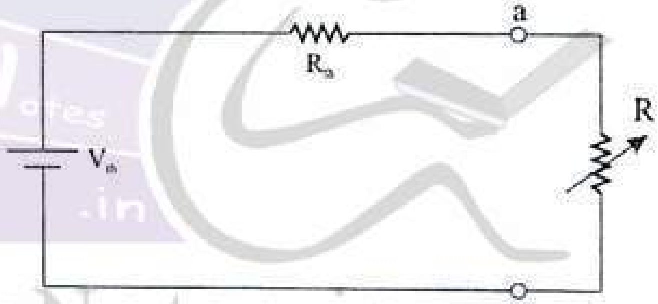

Fig 2.94

$P_{\max}=I^2R=8\cdot 44$ watts.

Example 2.11 : Use Thevenin's theorem to find current through 6Ω resistor in the circuit shown in fig 2.23

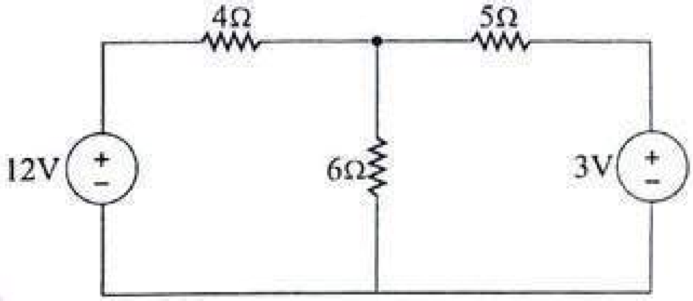

Fig 2.23

Solution :

Step-1: In this case load resistance $R_L = 6\Omega$, and load terminals AB is shown in fig 2.24. Remove R_L and apply KVL to the circuit shown in fig 2.25

$12 - 4I - 5I - 3 = 0$

$\Rightarrow I = 1A$

Then $V_{Th} = 12 - 4I = 12 - 4\times 1 = 8V$

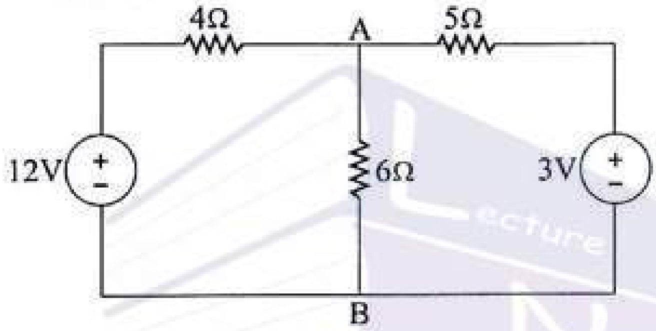

Fig 2.24

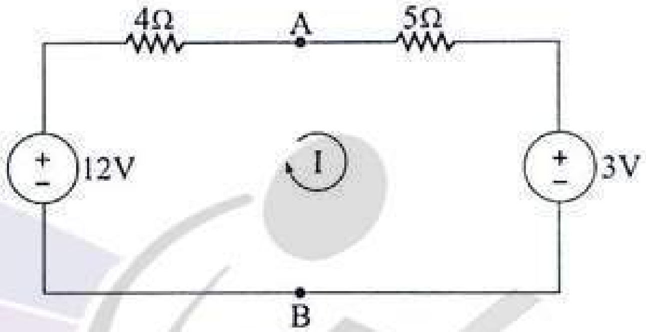

Fig 2.25

Step-2 : To determine R_{th}, the voltage sources are short circuited as shown in fig 2.26

The resistance seen at terminals AB is equal to the parallel combination of 4Ω and 5Ω resistance.

$$\therefore R_{Th} = \frac{4\times 5}{4+5} = 2.22\Omega$$

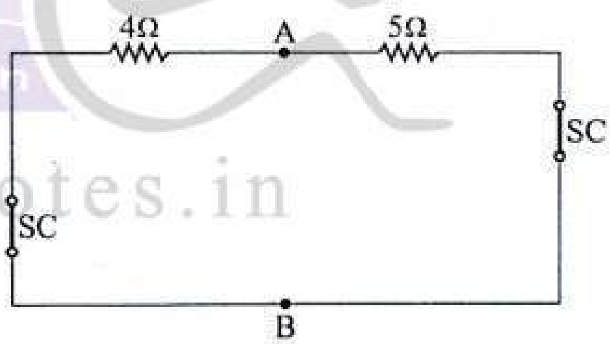

Fig 2.26

Step-3 :

The Thevenin's equivalent circuit is shown in fig 2.27.

Step-4 :

Current flowing through 6Ω is

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{2.22+6} = 0.973\,A$$

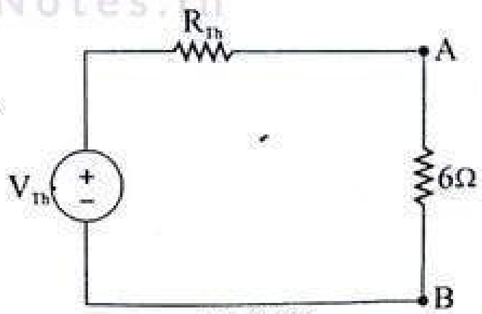

Fig 2.27

Example 2.17 : Use mesh analysis determine the currents I_1 and I_2 in the circuit shown in fig 2.39

Solution

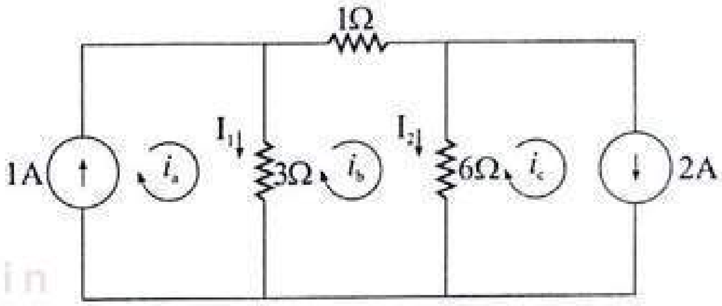

Fig 2.39

At mesh- a : $i_a = 1A$

At mesh- b : $3(i_b - i_a) + i_b \times 1 + 6(i_b - i_c) = 0$

At mesh- c : $i_c = 2A$

Solving we find that, $I_b = 15A$

$$\therefore I_1 = i_a - i_b = -05A$$

$$I_2 = i_b - i_c = -05A$$

Example 2.18 : Using node voltage analysis in the circuit of fig 2.40 find the current I through the voltage source. $R_1 = 100\Omega$, $R_2 = 5\Omega$, $R_3 = 200\Omega$, $R_4 = 50\ \Omega$, $V = 50V$, $I = 0.2A$

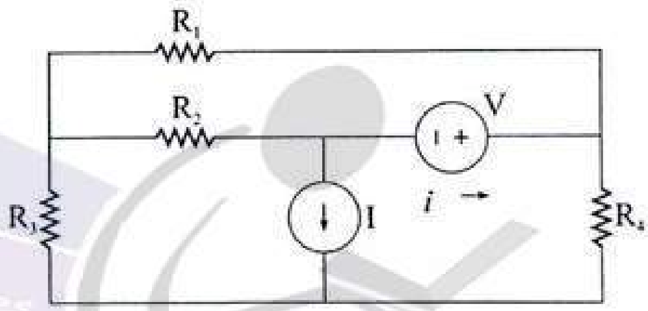

Fig 2.40

Solution : In fig 2.41, Apply KCL to node - 1,

$$\frac{O - V_1}{R_3} + \frac{V_2 - V_1}{R_2} + \frac{V_3 - V_1}{R_1} = 0$$

$$\Rightarrow \frac{-V_1}{200} + \frac{V_2 - V_1}{5} + \frac{V_3 - V_1}{100} = 0 \ \ldots\ldots\ (1)$$

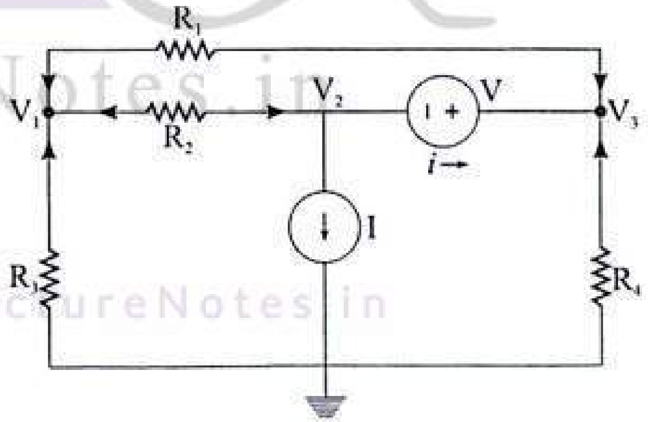

Fig 2.41

Apply KCL to node - 2,

$$\frac{V_1 - V_2}{R_2} - i - I = 0$$

$$\Rightarrow \frac{V_1 - V_2}{5} - i - 0.2 = 0 \ \ldots\ldots\ldots\ldots\ (2)$$

Apply KCL to node - 3,

$$\frac{V_1 - V_3}{R_1} + \frac{0 - V_3}{R_4} + i = 0$$

$$\Rightarrow \frac{V_1 - V_3}{100} - \frac{V_3}{50} + i = 0 \ \ldots\ldots\ldots\ldots\ (3)$$

For voltage source we have $V_3 - V_2 = 50$ ... (4)

Solving these equations we get $V_1 = -4553V$, $V_2 = -48.69V$, $V_3 = 131V$ *and* $i = 491mA$.

$\therefore$ Current through R_1 is, $I_1^1 = \frac{V_{S1}}{R_{eq}} = \frac{170}{4.99} = 34.08\,A$ (A to N)

Only voltage source V_{s2} in the circuit, suppress V_{S1} and V_{S3}

by replacing short circuits. as shown in fig 2.59

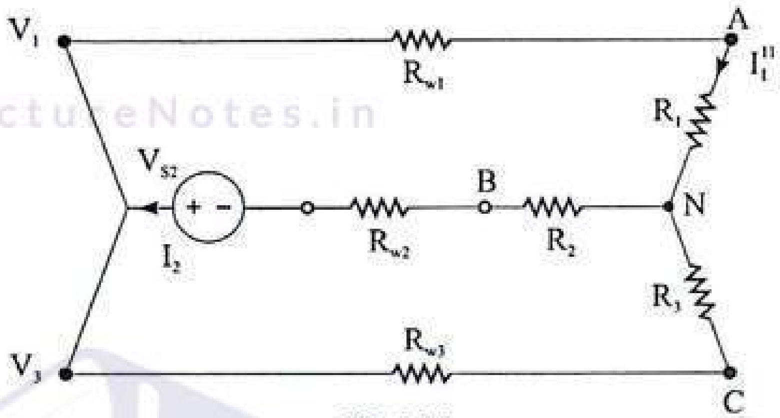

Fig 2.59

$R_{eq}=R_{eq2} \parallel R_{eq3} = 3 + \frac{2.6\times 11.7}{2.6+11.7} = 5.127\,\Omega$

Therefore $I_2 = \frac{V_{S2}}{R_{eq}} = 33.157\,A$

Applying current division rule,

Current through $R_1 = I_1^{11} = I_2 \times \frac{R_{eq3}}{R_{eq1}+R_{eq3}} = 27.12\,A$

Only voltage source V_{S3} in the circuit, suppress V_{S1} and V_{S2} by replacingshort circuits as shown in fig 2.60

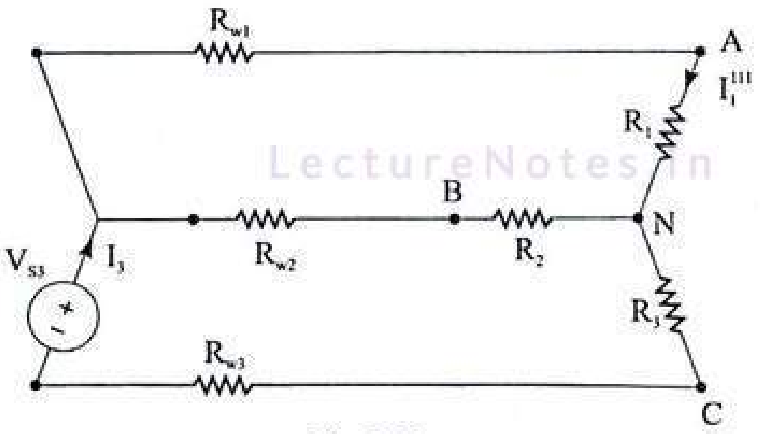

Fig 2.60

$R_{eq}=R_{eq3}+R_{eq2} \parallel R_{eq1} = 11.7 + \frac{3\times 2.6}{3+2.6} = 13.092\,A$

Example 2.32 For the circuit shown in fig 2.73 obtain Norton's equivalent circuit across ab.

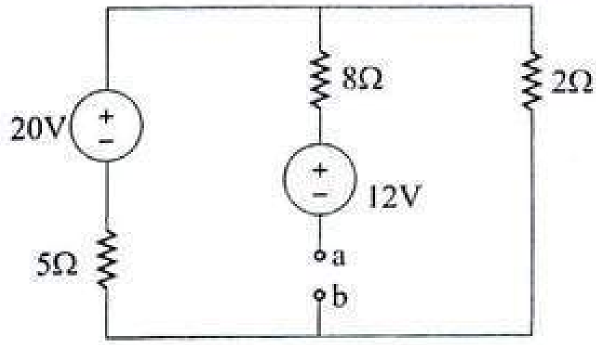

Fig 2.73

Solution : (1) To find I_N :

The terminals a and b are short circuited. Current flowing through the short circuited ab is Norton's equivalent current I_N.

Let us find I_N using node voltage method

Applying KCL to node -1,

$$\frac{20-V}{5}+\frac{12-V}{8}+\frac{0-V}{2}=0$$

$$\Rightarrow 160-8V+60-5V-20V=0$$

$$\Rightarrow 220=33V$$

$$\Rightarrow V=\frac{220}{33}=6.67V$$

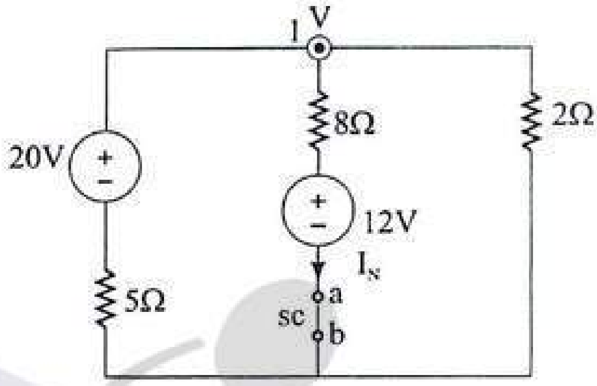

Fig 2.74

Therefore $I_N=\frac{-12+6.67}{8}=-0.667A=0.667A$ (direction of current from b to a).

(ii) To find R_N :

$R_N=8+(5 \parallel 2)=9.43\Omega$

Norton's equivalent circuit can be drawn as shown in fig 2.75

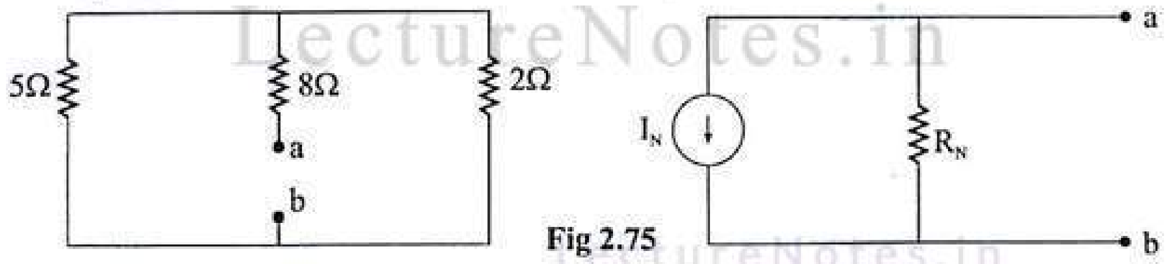

Fig 2.75

Example 2.33 Obtain the Norton equivalent circuit for the circuit shown in fig 2.76.

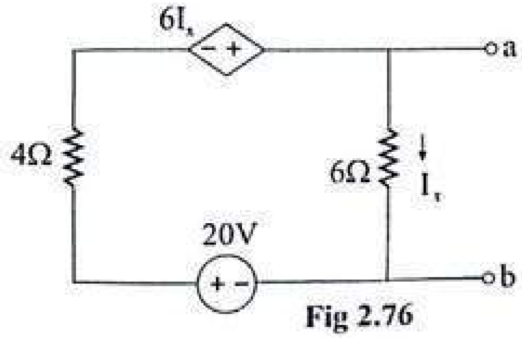

Fig 2.76

Example 2.12 : Find Thevenin's equivalent for the circuit shown in fig 2.28

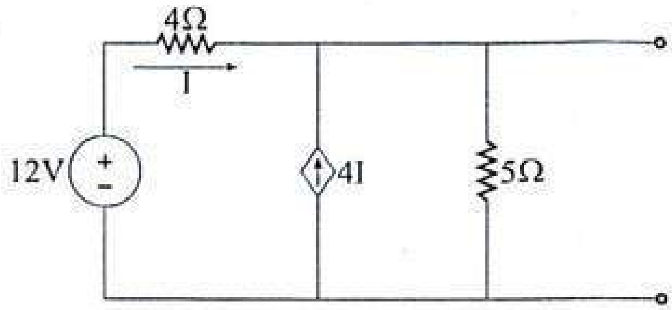

Fig 2.28

Solution :

Step-1 : First the current source 4I in parallel with 5Ω resistor is converted into a voltage source, $4I \times 5 = 20I$, in series with 5Ω resistor as shown in fig 2.29

Applying KVL,

$$12 - 4I - 5I - 20I = 0$$

$$\therefore \quad I = 0.414\ A$$

$$V_{Th} = 12 - 4I = 12 - 4 \times 0.414 = 10344V$$

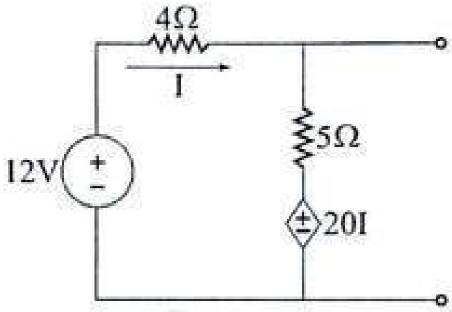

Fig 2.29

Step-2 : As a dependent current source is present in the circuit shown in fig. 2.24, R_{Th} Can not be found by short-circuiting the voltage source. For this, the terminals A and B are short-circuited as shown in fig 2.30

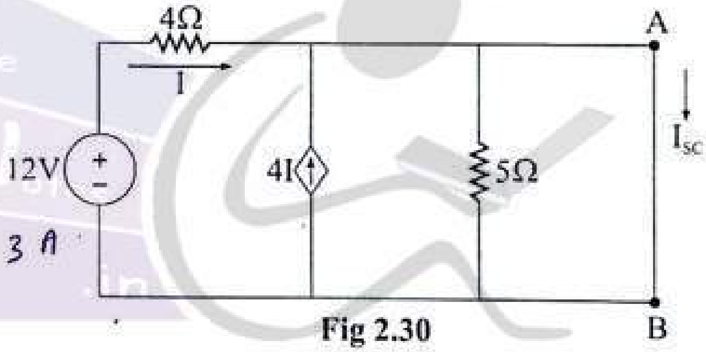

Fig 2.30

Apply KCL to node A, we get, $I + 4I = I_{sc}$

4Ω resistor is across 12 V source $\therefore I = \frac{12}{4} = 3A$

$$\Rightarrow I_{sc} = 5I$$ $= 5 \times 3 = 15A$

$$\therefore R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{10.344}{5I} = \frac{10.344}{5 \times 0.414} = 4.997\ \Omega$$

15 $= 0.6896\ \Omega$

2.4.3 Norton's Theorem

Norton's theorem states that ***any pair of terminals AB of a network with linear passive and active elements may be replaced by an equivalent current source I_N in parallel with an equivalent resistance R_N. The value of I_N (called Norton current) is equal to the current that would flow from A to B when the terminals A and B are short-circuited and R_N is the equivalent resistance looking into the network at AB with the independent active sources set to Zero i.e. with all the voltage sources shorted and all the current sources open-circuited leaving behind their internal resistance.***

These are the following steps for solving a network by Norton's theorem.

Step-I : Mark the pair of terminals AB of the network. Remove the load resistance R_L across the terminals AB and find the internal resistance (R_N) of the source network by deactivating the constant sources (for voltage sources, remove it by internal resistance and for current source delete the source by open circuit).

Example 2.19 : Using node voltage analysis in the circuit of fig 2.42 find the current I drawn from the independent voltage source.

$V=3V,\ R_1=\frac{1}{2}\Omega,\ R_2=\frac{1}{2}\ \Omega,$

$R_3=\frac{1}{4}\Omega\ \ R_4=\frac{1}{2}\Omega,\ R_5=\frac{1}{4}\ \Omega,\ I=05A$

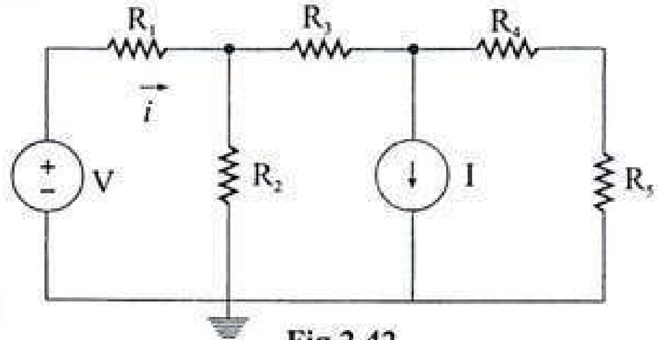

Fig 2.42

Solution : In fig 2.43,

Apply KCL to node -1

$$\frac{3-V_1}{0.5}+\frac{0-V_1}{0.5}+\frac{V_2-V_1}{0.25}=0 \text{ .... (1)}$$

Apply KCL to node -2

$$\frac{V_1-V_2}{0.25}-0.5+\frac{0-V_2}{0.75}=0 \text{ .... (2)}$$

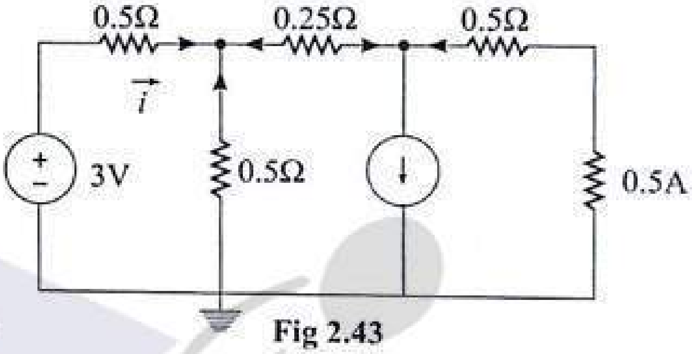

Fig 2.43

Solving these two equations we get,

$V_1=1.125V$ and $V_2=0.75V$

$$\therefore\ i=\frac{3-V_1}{R_1}=\frac{3-1.125}{0.5}=3.75A.$$

Example 2.20 Calculate the branch currents of the circuit shown in fig 2.44 by nodal voltage method.

Solution : In fig 2.45, Apply KCL to the node-1,

$$\frac{20-V_1}{5}+\frac{0-V_1}{10}+\frac{8-V_1}{2}=0$$

$\Rightarrow\ 80-8V_1=0$

$\Rightarrow\ V_1=10V$

$\therefore$ Current through

$$5\ \Omega=\frac{20-V_1}{5}=\frac{20-10}{5}=2A$$

Current through

$$2\ \Omega=\frac{8-V_1}{2}=\frac{8-10}{2}=-1A$$

Negative sign indicates that current through 2 Ω flows away from the node -1.

Current through $10\ \Omega=\frac{0-V_1}{10}=-1A$

Here also current through 10 Ωflows away from node -1.

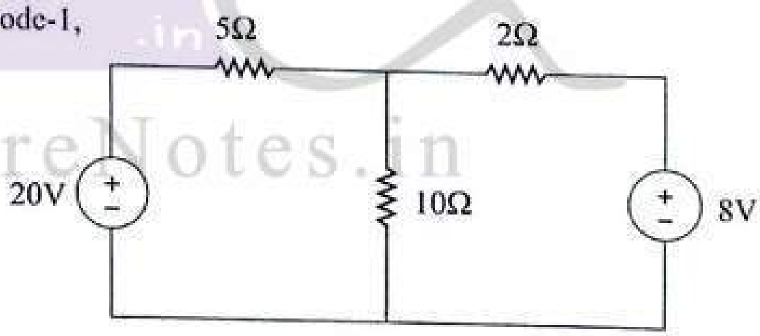

Fig 2.44

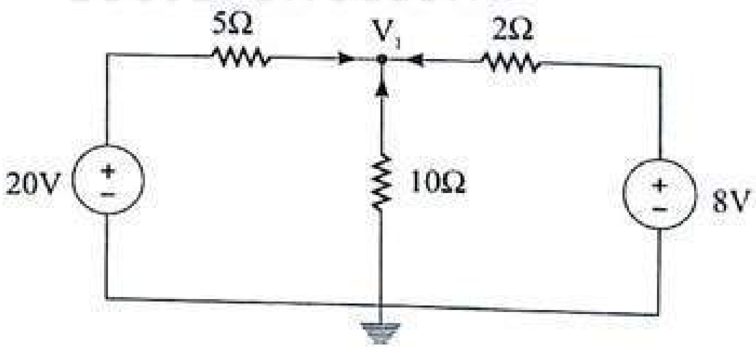

Fig 2.45

Therefore $I_3 = \frac{V_{S3}}{R_{eq}} = \frac{170}{13.092} = 12.985A$

Applying current division rule.

Current through R_1 is,

$$I_1^{III} = I_3 \times \frac{R_{eq2}}{R_{eq1} + R_{eq2}} = 12.985 \times \frac{3}{2.6+3} = 6.956A$$

Total current through $R_1 = I_1^{I} + I_1^{II} + I_1^{III} = 68.156A$

Example : 2.28 : Using superposition theorem, find the current through the voltage source'V', shown is fig 2.61

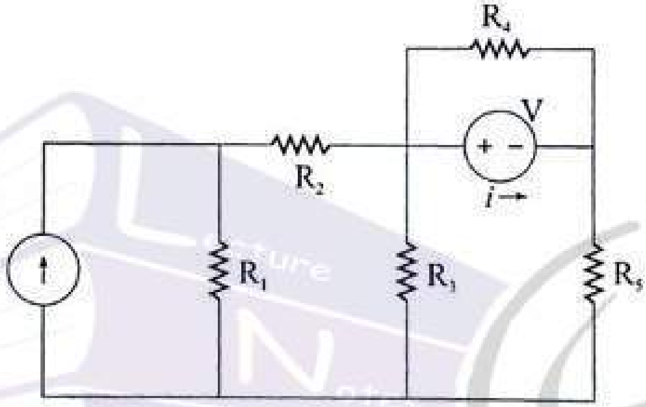

Fig 2.61

Given, I = 0.2$R_1 = 200\Omega$, $R_2 = 75\Omega$, $R_3 = 25\Omega$, $R_4 = 50\Omega$, $R_5 = 100\Omega$, V=10 volts.

Solution : Only current source (I) in the circuit, suppress voltage source 'v' by replacing it short circuit. convert current source (I) parallel with R_1, into voltage sourc connected in series with R_1 as shown in fig 2.62

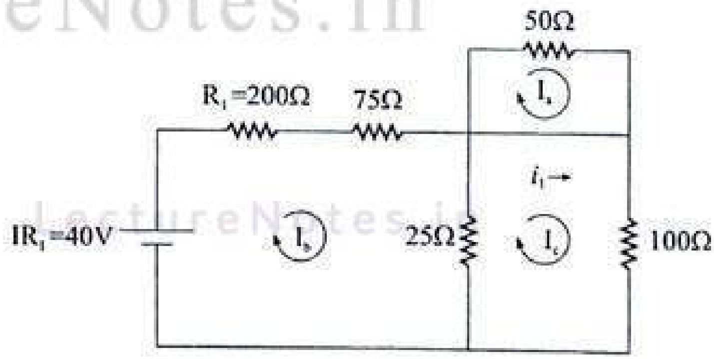

Fig 2.62

Applying KCL to mesh (a), $i_a = 0$

Mesh (b) $40 - 200i_b - 75i_b - 25(i_b - i_c) = 0$
$\Rightarrow \quad 40 - 300i_b + 25i_c = 0$ ....(1)

Mesh (c)) $-100i_c - 25(i_c - i_b) = 0$.
$\Rightarrow \; i_b = 5i_c$ ....(2)

Solving equations (1) and (2) we get,

$i_b = 0 \cdot 13559A$ and $i_c = 27 \times 10^{-3} A$

$\therefore \quad i_1 = i_c = 27 \times 10^{-3} A$

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and delivery.

Solution : (I) To find I_N :

The terminals a and b are short circuited. The current flowing through short circuited ab is called Norton current (I_N).

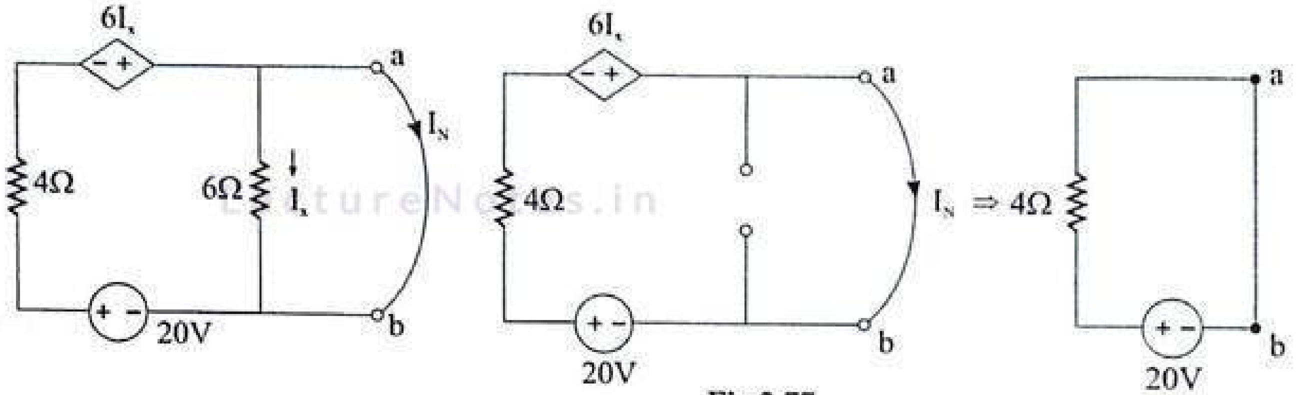


Fig 2.77

As the terminals a and b are short circuited, current through 6Ω resistance is zero i.e. $I_x = 0$.

Therefore value of controlled source is zero.

Norton current $I_N = \frac{20}{4} = 5A$

(ii) To find out R_N :

To find norton resistance R_N, First calculate the open circuit voltage across the terminals a and b.

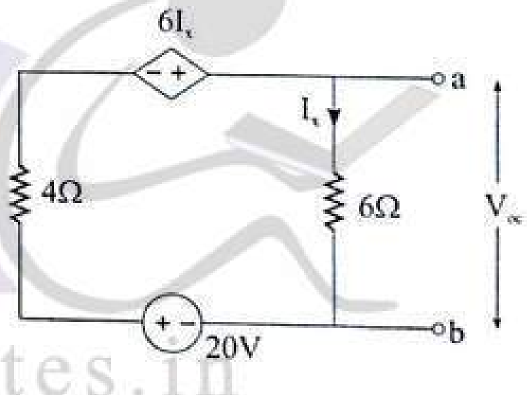


Fig 2.78

Apply KVL, $20 - 4I_x + 6I_x - 6I_x = 0$

$$\Rightarrow I_x = \frac{20}{4} = 5A$$

$\therefore$ Open circuit voltage $V_{oc} = 6I_x = 6 \times 5 = 30V$.

Norton resistance $R_N = \frac{V_{oc}}{I_N} = \frac{30}{5} = 6\Omega$

Example 2.34 Under no load condition a d.c generator has a terminal voltage of 120 V, when deliver its rated current of 40 ampere the terminal voltage drops to 112 Volts.

Find the Norton's equivalent circuit.

Solution : Under no load condition (i.e. open circuit) the terminal voltage is 120 volts.

Let R_a is the resistance of the armature.

Under load condition the terminal voltage is 112 Volts and load current I_L is 40 A.

Step-2 : Next, Short the terminals AB and find the short-circuit current flowing through the short terminals AB. Let this current be I_N.

Step-3 : Obtain Norton's equivalent circuit by playing R_N in parallel with I_N as shown in fig 2.31

Step-4 : Reconnect R_L across terminals AB and calculate the current flowing through it.

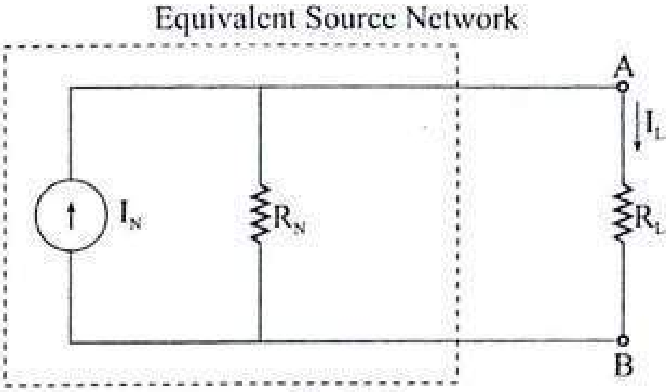


Fig 2.31

By Current division rule, $I_L = I_N \dfrac{R_N}{R_N + R_L}$

2.4.4 Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load.

Statement : ***It states that in dc circuits maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f sources replaced by their internal resistances.***

Proof : Let us consider a simple resistivie network in which a load resistance (R_L) is connected across terminals A and B of the network as shown in fig 2.32 (a).

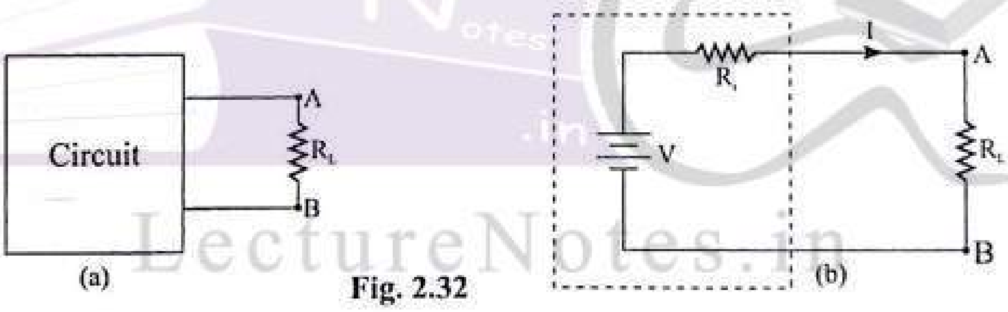


Fig. 2.32

The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin Voltage $(V_{Th} = V)$ in series with Thevenin resistance $(R_{Th} = R_i)$ as shown in fig 2.32 (b).

Clearly R_i is the resistance measured between terminals AB with R_L removed and emf sources replaced by their internal resistances. According to this theorem maximum power will be transfer from source to load when $R_L = R_i$. To proof this theorem let us assume that current I flows through R_L as shown in fig 2.32 (b).

$$\therefore \ I = \frac{V}{R_i + R_L}$$

Power across the load $(P_L) = I^2 R_L = \left(\dfrac{V}{R_i + R_L}\right)^2 R_L$

Example 2.21 Solve the circuit as shown in fig 2.46 by node voltage method.

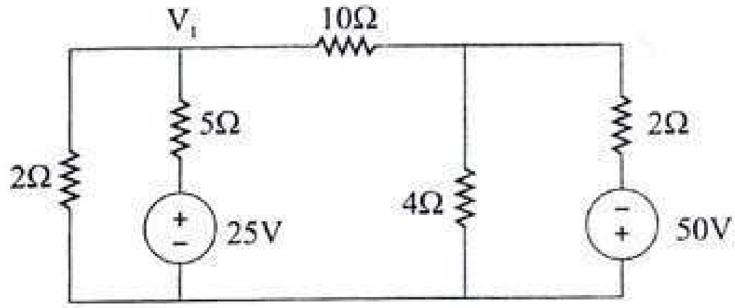


Fig 2.46

Solution : The circuit has two principal nodes numbered 1 and 2 and the third chosen as reference node as shown in fig 2.47.

Apply KCL to node-1,

$$\frac{0-V_1}{2}+\frac{25-V_1}{5}+\frac{V_2-V_1}{10}=0 \text{..........} (1)$$

Apply KCL to node-2

$$\frac{V_1-V_2}{10}+\frac{0-V_2}{4}+\frac{-50-V_2}{10}=0 \text{ ......} (2)$$

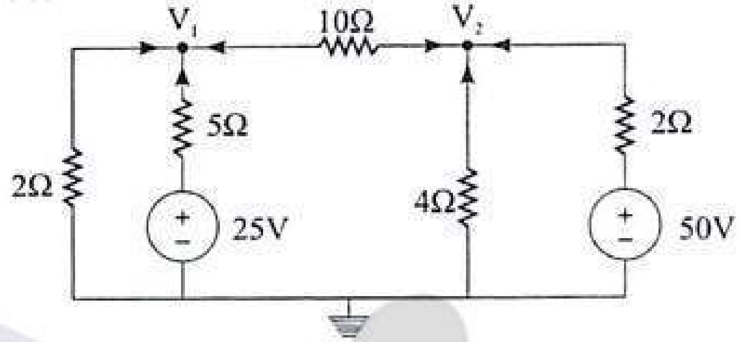


Fig 2.47

Solving these two equations we get,

$V_1 = 2.61V$ *and* $V_2 = -29.1V$

Current through

$$2\ \Omega = \frac{0-V_1}{2} = \frac{-2.61}{2} = -1.31A$$

Negative sign indicates that current through 2 Ω flows away from node-.

$$\text{Current through } 10\ \Omega = \frac{V_1-V_2}{10} = 3.17\ A$$

$$\text{Current through } 2\ \Omega = \frac{V_2+50}{2} = 10.45A$$

Example 2.22 : Calculate I_1 in fig 2.48 by mesh current method.

Solution : Applying KVL to mesh-1,

$$60 = 7I_1 + 10(I_1 - I_2)$$

$$\Rightarrow \ 19I_1 - 12I_2 = 60 \text{ ....} \quad (1)$$

Applying KVL to mesh -2,

$$0 = 12(I_2 - I_1) + 6(I_2 - I_3)$$

$$\Rightarrow -12\,I_1 + 18I_2 - 6I_3 = 0 \text{ ...}(2)$$

Applying KVL to mesh - 3,

$$0 = 6(I_3 - I_2) + 12I_3$$

$$\Rightarrow \ -6I_2 + 18I_3 = 0 \text{ ....} \ (3)$$

Solving these three equations we get $I_1 = 6A$.

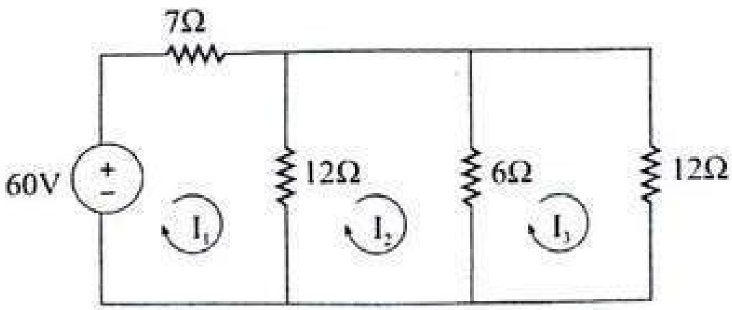


Fig 2.48

Only voltage source (v) in the circuit, suppress current source (I) by replacing it open circuit as shown in fig 2.63

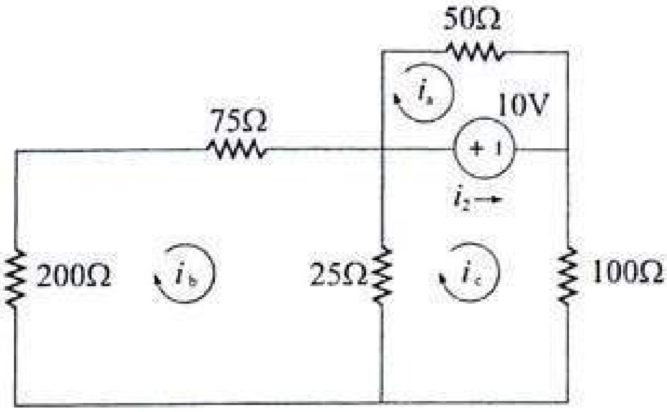


Fig 2.63

Applying KCl,

mesh (a) : $10-50i_a=0 \Rightarrow i_a=\frac{1}{5}A$

mesh (b) : $-200i_b-75i_b-25(i_b-i_c)=0$
$\Rightarrow -300i_b+25i_c=0 \rightarrow (3)$

mesh (c)) $-100i_c-25(i_c-i_b)-10=0$
$\Rightarrow -125i_c+25i_b-10=0 \rightarrow (4)$

Solving equations (3) and (4) we get,

$$i_b=-6.8\times10^{-3}A \text{ and } i_c=-81.6\times10^{-3}A$$

therefore $i_2=i_c-i_a=-281\times10^{-3}A$

Using superposition theorem, $i=i_1+i_2=-254\times10^{-3}A$.

Example 2.29 : Using superposition theorem find the current i drawn from the independent voltage source as shown in fig 2.64

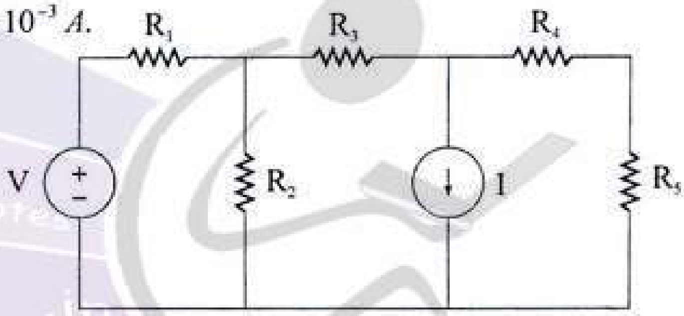


Fig 2.64

V = 3V, $R_1=0.5\Omega$, $R_2=0.5\Omega$, $R_3=0.25\Omega$, $R_4=0.5\Omega$, $R_5=0.25\Omega$, $I=0.5A$

Solution : Only current source (I) in the circuit, suppress voltage source (v) by replacing it short circuit as shown in fig 2.65

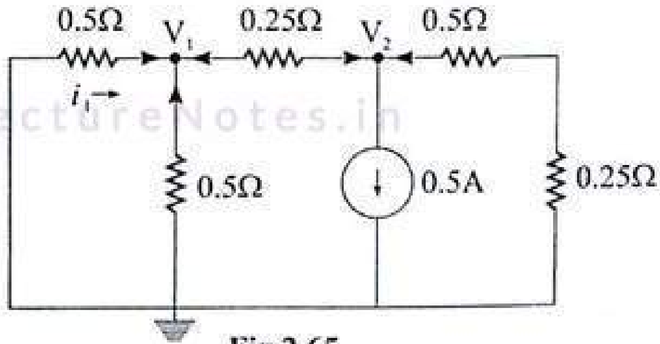


Fig 2.65

At node -1 : 2.67

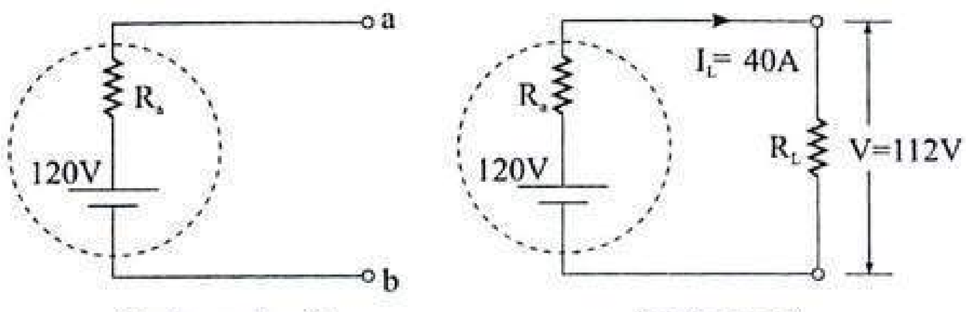
$$\frac{0-V_1}{0.5}+\frac{0-V_1}{0.5}+\frac{V_2-V_1}{0.25}=0$$

$$\Rightarrow -2V_1+V_2=0 \text{ ....(1)}$$

At node -2 :

$$\frac{V_1-V_2}{0.25}+\frac{0-V_2}{0.75}-0.5=0.$$

$$\Rightarrow 4V_1-5.333V_2=0.5 \text{ ....(2)}$$



(under no load) (under load)

Fig 2.79

$$\therefore R_a = \frac{120-112}{40} = 0.2\Omega$$

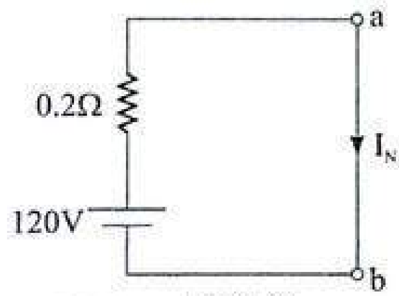


Fig 2.80

(i) To find Norton's current I_N :

To find I_N short the two terminals a and b .

$$\therefore I_N = \frac{120}{0.2} = 600\,A \text{ flows from a to b.}$$

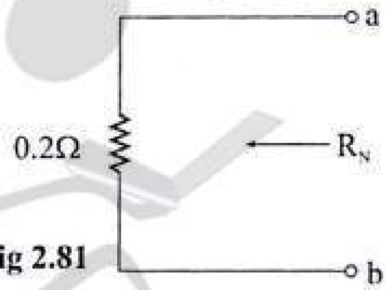


Fig 2.81

(ii) To find R_N :

To find Norton's resistance R_N, open circuited the two terminals a and b and short circuited the voltage source.

$$\therefore R_N = 0.2\Omega$$

(iii) Norton's equivalent circuit can be drawn as shown below. As I_N flows a to b, so its direction be upward.

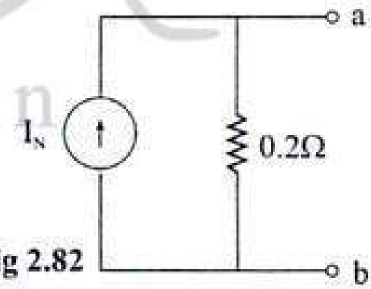


Fig 2.82

Example 2.35 Find current I shown in fig 2.83 by Norton's theorem.

Solution : (i) To find Norton's current I_N :

To find I_N short the two terminals a and b.

Let V be the Voltage at node -1 .

From the figure , V= 460 Volt.

Current flowing through

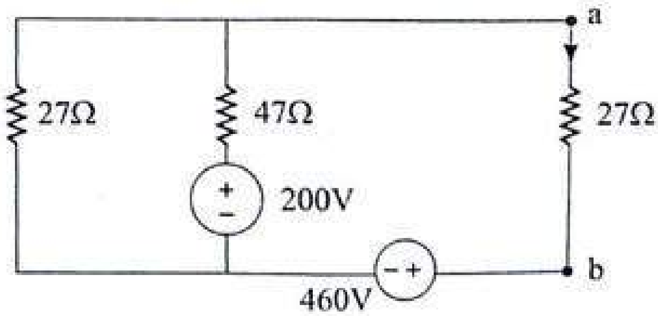


Fig 2.83

$$\frac{dP_L}{dR_L} = V^2\left[\frac{(R_i+R_L)^2-2R_L\ (R_i+R_L)}{(R_i+R_L)^4}\right]$$

For maximum P_L, $\frac{dP_L}{dR_L} = 0$

$$\Rightarrow V^2\left[\frac{(R_i+R_L)^2-2R_L\ (R_i+R_L)}{(R_i+R_L)^4}\right] = 0$$

$$\Rightarrow R_i + R_L = 2R_L$$

$$\Rightarrow R_L = R_i$$

Thus for maximum power transfer, $R_L = R_i$

The maximum power $= (P_L)_{max} = I^2 R_L$

$$= \left(\frac{V}{R_i + R_L}\right)^2 R_L = \frac{V^2}{4R_L}$$

The power delivered by the source is $(V\,I) = V\cdot\left(\frac{V}{R_i+R_L}\right) = V\left(\frac{V}{R_L+R_L}\right) = \frac{V^2}{2R_L}$

So the efficiency under maximum power transfer condition $= \frac{V^2/4R_L}{V^2/2R_L} = \frac{1}{2}$ (*or* 50%)

Note : Under the conditions of maximum power transfer, the efficiency is only 50% as one-half of the total power generated is dissipated in the internal resistance (R_i) *of the source.*

Example 2.13 : In the circuit shown in fig 2.33 the mesh currents are

$I_1 = 5A,\ I_2 = 3\ A,\ I_3 = 7\ A$. Determine the branch currents through $a. R_1\ \ b. R_2\ \ c. R_3$

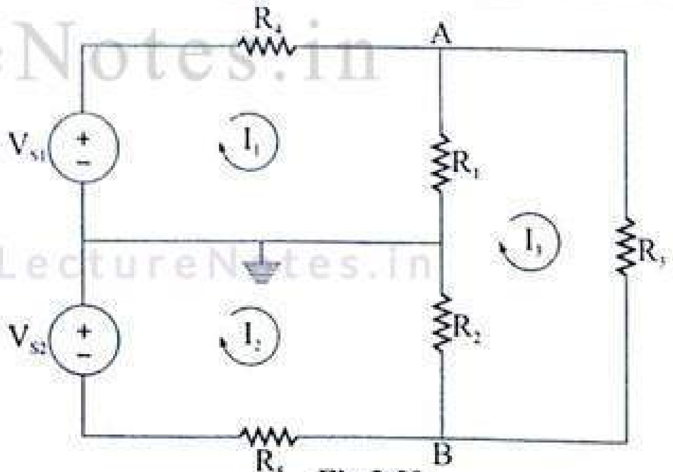


Fig 2.33

Solution :

a) Assume a direction of the current through R_1 i.e. from node A to node B. Applying KCL to node A, we get $I_1 - I_{R1} - I_3 = 0$

$$\Rightarrow I_{R1} = I_1 - I_3 = 5 - 7 = -2A$$

b) Assume a direction for the current through R_2 i.e. from node B to node A. Applying KCL to node B we get $-I_{R2} - I_2 + I_3 = 0$

$$\Rightarrow I_{R2} = I_3 - I_2 = 7 - 3 = 4A$$

c) Only one mesh current flows through R_3. If the current flows through R_3 is assured to flow in the same direction then, $I_{R1} = I_3 = 7A$

Example 2.23 : Find current I in the circuit shown in fig 2.49 (a)

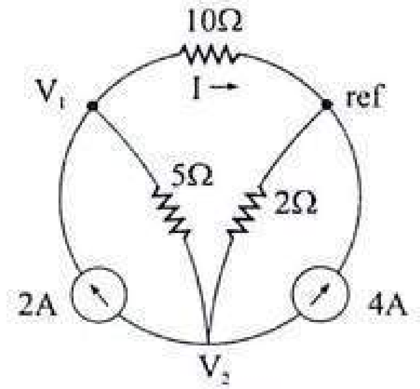


Fig 2.49 (a)

Solution : In figure 2.49 (b)

Apply KCL to node -1,

$$2+\frac{V_2-V_1}{5}+\frac{0-V_1}{10}=0$$

$$\Rightarrow \quad 3V_1-2V_2=20 \text{ ....(1)}$$

Apply KCL to node -2,

$$-2-4+\frac{V_1-V_2}{5}+\frac{0-V_2}{2}=0$$

$$\Rightarrow \quad 2V_1-7V_2=60 \text{ ....(2)}$$

Solve equations (1) and (2) we get,

$V_1=1.18V$ and $V_2=-8.235V$

$$\therefore \; I_1=\frac{V_1}{10}=\frac{1.18}{10}=0.118\,A$$

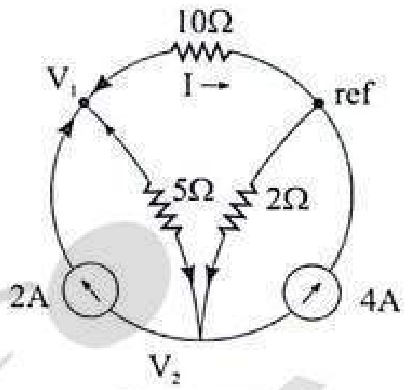


Fig 2.49 (b)

Example 2.24 : Find the voltage V_{ab} in the network shown in fig 2.50

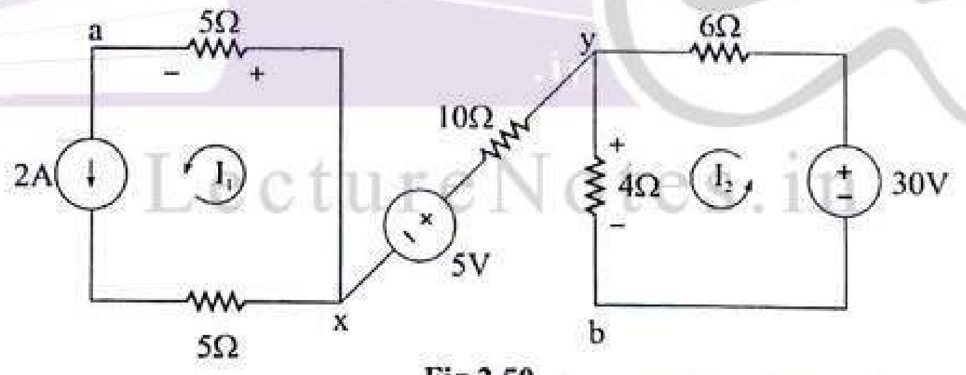


Fig 2.50

Solution : The two closed loops are independent and no current can pass through the connecting branch.

$$I_1=2\,A,\; I_2=\frac{30}{6+4}=3\,A$$

$$V_{ab}=V_{ax}+V_{xy}+V_{yb}=-I_1(5)-5+I_2(4)=-3V$$

Solving equations (1) and (2) we get,

$$V_1 = -0.074V \text{ and } V_2 = -0.15\text{ volt.}$$

Therefore $i_1 = \frac{0-V_1}{0.5} = \frac{0-(-0.074)}{0.5} = 0.148\,A.$

Only voltage source (v) in the circuit and suppress current source by replacing it open circuit as shown in fig 2.66

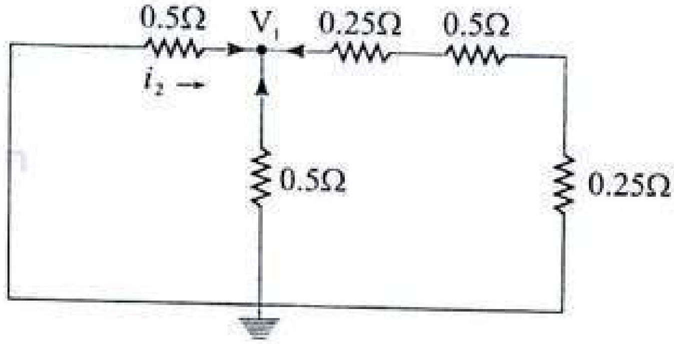


Fig 2.66

At node -1 :

$$\frac{3-V_1}{0.5} + \frac{0-V_1}{0.5} + \frac{0-V_1}{1} = 0$$

$$\Rightarrow V_1 = 1.2V$$

Therefore, $i_2 = \frac{3-V_1}{0.5} = \frac{3-1.2}{0.5} = 3.6\,A$

Using Principle of superposition, $i = i_1 + i_2 = 3.748\,A$

Example 2.30 : Find the Thevenin equivalent resistance seen by resistor R_5 the Thevenin (open - circuit) voltage and the Norton (short-circuit) current when R_5 is the load in fig 2.67

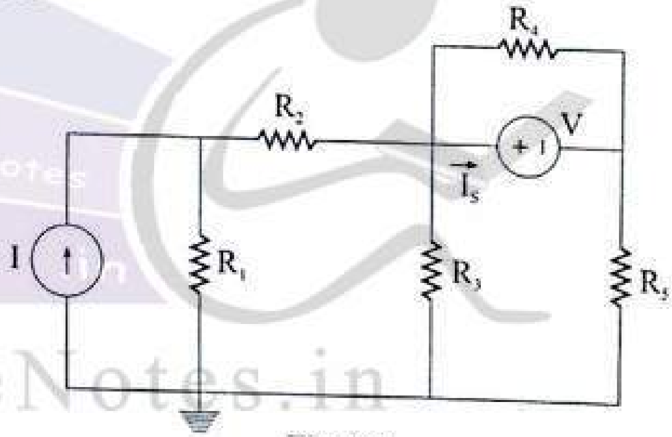


Fig 2.67

$I = 0.2A,\ R_1 = 200\Omega,\ R_2 = 75\Omega,\ R_3 = 25\Omega,\ R_4 = 50\Omega,\ R_5 = 100\Omega,\ V = 10V$

Solution : (1) Remove the load resistance R_5, short circuit the voltage source and open circuit the current source as shown in fig 2.68

Thevenin equivalent

resistance $R_{th} = (R_1 + R_2) \,||\, R_3$

$\therefore\ R_{th} = (200+75) \,||\, 25 = 275 \,||\, 25 = 22.916\Omega$

(2) Remove the load resistance R_5.

Apply KCL to three nodes as shown in fig 2.69

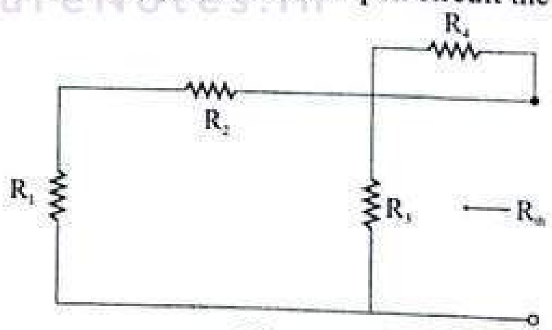


Fig 2.68

$27\,\Omega$ is $I_1 = \frac{V-0}{27} = \frac{460-0}{27} = 17.03\,A$

Current flowing $47\,\Omega$ is

$$I_2 = \frac{V-200}{47} = \frac{460-200}{47} = 5.53\,A$$

Therefore $I_N = I_1 + I_2 = 17.03 + 5.53 = 22.56\,A$, flowing from b to a.

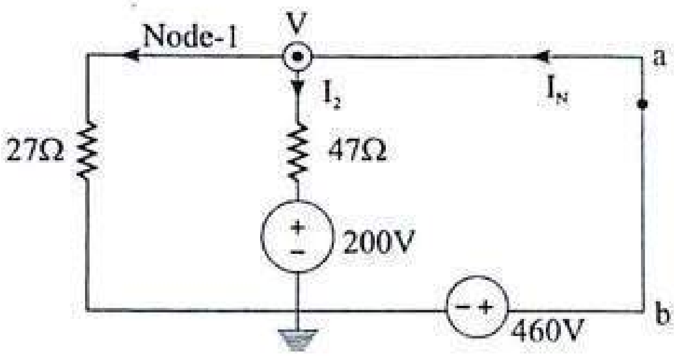


Fig 2.84

(ii) To find Norton's resistance R_N :

To find R_N, short the voltage sources and open the two terminals a and b.

$R_N = 27 \parallel 47 = 17.1\Omega$

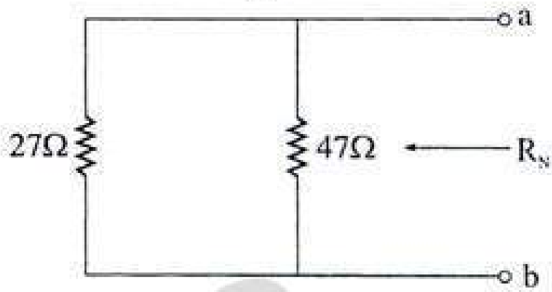


Fig 2.85

(iii) To find **I** :

The Norton's equivalent circuit shown in figure 2.86

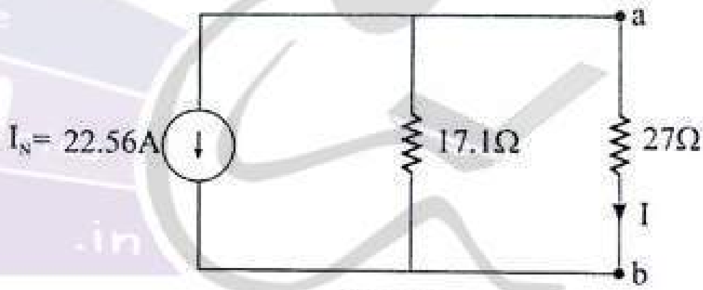


Fig 2.86

As I_N flows from b ot a, so its direction is down ward as shown in figure.

Applying current division rule,

$$I = -22.56 \times \frac{17}{17+27} = -8.77\,A$$

Here Norton current I_N is considered as negative (i.e -22.56 A) because this current is divided into two resistors 17.1Ω and 27Ω & current through 27 Ω resistance flows a to b.

Example 2.36 Find the value of the adjustable resistance R which results in maximum power transfer across the terminals 'ab' of the circuit shown in fig 2.87.

Solution : Let us find Thevenin's equivalent circuit across 'ab'.

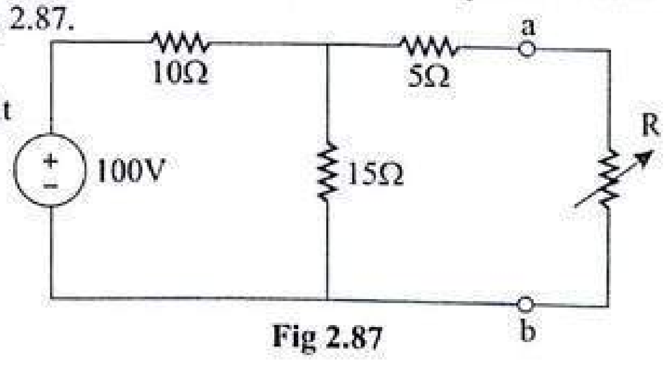
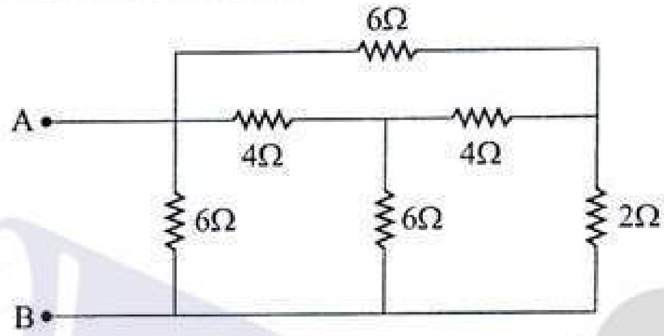
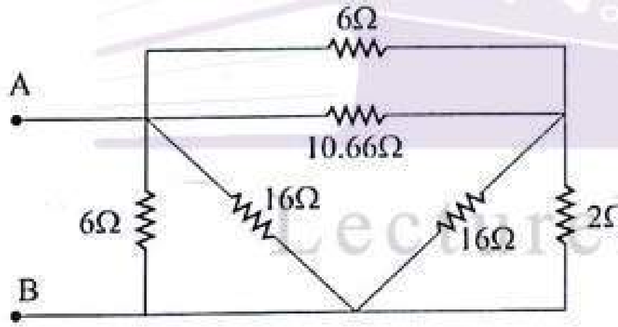


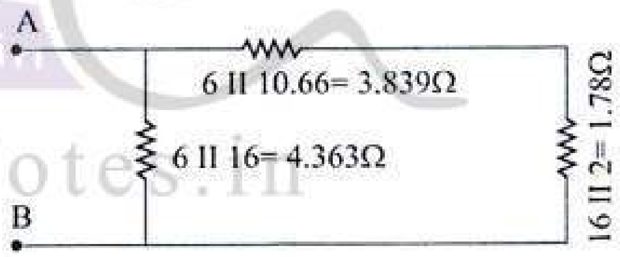
Fig 2.87

(1) Find the resistance R_{AB} using star- Delta transformation, **(1st semester 2004)**



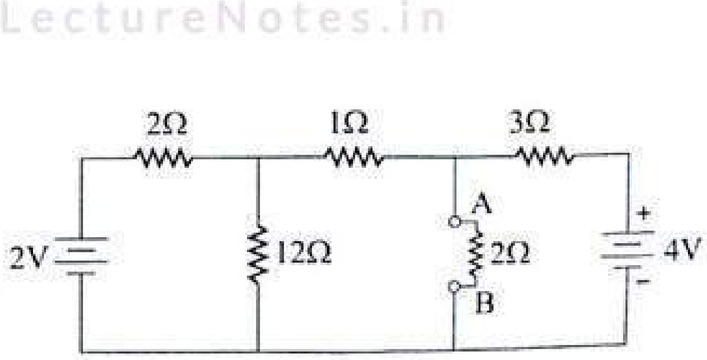
Solution :-





$$\therefore R_{AB} = \frac{(4.363)(3.839+1.78)}{4.363+3.839+1.78} = 2.456\Omega$$

(2) Use (a) Thevenin's theorem and (b) the Principle of Superposition to find the Current in a 2 ohm resistor connected between A and B in the circuit shown in fig. **(2nd semester 2004)**



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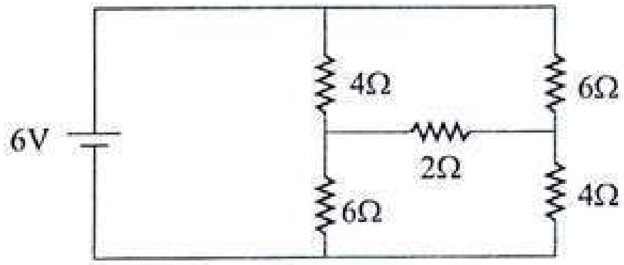
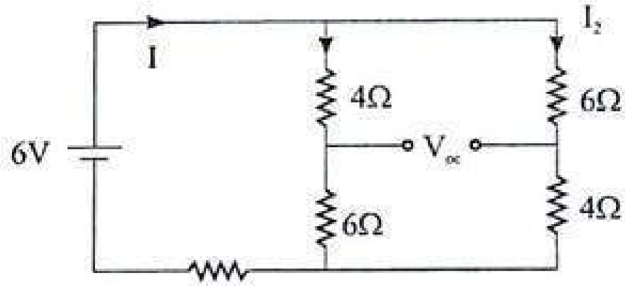
(7) Apply Thevenin's theorem to find current though 2ohm resistance as shown in figure.

(1st semester 2005)



Solution :- Removing 2Ω resistance, the open circuit voltage across its terminals is found out in the network as shown below.



Total resistance of the circuit $= \frac{10\times 10}{10+10} = 5\Omega$

Current from 6V battery is, $I = \frac{6}{5} = 1.2\,A$

According to current division rule,

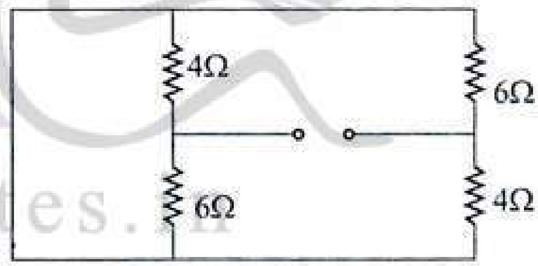
$$I_1 = I\left(\frac{10}{10+10}\right) = 0.6\,A$$

Also $I_2 = I - I_1 = 1.2 - 0.6 = 0.6\,A$

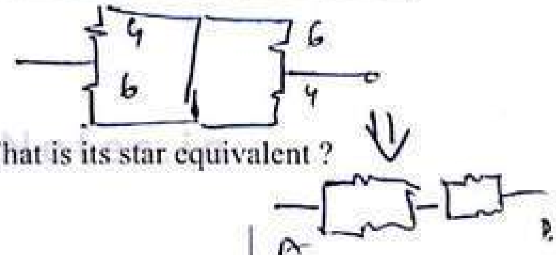
Applying KVL, $-4I_1 - V_{OC} + 6I_2 = O$

$\Rightarrow V_{OC} = -4\times 0.6 + 6\times 0.6 = 1.2\,V$

Deactivating the voltage source, thevenin's equivalent resistance (R_{Th}) can be obtained as shown below.



~~$R_{Th} = (6+4) \,||\, (6+4) = 5\Omega$~~ 6||4 + 6||4 = 2.4 + 2.4 = 4.8

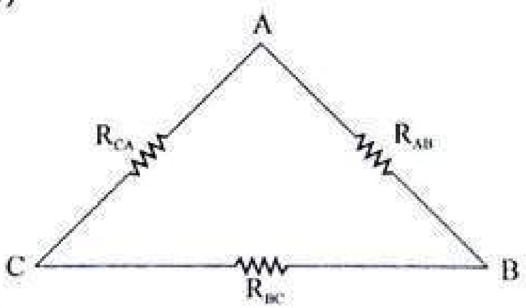


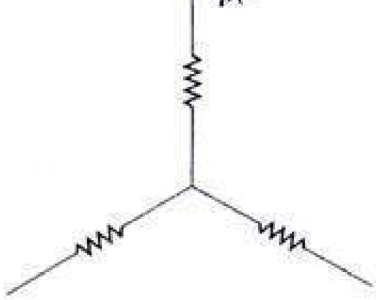
Current through 2Ω resistance is,

$$I_L = \frac{V_{OC}}{R_{Th}+R_L} = \frac{1.2}{5+2} = 0.1714\,A$$

(8) There numbers of 6ohm resistors are connected in Delta. What is its star equivalent ?

(2nd Semester 2005)





Apply KCL to node - 3,

$$\frac{V_2 - V_3}{30} + \frac{O - V_3}{40} + \frac{V_1 - V_3}{20} = O \quad \Rightarrow 6V_1 + 4V_2 - 13V_3 = O \quad .....(3)$$

Solving there equations we get,

$V_1 = 10.97$ volts, $V_2 = 6.74$ volts and $V_3 = 7.136$ volts.

Current through 40 Ω is $I^1 = \frac{V_3}{40} = \frac{7.136}{40} = 0.1784\,A$

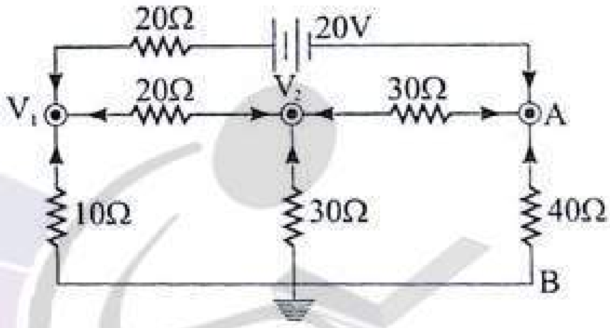
Direction of I^1 is from A to B.

(ii) Keep 20V voltage source and short the other voltage source.

Apply KCL to node-1,

$$\frac{O - V_1}{10} + \frac{V_2 - V_1}{20} + \frac{V_3 + 20 - V_1}{20} = O$$

$$\Rightarrow 4V_1 - V_2 - V_3 = 20 \quad .... (4)$$



Apply KCL to node-2,

$$\frac{V_1 - V_2}{20} + \frac{O - V_2}{30} + \frac{V_3 - V_2}{30} = O$$

$$\Rightarrow 3V_1 - 7V_2 + 2V_3 = O \quad .... (5)$$

Apply KCL to node-3,

$$\frac{V_1 - 20 - V_3}{20} + \frac{V_2 - V_3}{30} + \frac{O - V_3}{40} = O$$

$$\Rightarrow 6V_1 + 4V_2 - 13V_3 = 120 \quad ....(6)$$

Solving equations, (4), (5) and (6) we get,

$V_1 = 2.55$ volts, $V_2 = -1.321$ volts and $V_3 = -8.4581$ volts.

Current through 40Ω is $I^{11} = \frac{V_3}{40} = -0.21145$ A

Direction of current I^{11} is from B to A (as V_3 is lower potential compared to potential of reference node).

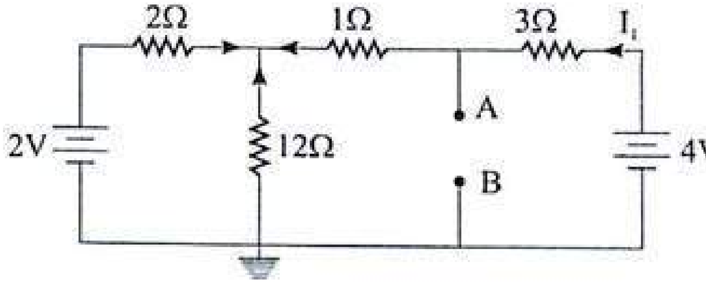
According to principle of superposition current through 40Ω is,

I= $I^1 - I^{11}$ ($I^1 and\, I^{11}$ are opposite in direction)

=0.178-0.21145

=-0.03345 A

Negative sign indicates that direction of I is along the direction of I^{11} i.e. from B to A.



Solution :-(a) Thevenin's theorem :

Here the load resistance is $R_L = 2\Omega$. Remove the load resistance and make AB open.

Applying KCL to the node,

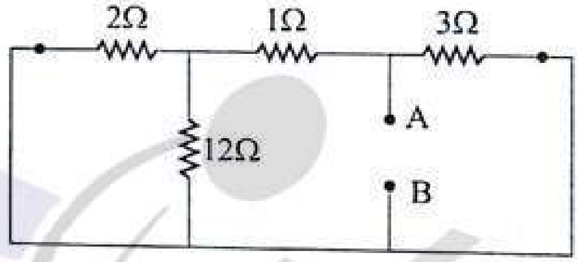
$$\frac{2-V}{2}+\frac{0-V}{12}+\frac{4-V}{4}=0$$

$$\Rightarrow \; V = 2.4 \text{ volts.}$$

$$\therefore I_1 = \frac{4-V}{4} = \frac{4-2.4}{4} = 0.4\,A$$

Thevenin's voltage $V_{Th} = V_{AB} = 4 - 3 \times 0.4 = 2.8$ volts.

When we calculate R_{Th} then short the two voltage sources as shown.



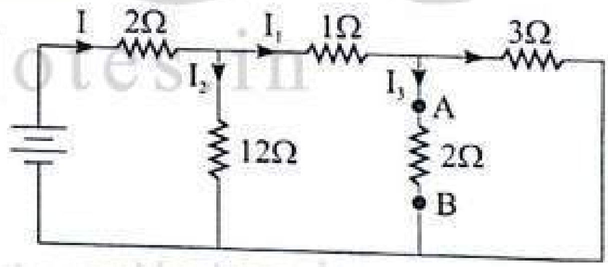
$$\therefore R_{Th} = [(2 \parallel 12)+1] \parallel 3 = 1.426\Omega$$

According to Thevenin's theorem, current through load resistance 2 Ω is

$$I = \frac{V_{Th}}{R_{Th}+R_L} = \frac{2.8}{1.426+2} = 0.8187\,A.$$

(b) Superposition theorem :

(I) First Keep the Voltage source 2V and short the other voltage source.



Total resistance of the circuit is

R=[{(3 ∥ 2)+1} ∥ 12] +2 =3.86 Ω

Total current flows through the circuit is

$$I = \frac{V}{R} = 0.518\,A$$

According to current division rule,

$$I_1 = 0.518\left(\frac{12}{12+2.2}\right) = 0.4377\,A.$$

$\therefore$ Current through 2 Ω resistance is, $I_3 = I_1\left(\frac{3}{3+2}\right) = 0.2626$ A

Solution : $R_A = \frac{R_{AB} . R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{6 \times 6}{6+6+6} = 2\Omega$

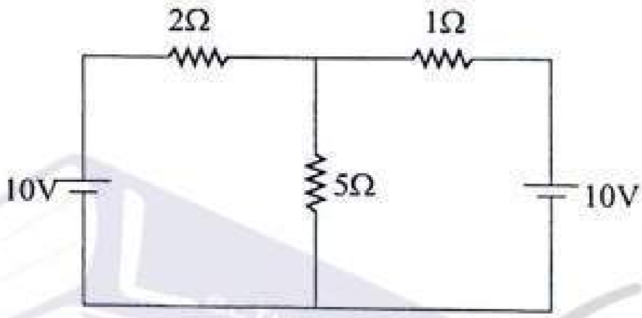
$$R_B = R_C = 2\Omega$$

(9) Does the equation Y = mx+c follow the principle of superposition when x and y are input and output voltage and m & c are constants ? ***(2nd semester 2005)***

Solution :- The equation Y = MX +C does not follow the principle of superposition, as it contains an intercept 'C' and does not obey additive property.

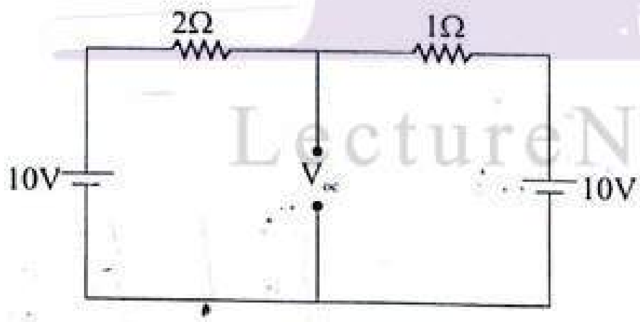
(10) Determine the current in the 5Ω resistance by application of Thevenin's theorem.

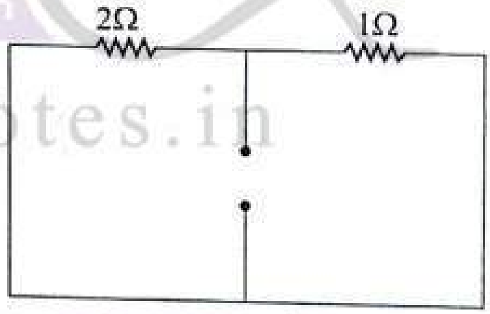
(2nd semester 2005)



Solution : Removing 5Ω resistance, the open circuit voltage (V_{OC}) across its terminals is found out in the network show below.

$$V_{OC} = V_{Th} = 10 \ volt$$





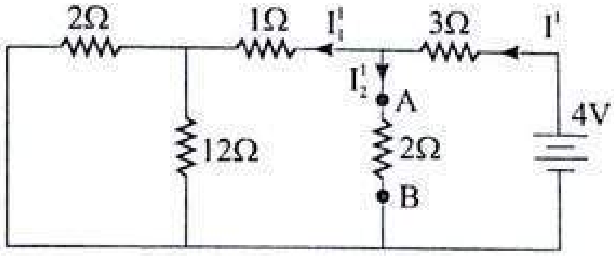
Deactivating the voltage source, Thevenin's equivalent resistance (R_{Th}) can be obtained as shown below.

$$R_{Th} = 2 || 1 = \frac{2}{3} = 0.667\,\Omega$$

Current through 5Ω resistance is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{102}{0.667+5} = 1.76 \text{ A}$$

(ii) Then keep the voltage source 4 V and short the other voltage source.



Total resistance of the circuit is

$$R^1 = [\{(2 \| 12) + 1 \| 2\}] + 3 = 4.1514\Omega$$

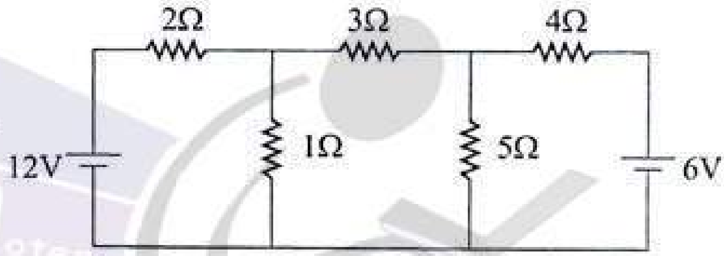
Total current flows through the circuit, $I^1 = \frac{4}{4.151} = 0.9636$

According to current division rule,

$$I_2^1 = I^1\left(\frac{2.714}{2.714+2}\right) = 0.5543 \text{ A}$$

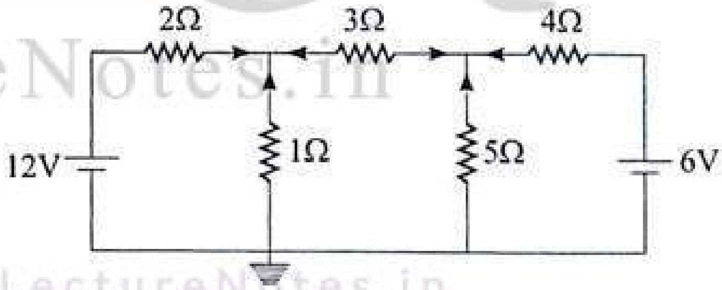
After knowing I_3 and I_2^1 then total current flows through 2Ω resistance $= I_3 + I_2^1 = 0.2626 + 0.5543 = 0.817$ A.

(3) Use the mesh or node analysis to find currents in the different branches of the circuit shown.



(Supplementary Exam July - 2004).

Solution : Let V_1 and V_2 are the voltages of two nodes as shown in figure.



Applying KCL to node - 1, we get

$$\frac{12-V_1}{2} + \frac{{}_1O-V}{1} + \frac{V_2-V_1}{3} = O$$

$$\Rightarrow 36\text{-}3V_1 - 6V_1 + 2V_2 - 2V_1 = O$$

$$\Rightarrow -11V_1 + 2V_2 = 36 \quad \text{..............(1)}$$

Again applying KCL to node - 2, we get

$$\frac{V_1-V_2}{3} + \frac{O-V_2}{5} + \frac{6-V_2}{4} = O$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = O$$

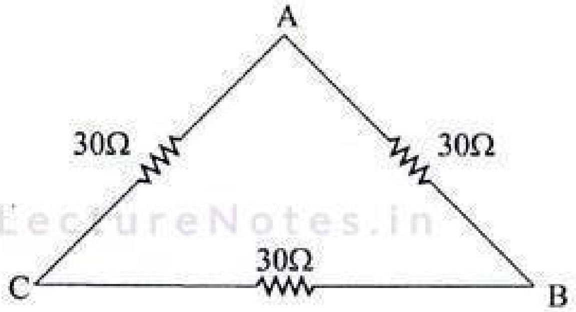
$$\Rightarrow 20V_1 - 47V_2 = -90 \quad \text{......(2)}$$

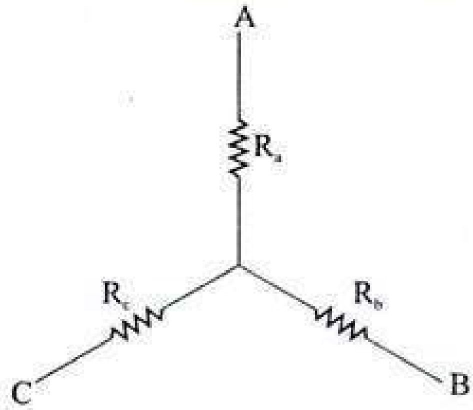
Solving equations (1) and (2) we get $V_1 = 3.924$ volt and $V_2 = 3.584$ volt.

Current through 2Ω resistance $= \frac{12-V_1}{2} = \frac{12-3.924}{2} = 4.038A$

(11) Three resistance of 30 ohms each are connected in delta. Find out the star equivalent of this combination . **(1st semester 2006)**

Solution :-



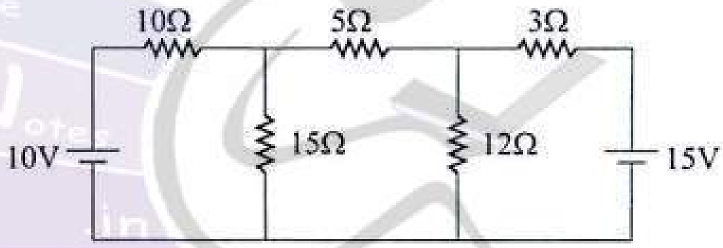


$$\therefore \quad R_A = \frac{R_{AB}.R_{CA}}{R_{AB}+R_{BC}+R_{CA}} = \frac{(30)(30)}{30+30+30} = 10\Omega$$

Similarly $R_B = R_C = 10\Omega$

(12) Use superposition principle to find out the current in 15 ohm resistor.

(1st semester 2006)



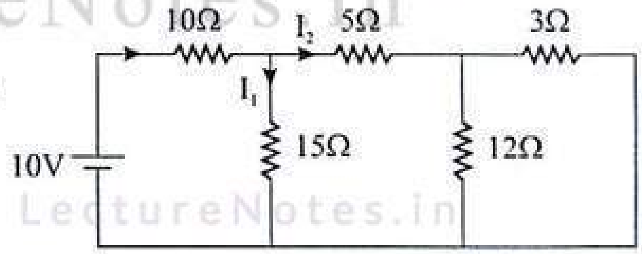
Solution: (I) First keep the voltage source 10V and short the other voltage source.

Total resistance of the circuit is

R = [{(12 || 3)}+5} || 15]+10 =14.955 ohm

Total current flows from voltage source,

$$I = \frac{10}{14.955} = 0.668 \text{ A}$$



According to current division rule, $I_1 = 0.668 \times \frac{7.4}{15+7.9} = 0.22$ A

(ii) Then keep the voltage 15V and short the other voltage source.

Total resistance of the circuit is R^1 = [{(10 || 15)+5} || 12]+3 =8.739 ohm

Total current flows from voltage source, $I^1 = \frac{15}{8.739} = 1.71\,A$

Current through 1Ω resistance $= \dfrac{O - V_1}{1} = -3.924\,A$

Current through 3Ω resistance $= \dfrac{V_1 - V_2}{3} = 0.1133\,A$

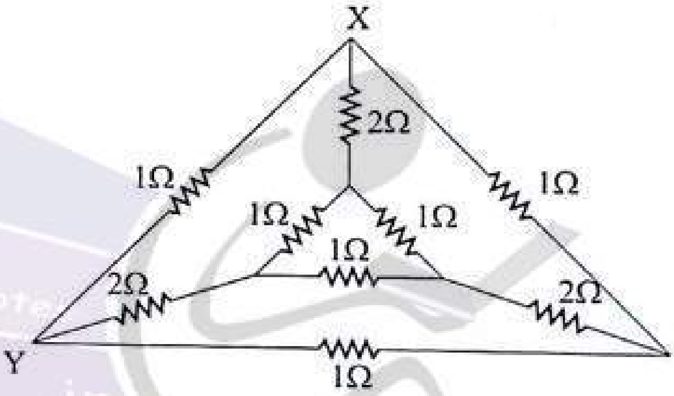
Current through 5Ω resistance $= \dfrac{O - V_2}{5} = -0.7168\,A$

Current through 4Ω resistance= $\dfrac{6 - V_2}{4} = 0.604\,A$

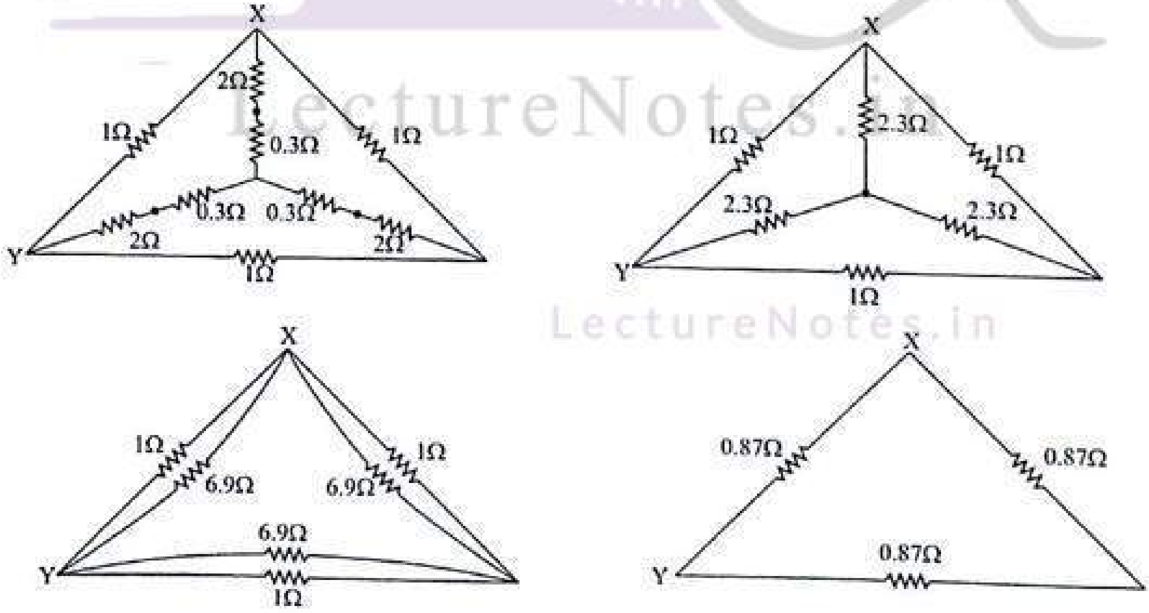
As currents through 1Ω and 5Ω are negative, so actually their directions are opposite to the assumptions.

(4) Determine the resistance between the points X and Y for the network given below :

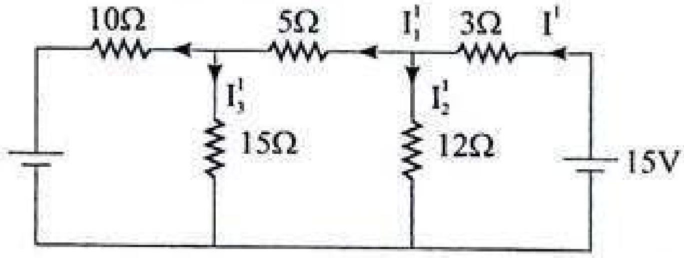
(Supplementary Exam 2004)



Solution :- The resistance between points X and Y can be obtained by star - delta conversion.



The resistance X and Y = (0.87+0.87) II (0.87) = 0.58 Ω

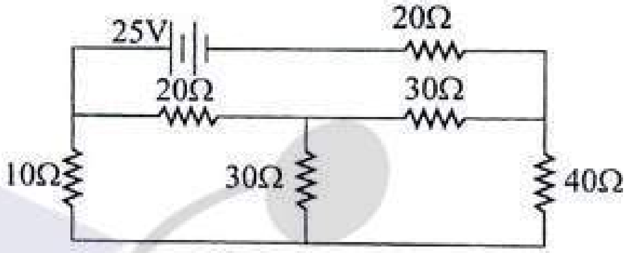


According to current division rule, $I_1^1 = 1.71 \times \frac{12}{12+11} = 0.895$ A

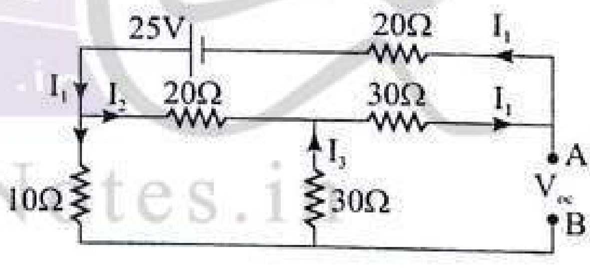
Again, $I_3^1 = I_1^1(\frac{12}{15+10}) = 0.895(\frac{10}{25}) = 0.358\,A$

∴ Total current flows through 15Ω resistance is $= I_1 + I_3^1 = 0.22 + 0.358 = 0.57$ A

(13) Using Thevenin's theorem, find the current flowing in the 40 ohm resistor as shown in figure. **(1st semester 2008)**



Solution : Remove 40Ω resistance, the open circuit voltage (V_{OC}) across its terminals is found out in the network shown.



Equivalent resistance across 25volt is

R_{eq} = [20 II (10+30)]+30+20 =63.33Ω

Current flows from voltage source is

$I_1 = \frac{25}{63.33} = 0.3947\,A$

According to current division rule, $I_3 = I_1(\frac{20}{10+30+20}) = 0.131\,A$

∴ $V_{Oc} = -30I_3 - 30I_1$ = -30(0.131)-30(0.3947)

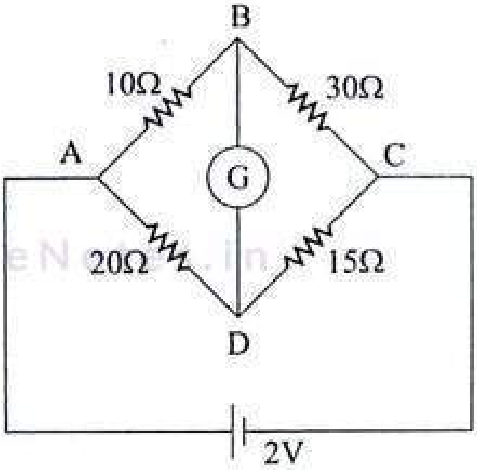
= -15.771 volt

= 15.771 V (with B positive)

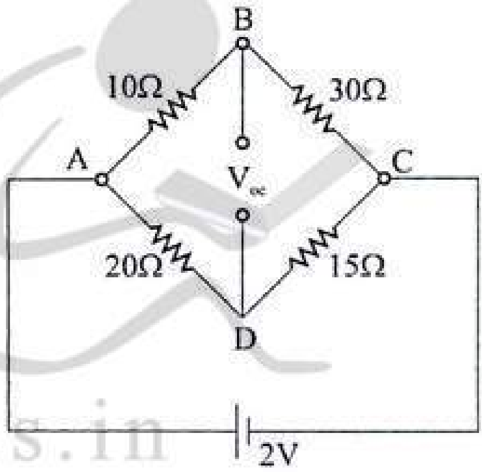
∴ Thevenin's voltage $= V_{Th} = V_{OC}$ = 15.771 volt (with B positive)

Deactivating the voltage source,m Thevenin's equivalent resistance (R_{Th}) can be obtained as shown below.

(5) Find the current through the galvanometer of 40Ω resistance by applying Thevenis's theorem. Hence find the value of Norton's current source. **(Supplementary Exam 2004)**



Solution : Removing 40Ω resistance, the open circuit voltage (V_{OC}) across its terminal is found out in the network shown below.



Current through 10Ω resistance $= \frac{2}{10+30} = 0.05A$

Voltage across 10Ω resistance is $V_1 = 0.05 \times 10 = 0.5V$

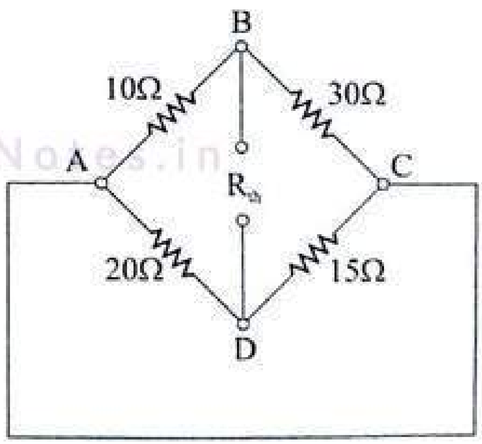
Current through 20Ω resistance is $= \frac{2}{35} = 0.057A$

Voltage across 20Ω resistance is $V_2 = 0.057 \times 20 = 1.14V$

$\therefore \quad V_{BD} = V_2 - V_1 = 1.14 - 0.5 = 0.642$ volts.

$\therefore \quad V_{OC} = V_{Th} = V_{BD} = 0.642$ volts.

Deactivating the voltage source, thevenin's equivalent resistance (R_{Th}) can be obtained as shown below.



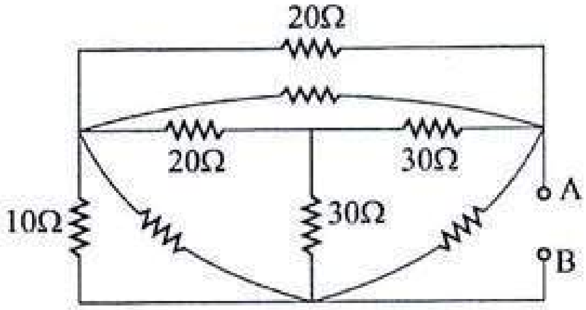
$R_{Th} = (10//30) + (20 \parallel 15) = 16.071\Omega$

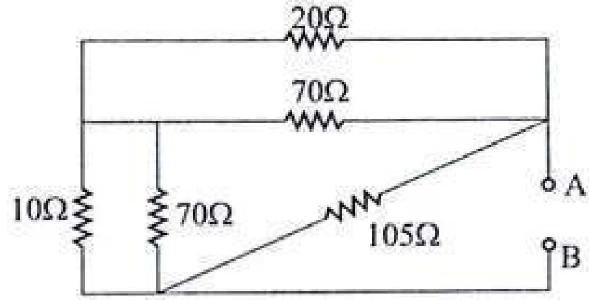
$\therefore$ Current through 40Ω resistance is,

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{0.642}{16.071 + 40} = 0.0114A$$

The Thevenin's equivalent circuit can be drawn as shown below,

Converting the voltage source V_{Th} in series with R_{Th}, the Norton's equivalent circuit can be drawn below. (Using source conversion techniques)





$\therefore R_{Th} = [(20 \parallel 70)+(10 \parallel 70)] \parallel 10^5 = 19.73\Omega$

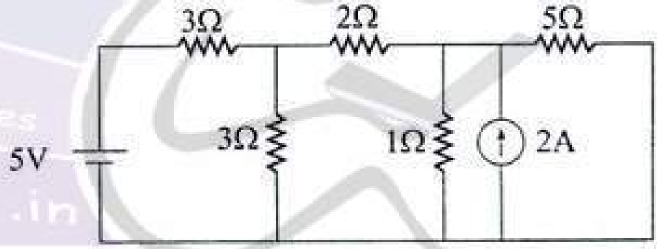
Current through 40Ω resistance is, $I_L = \dfrac{V_{Th}}{R_{Th}+R_L} = \dfrac{15.771}{19.73+40}$

$$=0.264 \text{ A}$$

(14)(a)Applying nodal analysis to the circuit given below, calculate the voltages at all possible nodes.

(b) Applying super position principle to the above circuit, calculate the current through 2Ω resistor.

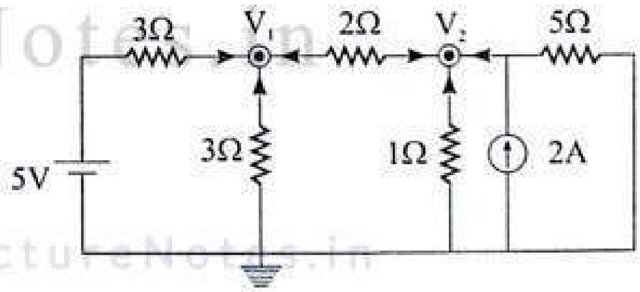
(1st semester 2009)



Solution (a) Applying KCL node-1,

$$\frac{5-V_1}{3}+\frac{0-V_1}{3}+\frac{V_2-V_1}{2}=0$$

$$\Rightarrow 7V_1 - 3V_2 = 10 \quad ....(1)$$



Applying KCL to node

$$\frac{V_1-V_2}{2}+\frac{O-V_2}{1}+2+\frac{O-V_2}{5}=0$$

$$\Rightarrow 5V_1 - 17V_2 = -20 \quad ....(2)$$

Solving eqn (1) and (2) we get

$V_1 = 2.211$ volts and $V_2 = 1.8269$ volts.

(b) (i) First keep the voltage source and open the current source.

Total resistance of the circuit is, R= [{(5 || 1)+2} || 3]+3 =4.457Ω

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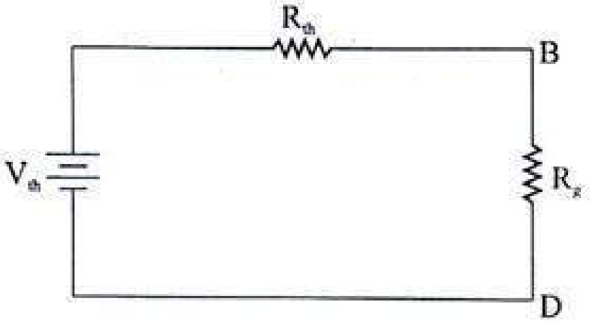
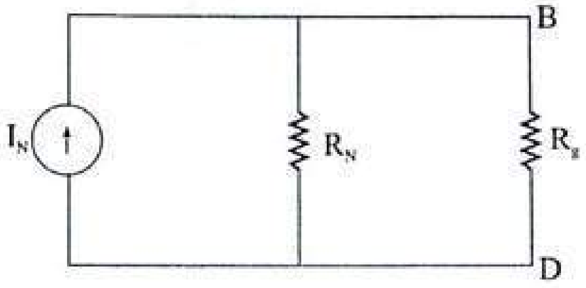


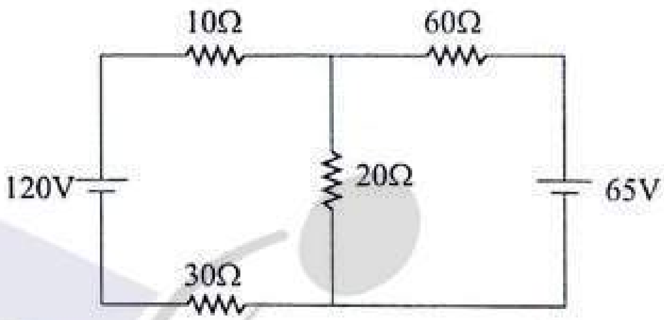


Where $I_N = \frac{V_{Th}}{R_{Th}} = \frac{0.642}{16.071} = 0.0399$ A

$R_N = R_{Th} = 16.071\Omega$

(6) Use superposition theorem to find the current through 20ohm resistance as shown below.

(1st semester 2005)



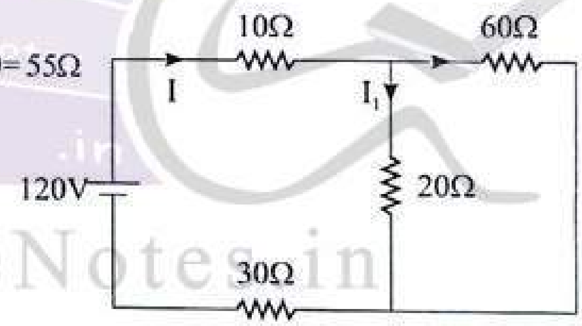
Solution :- (i) First keep 120V voltage source and short the other voltage source.

Total resistance of the circuit R = (60||20)+10+30= 55Ω

Current flows battery is, I = $\frac{V}{R} = \frac{120}{55}$ =2.18 A

According to current division rule,

$I_1 = I(\frac{60}{60+20})$=1.635 A

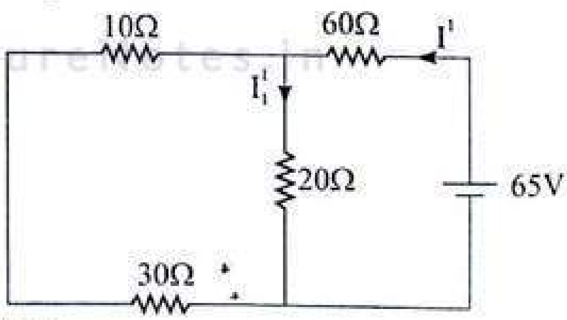


(ii) Then keep 65V voltage source and short the other voltage source.

Total resistance of the circuit is

$R^1 = (40 \parallel 20) + 60 = 73.34\Omega$

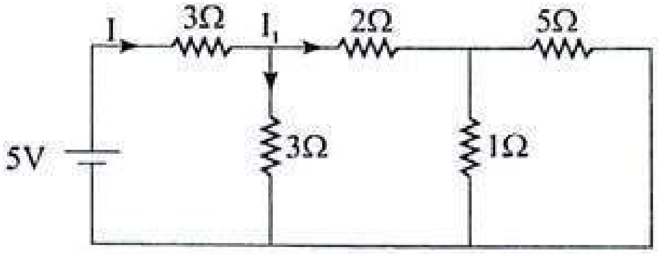
Current flows from battery is $I^1 = \frac{65}{73.34}$ =0.886 A



According to current division rule, $I_1^1 = I^1(\frac{40}{40+20}) = 0.59$ A

Knowing I_1 and I_1^1 the current through 20Ω resistance is = $I_1 + I_1^1$= 1.635 +0.59 =2.225 A

Current flows from battery,



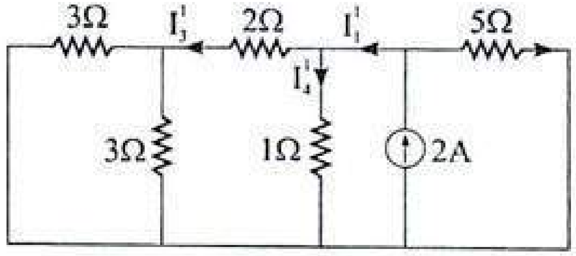
$$I = \frac{5}{4.457} = 1.121\text{A}$$

According to current division rule,

$$I_1 = I\left(\frac{3}{3+2.833}\right) = 0.576 \text{ A}$$

(ii) Then keep the current source and short the voltage source.

According to current division rule,



$$I_1^1 = 2 \times \frac{5}{5 + [(3\|3)+2]\|1} = 1.73 \text{ A}$$

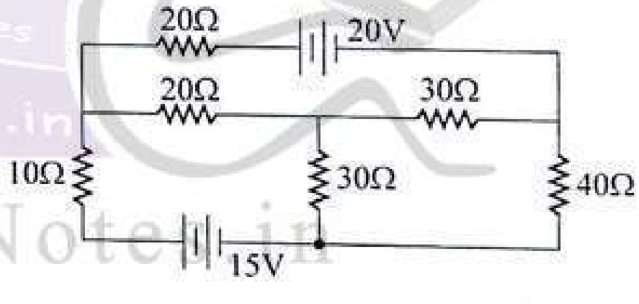
$$I_3^1 = 1.73 \times \frac{1}{1+3.5} = 0.3844 \text{ A}$$

Current through 2 ohms resistor

$= I_1 - I_3^1$ (I_1 and I_3^1 direction are opposite in direction)

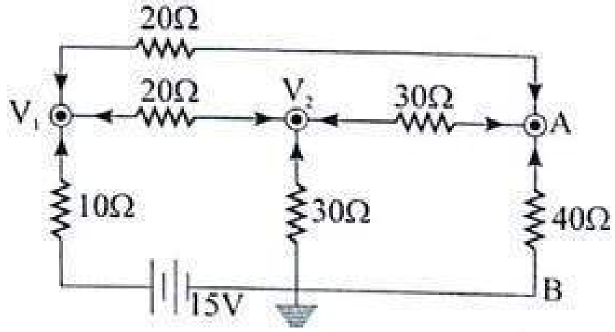
$= 0.576 - 0.3844 = 0.1916$ A

(15) Using superposition theorem, find the current flowing in the 40 ohm resistor as shown in figure. **(2nd semester 2009)**



Solution :- (i) 1st keep the 15V voltage source and short the other voltage source.

Apply KCL node -1,



$$\frac{15 - V_1}{10} + \frac{V_2 - V_1}{20} + \frac{V_3 - V_1}{20} = O$$

$$\Rightarrow 4V_1 - V_2 - V_3 = 30 \quad .....(1)$$

Apply KCL to node -2,

$$\frac{V_1 - V_2}{20} + \frac{O - V_2}{30} + \frac{V_3 - V_2}{30} = O$$

$$\Rightarrow 3V_1 - 7V_2 + 2V_3 = O \quad ......(2)$$

Do Your Self

T2.1 Find currents I_3, I_4 and I_6 Figure T2-1

[I_3=2A; I_4=-1A; I_6=3A]

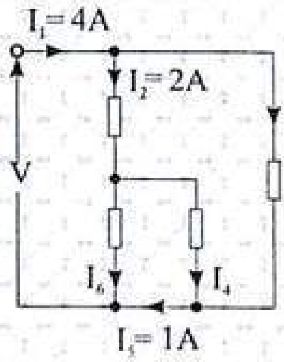


Fig T2.1

T2.2 For the networks shown in Figure, T2-2 find the values of the currents marked.

[(a) I_1=4A, I_2=-1A, I_3=13A
(b) I_1=40A, I_2-60A, I_3=120A
I_4,=100A, I_5=-80A]

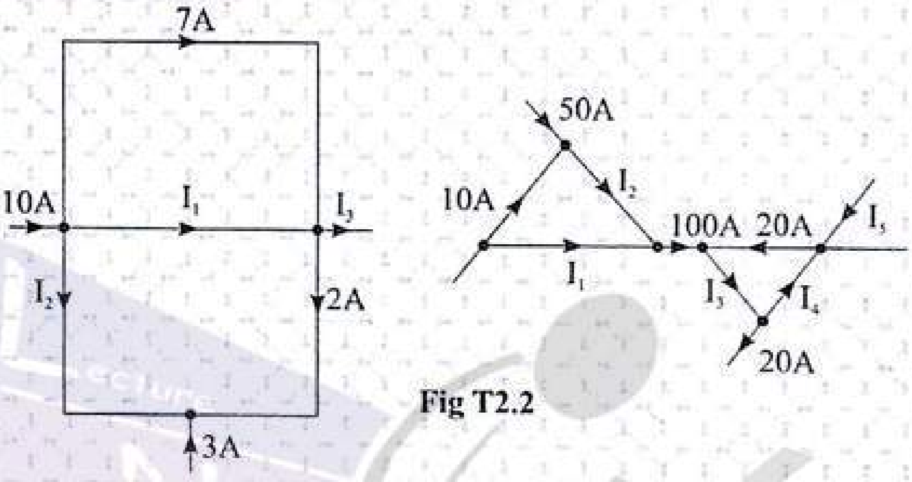


Fig T2.2

T2.3 Calculate the currents I_1 and I_2 in Figure T2.3

[I_1=0.8A, I_2=0.5A]

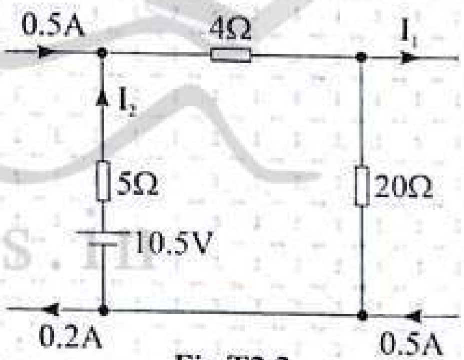


Fig T2.3

T2.4 Find the current flowing in the 3 Ω resistor for the network shown in Figure T2.4 Find also the p.d across the 10 Ω and 2Ω resistors.

[2.715A, 7.410V, 3.948V]

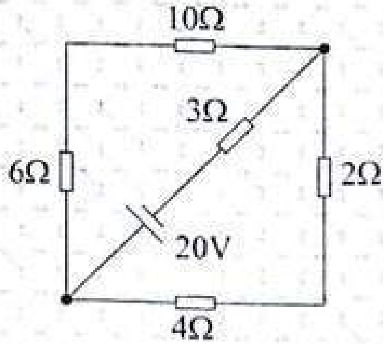


Fig T2.4

T2.26 Replace the network of Fig. to T2.26 the left of terminals ab by its Thevenin's equivalent circuit. Hence, determine I.

[-1.68A]

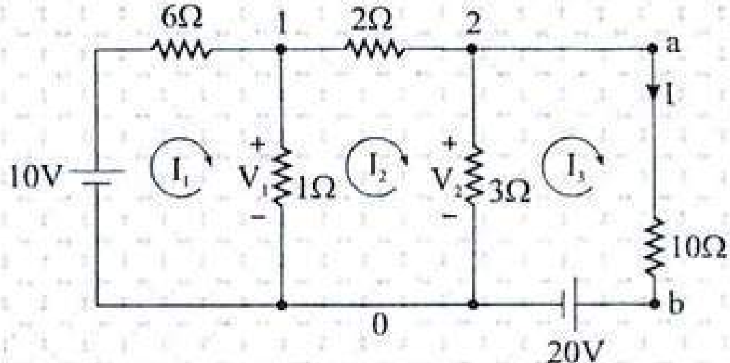


Fig T2.26

T2.27 Determine the current flowing in the 6Ω resistance of the network shown in Figure T2.27 by using Norton's theorem.

[2.5mA]

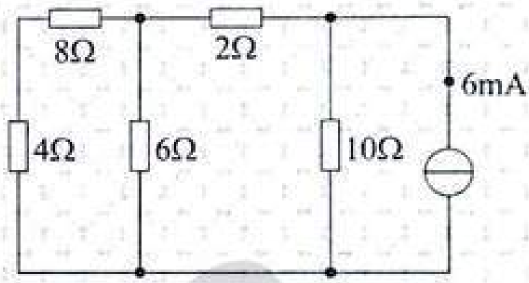


Fig T2.27

T2.28 A d.c source has an open-circuit voltage of 20Vand an internal resistance of 2 Ω. Determine the value of the load resistance that gives maximum power dissipation. Find the value of this power.

[2Ω, 50W]

T2.29 A d.c source having an open-circuit voltage of 42V and an internal resistance of 3Ω is connected to a load of resistance RL. Determine the maximum power dissipated by the load.

[147W]

T2.30 A voltage source comprising six 2V cells, each having an internal resistance of 0.2Ω, is connected to a load resistance R. Determine the maximum power transferred to the load.

[30W]

T2.31 The maximum power dissipated in a 4Ω load is 100 W when connected to a d.c voltage V and internal resistance r. Calculate (a) the current in the load, (b) internal resistance r, and ©) voltage V.
[(a) 5A (b) 4Ω ©) 40V]

T2.32 Determine the value of the load resistance RL shown in Figure that gives maximum power dissipation and find the value of the power.

[RL=1.6Ω, P=57.6W]

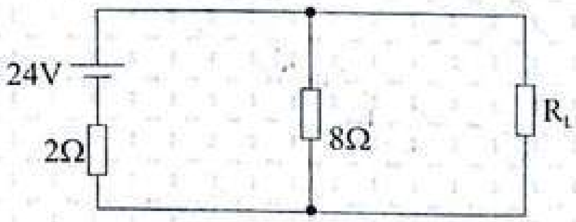


Fig T2.28

T2.5 For the network shown in Figure T2.5 find:

(a) the current in the battery,

(b) the current in the 300Ω resistor,

c) the current in the 90Ω resistor, and

(d) the power dissipated in the 150 Ω resistor.

[(a) 60.38mA (b) 15.10mA ©) 45.28mA (d) 34.20mW]

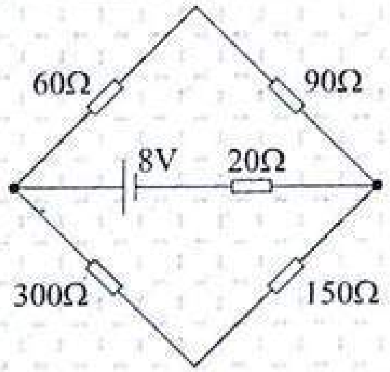


Fig T2.5

T2.6 For the bridge network shown in Figure T2.6 find the currents I_1 to I_5.

[I_1=1.26A, I_2=0.74A, I_3=0.16A, I_4=1.42A, I_5=0.58A]

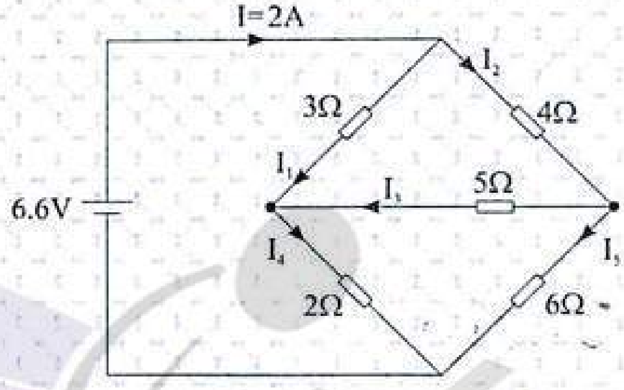


Fig T2.6

T2.7 Find the current in, and the voltage across, the2 Ω resistance in Fig T2-2using mesh current method.

[5A;10V]

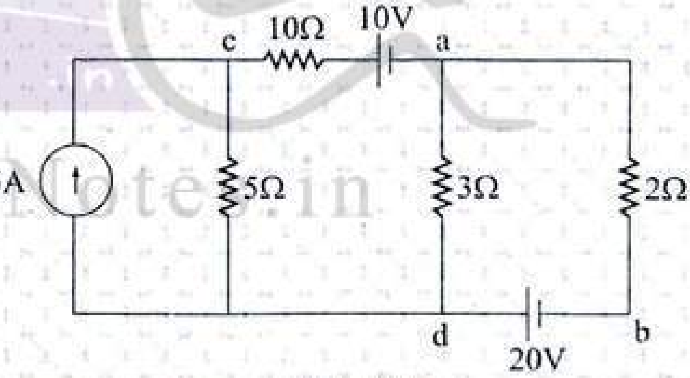


Fig T2.7

T2.8 By means of nodal analysis & mesh current method, find the current delivered by the 10-V source and the voltage across the 10Ωresistance in the circuit shown in Figure T2.8

[1.132A;-9.34V.]

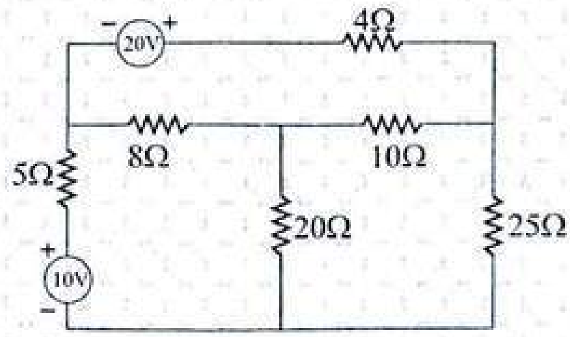


Fig T2.8

T2.9 For the network shown in Figure T2-9 find the current delivered by the 10-V source and the voltage across the 3Ω resistor by means of mesh-current analysis.

[Absorbing the current ~~20/9~~A., ~~7.78~~V.]

0.5556A, -4.444V

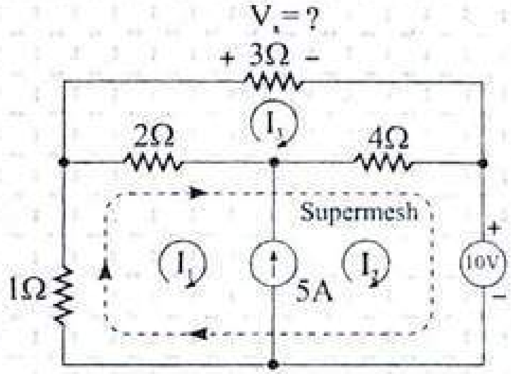


Fig T2.9

T2.10 Consider the circuit in Figure T2-10 which include a controlled source, and find the current in the 5-V source and the voltage across the 5-Ω resistor by using (a) the loop-current method and (b) the node-voltage method.

[0.273 A;-1.563V.]

-0.273A

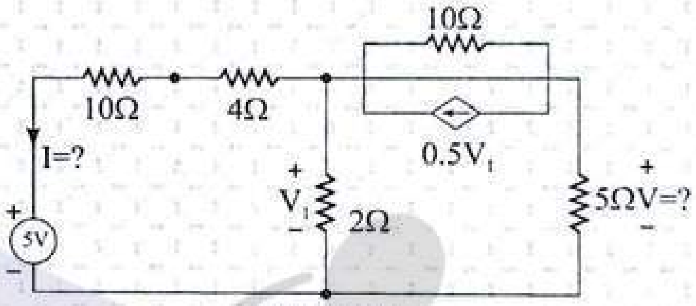


Fig T2.10

T2.11 Use the superposition theorem to find currents I_1,and I_2 and I_3 of Figure T2.11

[I_1=2A, I_2=3A, I_3=5A]

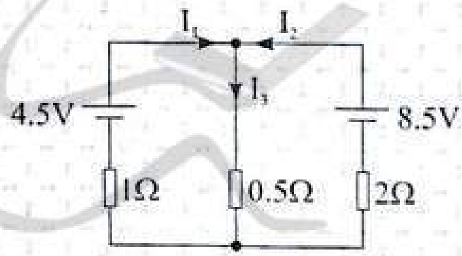


Fig T2.11

T2.12 Use the superposition theorem to find current in the 8 Ω resistor of Figure T2.12

[0.385A]

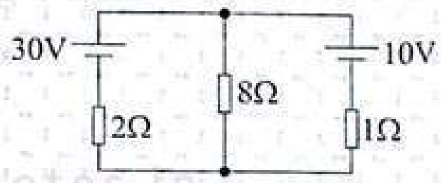


Fig T2.12

T2.13 Use the superposition theorem to find current in each branch of the network shown in figure T2-13

[10V battery discharges at 1.429A4V battery charges at 0.857A Current through 10 Ω resistor is 0.571A]

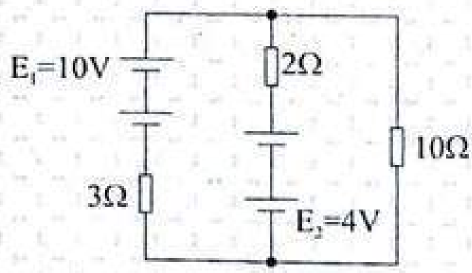


Fig T2.13

T2.14 Use the superposition theorem to determine the current in each branch of the arrangement shown in Figure T2-14

[24 V battery charges at 1.664A 52V battery discharges at 3.280A Current in 20 Ω resistor is 1.616A]

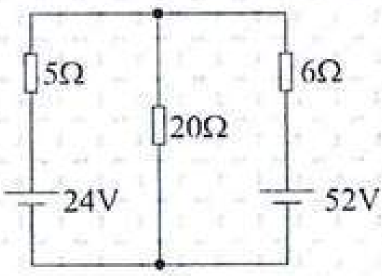


Fig T2.14

T2.15 Determine the voltage across the 20 Ω resistor in the following circuit of Figure T2-15 with the application of superposition.

[32.25 V] [96 V]

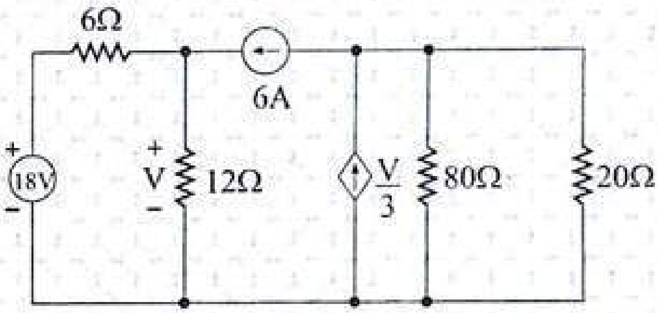


Fig T2.15

T2.16 Use delta-wye transformation for network reduction and determine the current through the 12Ω resistor in the circuit of Figure T2.16

[4.5A]

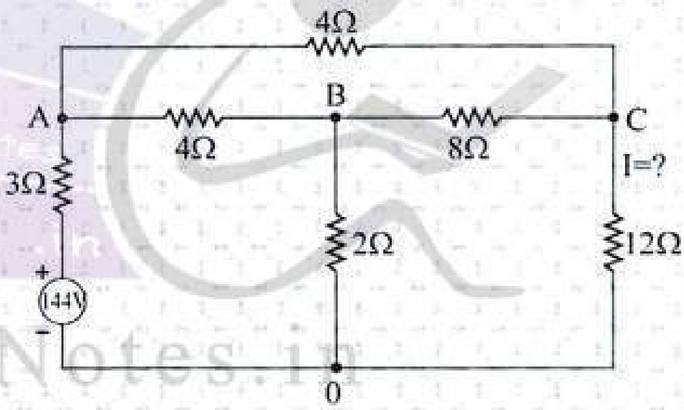


Fig T2.16

T2.17 A resistive network is shown in Fig.T2.17 Determine the value of the equivalent resistance at the terminals AB.

[4Ω]

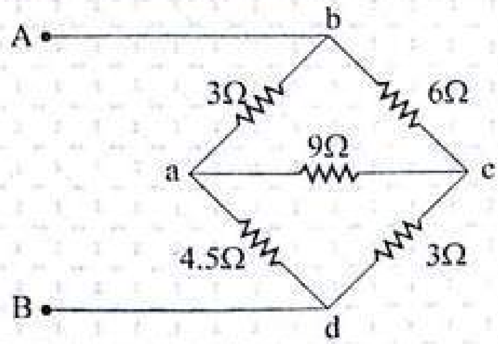


Fig T2.17

T2.18 Determine the power supplied to the network of Fig. T2.18.

[4705.9W]

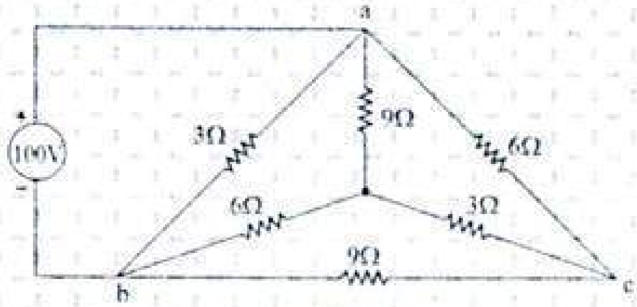


Fig T2.18

T2.19 Calculate the current in the 2ohm resistance of the network shown in Fig. T2.19

[5A]

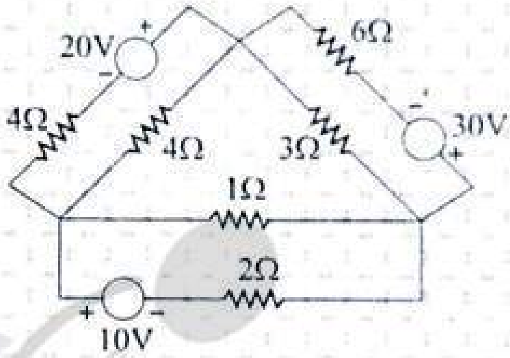


Fig T2.19

T2.20 Use Thevenin's theorem find the current flowing in the 14Ω resistor of the network shown in Figure T2.20 Find also the power dissipated in the 14 Ω resistor.

[0.434A,2.64W]

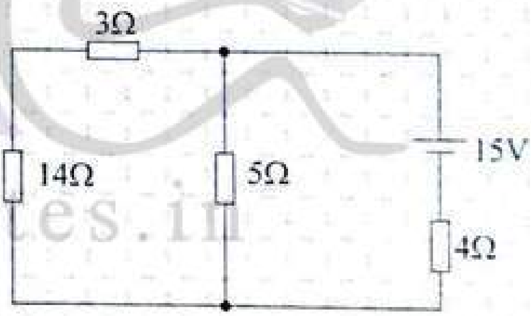


Fig T2.20

T2.21 Use Thevenin's theorem find the current flowing in the 6 Ω resistor shown in FigureT2.21 and the power dissipated in the 4Ω resistor.

[2.62A,42.07W]

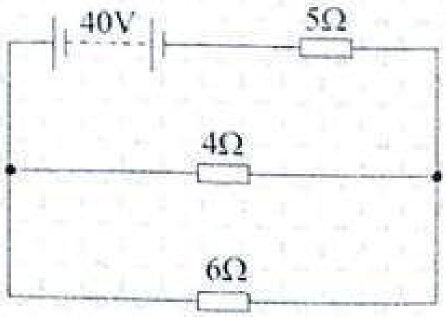


Fig T2.21

T2.22 In the network shown in Figure T2.22 the battery has negligible internal resistance. Find, using Thevenin's theorem, the current flowing in the 4Ω resistor.

[0.918A]

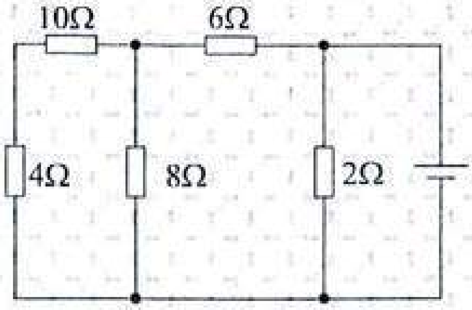


Fig T2.22

T2.23 For the bridge network shown in figure T2.23, find the current in the 5Ω resistor, and its direction, by using Thevenin's theorem.

[0.153A from B to A]

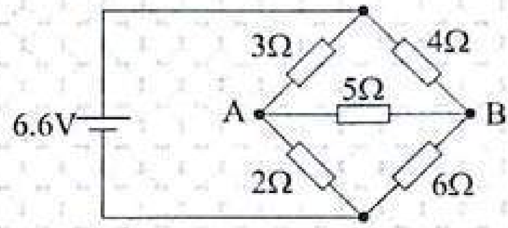


Fig T2.23

T2.24 Consider the circuit shown in Figure T2.24 Reduce the portion of the circuit to the left of terminals a-b to (a) A Thevenin's equivalent and (b) A Norton equivalent. Find the current through R=16 Ω, and Comment on whether resistance matching is accomplished for maximum power transfer.

[128V,16Ω, 8A, 16Ω, 4A)

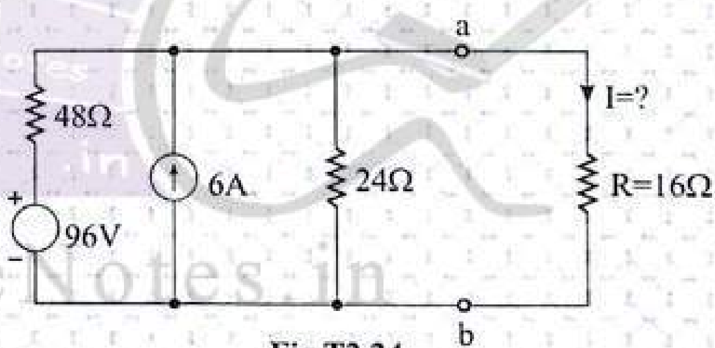


Fig T2.24

T2.25 Consider the circuit of Figure, including a dependent source, Obtain the Thevenin's equivalent at terminals a-b.

[5V, 200Ω]

5V, 100 Ω

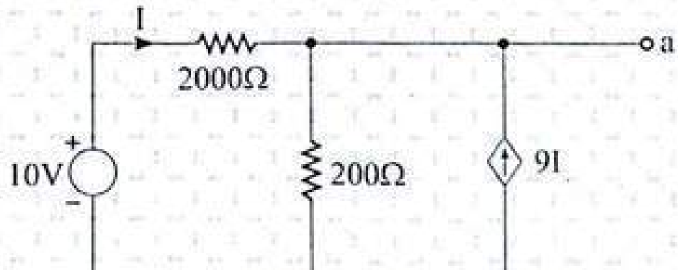


Fig T2.25

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Basic Electrical Engineering

Topic:

Principles Of Electro-Mechanism

Contributed By:

Dr. RAMAPRASAD PANDA

Silicon Institute Of Technology

Principles of Electro-mechanism

Chapter - 7

Energy available in many forms is often converted to and from electrical form because electric energy can be transmitted and controlled simply, reliable and efficiently. Among the energy conversion devices, electromechanical energy converters are most important. Electro mechanical energy conversion involves the interchange of energy between an electric system and a mechanical system while using magnetic field as a means of conversion.

When energy is converted from mechanical to electrical form the device is displaying generator action while a motor action involves converting electrical energy to mechanical energy. In this chapter, concept of magnetic circuit and how a magnetic field serves as a medium of converting energy from one form to the another. a system (physical or electrical) is subjected to a change, then the initial circuit behaviour (transient response) differs significantly from the steady state behaviour. When we analyse a time varying system, we get a differential equation, solution of which gives the output of the system as a function of time. While solving the differential equation we find complementary function which is transient response and particular integral representing steady state response of the system. The total solution gives the total response of the circuit.

In this chapter we have analysed the total response of an electrical circuit with DC excitation only.

7.1 Magnetism

The word 'magnet' comes from the ancient Greek City of Magnesia (The modern town Mania in western Turkey),where the natural magnets called lodes tones were found.

The power of a magnet by which it attracts certain substances is called magnetism and the materials which are attracted by a magnet are called magnetic materials. The fundamental nature of magnetism is the interaction of moving electric charges.

These are the following characteristics of magnets:

(I) A magnet has two poles i.e north pole and south pole and the pole strengths of two poles are same.

(ii) The two poles of a magnet can not be isolated (i.e separated out)

(iii) Between magnets, like poles repel and unlike poles attract.

(iv) Magnet always attract iron and its alloys.

(v) Magnetic field, in turn, exerts forces on other moving charges and current carrying conductors.

(vi) Magnet can be magnetically saturated.

(vii) Magnet can be demagnetized by beating, mechanical jerks, heating and with lapse of time.

(viii) It produces magnetism in other materials by induction.

(viv) The magnetism of materials is mainly due to the spin motion of its electrons.

(vv) On bending a magnet its pole strength remains unchanged but its magnetic moment changes.

7.2 Coulomb's Law of Magnetic Force

This law states that, the force of attraction or repulsion in between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of distance between them.

Consider two poles of magnetic strength m_1 *and* m_2 placed at a distance d apart in a medium as shown in fig.7.1.

According to coulomb's law, the force between two poles is given by ;

$$F \alpha m_1 m_2$$

$$\alpha \frac{1}{d^2}$$

$$F \alpha \frac{m_1 m_2}{d_2}$$

$$\Rightarrow F = K \frac{m_1 m_2}{d_2}$$

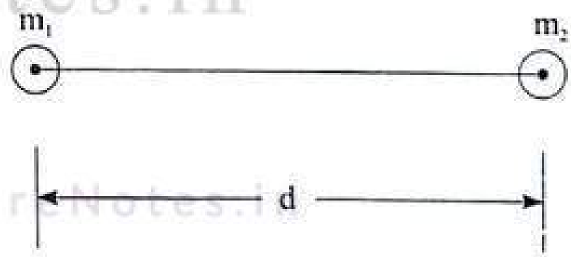


Fig 7.1

Where K is a constant whose value depends upon the surrounding medium. The value of K is given by,

$$K = \frac{1}{4\pi\mu_0\mu_r}$$

Where μ_0 = Absolute permeability of medium = Permeability of air or vacuum.

μ_r = Relative permeability of the medium.

The value of $\mu_0 = 4\pi \times 10^{-7} \frac{H}{m}$ and the value of μ_r is different for different media.

$$\therefore F = \frac{m_1 m_2}{4\pi\mu_0\mu_r d^2}$$

If $m_1 = m_2 = m$ (say), d = 1 metre, $F = \frac{1}{4\pi\mu_0} N$ and the medium is air (i.e $\mu_r = 1$)

then $m^2 = 1$

$$\Rightarrow m = \pm\, weber$$

Hence, a unit magnetic pole may be defined as that pole which when placed in air or vacuum at a distance of one metre from a similar and equal pole repels it with a force of $\frac{1}{4\pi\mu_0}$ newton.

Note:

- The phenomenon of magnetizing an unmagnetized substance by the process of magnetic induction in called magnetization
- The process of protecting any apparatus from the effect of earth's magnetic field is known as magnetic shielding.
- The phenomenon of decreasing or spoiling magnetic strength of a material is known as demagnetization.
- The state of a material after which the increase in its magnetic strength stops in known as magnetic saturation.

7.3 Magnetic Field

It is a space surrounding a magnet within which there is a force acting on hypothetical unit north pole.

The magnetic field around a magnet is represented by imaginary lines called magnetic lines of force. Inside the magnet each line of force passes from s-pole to n-pole, thus forming a closed loop or magnetic circuit as shown in fig 7.2.

The magnetic field is strongest near the pole and goes on decreasing in strength as we move away from the magnet. Magnetic lines of force have no real existence and are purely imaginary, but they are use ful concept to describe the various magnetic effects.

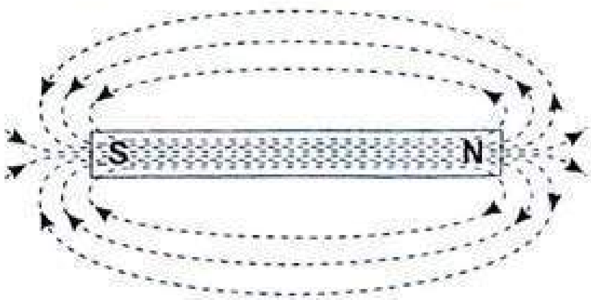


Fig 7.2

7.4. Magnetic Flux

It is the number of magnetic lines of force come out of the north pole or enter the south pole of a magnet. It is represented by ϕ.

Its C.G.S unit is maxwell and S.I unit is weber

I weber (wb)= 10^8 maxwell.

7.5 Magnetic flux density (B)

It is defined as the flux passing per unit area through a plane at right angles to the flux.

It is represented by B. Its C.G.S unit is gauss and S.I unit is tesla (T) or $\frac{wb}{m^2}$. One tesla $= 10^4$ gauss.

Flux density, $B = \frac{\phi}{A} \frac{wb}{m^2}$

Where ϕ = Flux in wb

A = area in m^2 normal to flux,

Magnetic flux density is a measure of field concentration i.e amount of magnetic flux in each square metre of the magnetic field.

7.6 Magnetic intensity or Magnetising force (H)

Magnetic intensity at any point in a magnetic field is defined as the force acting on a unit n-pole placed at that point. It is represented by H. Its S.I unit is Nwb^{-1}.

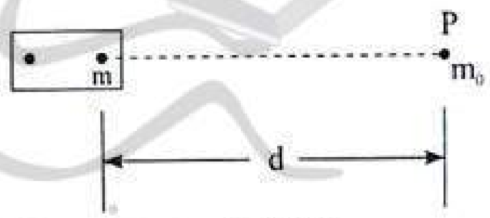


Fig 7.3

Consider a bar magnet of pole strength m. P is any point in the field of bar magnet at a distance 'd' as shown in fig 7.3 at which magnetic intensity is to be calculated. Imagine a north pole of pole strength m_0 placed at point P.

According to coulomb's law, force acting on pole strength m_0 due to bar magnet is,

$$F = \frac{mm_0}{4\pi\mu_0 d^2}$$

$\therefore$ Force acting on unit pole strength due to bar magnet = $\frac{F}{m_0}$

Magnetic intensity at P is, H = force acting on unit n-pole placed at P,

$$= \frac{F}{m^0}.$$

$$= \frac{mm_0}{4\pi\mu_0 d^2} \Big/ m_0 \quad = \frac{m}{4\pi\mu_0 d^2} \; Nwb^{-1}$$

Magnetic intensity is a vector quantity. Its direction is along the direction of force acting on unit pole strength (unit north pole).

7.7 Permeability

Permeability of a material is defined as its conducting power for magnetic lines of force. The greater the permeability of a material, the greater is its conducting power for magnetic lines of force (or magnetic flux) and vice versa. It is represented by μ.

$\mu = \mu_0 \mu_r$

Where μ= actual permeability of the material

μ_0 = absolute permeability of the material = Permeability of air or vacuum.

μ_r = relative permeability of the material

The value of μ_0 is $4\pi \times 10^{-7} Hm^{-1}$. The value of μ_r for air or vacuum is 1. The value of μ_r for all non-magnetic material is also 1. However, μ_r of magnetic material is very high. For example, soft iron (i.e pure iron) has a μ_r of 8000. Due to high relative permeability of magnetic materials (e.g iron, steel and other magnetic alloys), they are widely used for the cores of all electro magnetic equipment. It should be noted that the value of μ_r is not constant for a given magnetic material. It varies considerably with flux density (B) in the material.

7.8 Relation between B and H

The flux density B produced in a material is directly proportional to the applied magnetising force H. The greater the magnetising force, the greater is the flux density and vice versa.

$\therefore B \alpha H$

$\Rightarrow B =$ constant $\times H$

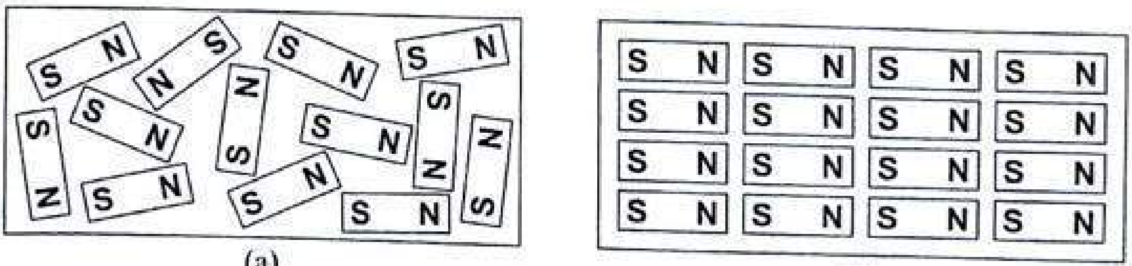
$\Rightarrow B = \mu H$

$\Rightarrow B = \mu_0 \mu_r H$

For air, $B = \mu_0 H \qquad \because \mu_r = 1$

7.9 Molecular theory of Magnetism

According to this theory, molecules of all substances are basically magnets, each having a north and south pole. Every substance consists of a very large number of tiny magnets called molecular magnets.



(a) (b)

Fig 7.4

Before a piece of iron has been magnetised the molecular magnets lie in disorderly positions as shown in fig 7.4 (a), so that poles of molecular magnets neutralise with each other. Hence iron piece does not show any magnetism (i.e no poles are developed at the ends).

When magnetising force (H) is applied to the iron piece (by passing electric current) then molecular magnets are line up in an orderly manner, with n-pole of one molecular magnet facing the s-pole of other as shown in fig 7.4 (b) As a result the magnetic fields of molecular magnets add each other and two definite n-pole and s-pole are developed at the ends of the iron bar. Hence the iron piece gets magnetised.

7.10 Magnetic circuit

The closed path followed by magnetic flux is called a magnetic circuit.

A magnetic circuit provides a path for magnetic flux, just as an electric circuit provides a path for the flow of electric current. A magnetic circuit consists of materials having high permeability. It is because these materials offer very small opposition to the flow of magnetic flux.

Consider a coil of N turns wound on an iron core as shown in fig 7.5. When current I passes through the coil then magnetic flux (ϕ) is set up in the core. The flux follows the closed path PQRSP. This path is called magnetic circuit.

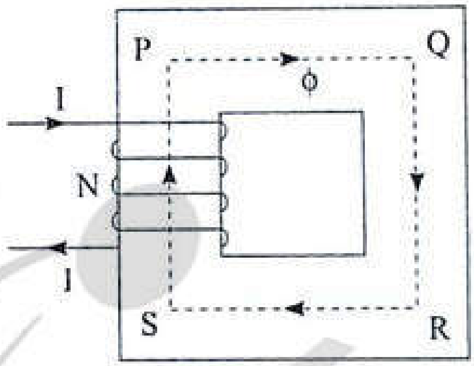


Fig 7.5

7.11 Magneto motive force (mmf)

The amount of work done required to carry a unit magnetic pole once through the entire magnetic field called magneto motive force.

It drives magnetic flux through a magnetic circuit It is equal to the product of current and number of turns of the coil. Its unit is ampere-turns (AT).

Magnetomotive force (or mmf) = NI ampere-turns (or AT)

7.12 Analysis of magnetic circuit

Consider a magnetic circuit of area of cross-section 'a' mean length ℓ and relative permeability of core material μ_r as shown in fig 7.6

A coil of N turns wound on an iron core as shown. When current I passes through the coil then magnetic flux ϕ is set up in the core.

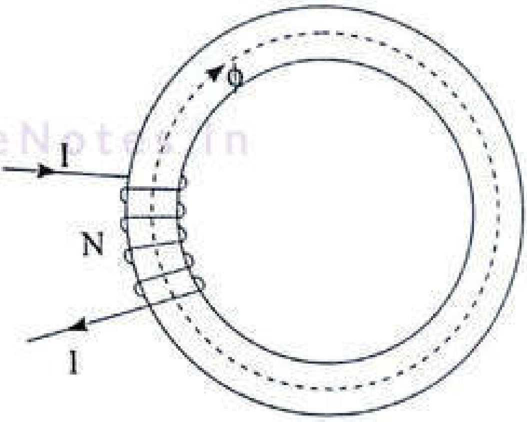


Fig 7.6

Flux density in the core is, B= $\frac{\phi}{a}$ $\frac{wb}{m^2}$

Magnetising force in the core,

$$H=\frac{B}{\mu}=\frac{B}{\mu_0\mu_r}=\frac{\phi}{a\mu_0\mu_r} \quad \text{...............(1)}$$

But magnetising force is defined as the ratio of mmf to mean length of magnetic path.

$$\therefore \quad H = \frac{mmf}{mean\ length\ of\ magnetic\ path} = \frac{NI}{l}\ \frac{AT}{m} \quad \text{.......................... (2)}$$

From equation (1) and (2) we get,

$$\frac{\phi}{a\mu_0\mu_r} = \frac{NI}{l}$$

$$\Rightarrow \phi = \frac{NI(a\mu_0\mu_r)}{l} = \frac{NI}{l/a\mu_0\mu_r}$$

$$\Rightarrow \quad \phi = \frac{NI}{S}$$

Where $S = \frac{l}{a\mu_0\mu_r} =$ reluctance of the magnetic circuit.

$$\text{Flux } (\phi) = \frac{NI}{S} = \frac{mmf}{reluctance} \quad \text{.................. (3)}$$

Equation (3) is sometimes referred to as ohm's law of magnetic circuit.

7.13 Reluctance

The opposition that the magnetic circuit offers to flux is called reluctance.

It depends upon length, area of cross section of magnetic circuit and permeability of the material of the circuit. It is represented by S.

$$\therefore \quad S = \frac{l}{a\mu_0\mu_r}$$

Its S.I unit is AT/wb. Reluctance in a magnetic material varies inversely as relative permeability μ_r, so magnetic materials (e.g iron, steel, cobalt etc) have low reluctance and non-magnetic materials (wood, brass etc) have high reluctance, It is because magnetic material has high μ_r and non- magnetic material has low μ_r.

The reciprocal of reluctance is called ***permeance***. It is represented by Λ.

$$\text{Permeance } (\Lambda) = \frac{1}{reluctance\ (s)} = \frac{a\mu_0\mu_r}{l}$$

Its S.I. unit is $\frac{Wb}{AT}$ or Henrys (H).

7.14. Series Magnetic Circuit

Consider a series magnetic circuit consists of three different dimensions and materials as shown in fig 7.7

In series magnetic circuit flux (ϕ) through each part is same. The total reluctance in series magnetic circuits is equal to the sum of reluctances of individual parts.

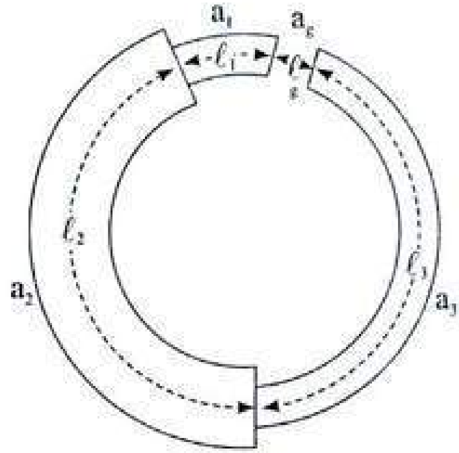


Fig 7.7

$$\therefore \text{Total reluctance (S)} = S_1 + S_2 + S_3 + S_g$$

$$S = \frac{l_1}{a_1\mu_0\mu_{r_1}} + \frac{l_2}{a_2\mu_0\mu_{r_2}} + \frac{l_3}{a_3\mu_0\mu_{r_3}} + \frac{l_g}{a_g\mu_0}$$

($\because$ *for air* $\mu_r = 1$

Total mmf = (Flux) (Total reluctance)

$$= (\phi)\left[\frac{l_1}{a_1\mu_0\mu_{r_1}} + \frac{l_2}{a_2\mu_0\mu_{r_2}} + \frac{l_3}{a_3\mu_0\mu_{r_3}} + \frac{l_g}{a_g\mu_0}\right]$$

$$= \frac{\phi l_1}{a_1\mu_0\mu_{r_1}} + \frac{\phi l_2}{a_2\mu_0\mu_{r_2}} + \frac{\phi l_3}{a_3\mu_0\mu_{r_3}} + \frac{\phi l_g}{a_g\mu_0}$$

$$= \frac{B_1 l_1}{\mu_0\mu_{r1}} + \frac{B_2 l_2}{\mu_0\mu_{r2}} + \frac{B_3 l_3}{\mu_0\mu_{r3}} + \frac{B_g l_g}{\mu_0}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

Whereas $\dfrac{B}{\mu_0\mu_r} = H$

7.15 Parallel Magnetic Circuit

A magnetic circuit which has more than one path for flux is called parallel magnetic circuit . In parallel magnetic circuit the mmf across each path is same.

The concept of parallel magnetic circuit is illustrated in fig 7.8. When current I flows through the coil, flux ϕ_1 is set up. This flux ϕ_1 is divided into two parts i.e ϕ_2 and ϕ_3 along two magnetic paths BE and BCDE.

The flux ϕ_2 passes along path BE and flux ϕ_3 passes along path BCDE.

$$\phi_1 = \phi + \phi_3$$

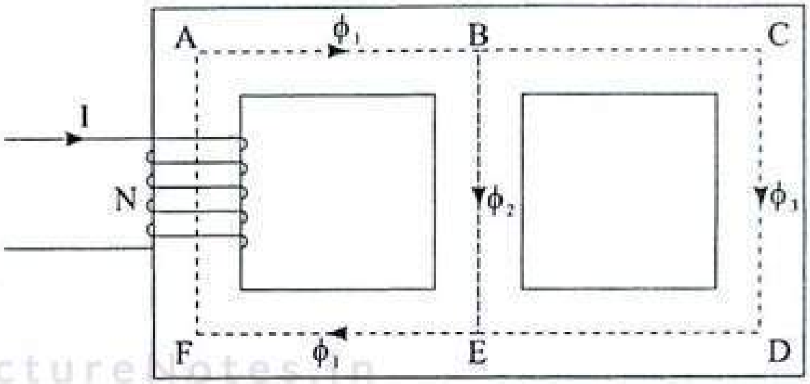


Fig 7.8

As two paths BE and BCDE are parallel, so mmf required for path BE is equal to mmf required for path BCDE.

Total mmf required = mmf of path EFAB + mmf of path BE or path BCDE

$$\therefore mmf = (flux)(reluctance) = \phi S$$

$$\Rightarrow NI = \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$$

$$\phi_2 S_2 = \phi_3 S_3$$

Where S_1 = reluctance of path EFAB

S_2 = reluctance of path BE

S_3 = reluctance of path BCDE

$\phi_2 S_2$ = mmf of path BE

$\phi_3 S_3$ = mmf of path BCDE.

7.16 Leakage flux

Magnetic flux is classified into two groups. (I) Useful flux (ii) Leakage flux.

The total magnetic flux produced is equal to the sum of the useful flux and the leakage flux.

The part of the total magnetic flux which has its path wholly within the magnetic circuit is called useful flux. The magnetic flux having its path partly in magnetic circuit and partly in air is called leakage flux. These fluxes are shown in fig 7.9

The ratio of total flux produced to the useful flux is called ***leakage factor*** or ***leakage coefficient***.

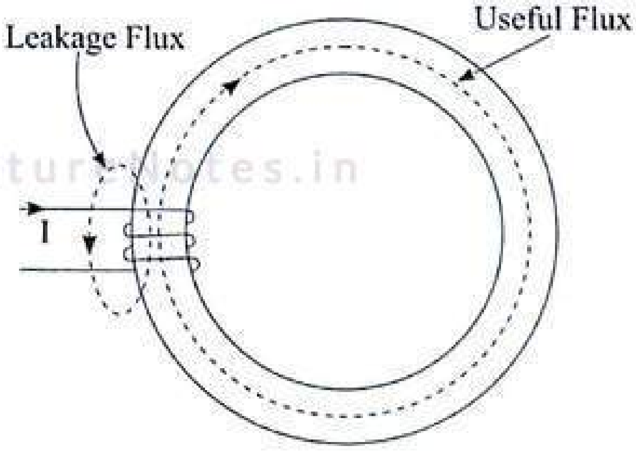


Fig 7.9

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and delivery.

$\therefore$ Leakage factor $(\lambda) = \dfrac{total\ flux\ produced}{useful\ flux}$

The value of λ is greater than one.

7.17 Magnetic Fringing

Consider a ring provided with an air gap. when the flux lines cross the air gap, they tend to bulge out across the edges of the air gap. This effect is called fringing. It is shown in fig 7.10

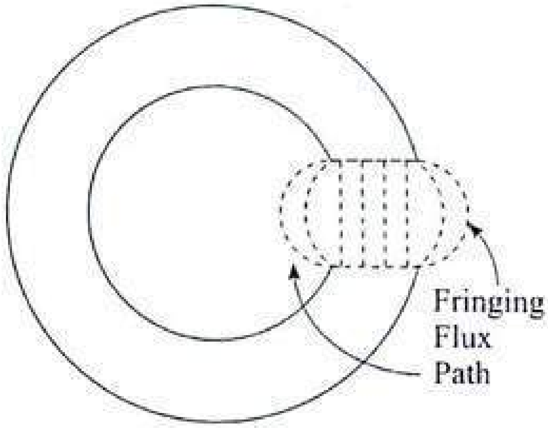


Fig 7.10

The effect of fringing is to increase the gap area than that of ring. Hence the flux density in the air gap is reduced. The effective increase depends upon the length of the air gap. If the gap length is small compared with the gap width the effect of fringing can be neglected.

7.18 B-H Curve

The graph between the flux density B and field intensity H of a magnetic material is called B-H curve (or magnetization curve). A typical B-H curve for an iron specimen is shown in fig 7.11

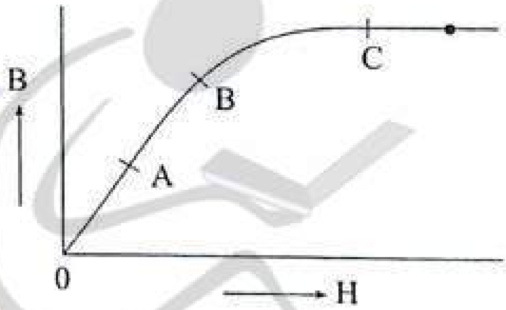


Fig 7.11

- In the region OA magnetic field strength H is too weak to cause any appreciable alignment of molecular magnets (or domains). Consequently, the increase in flux density B is small.
- In the region AB, more and more domains get aligned as H increases. Consequently, B increases linearly with H.
- In the region BC, only a few domains are left unaligned. Consequently, the increase in B with H is very small.
- Beyond C, no more domains are left unaligned and the iron material is said to be magnetically saturated.
- The point 'C' is called the point of saturation for the material i.e knee point.
- The slope of B-H curve represents permeability μ.

$\therefore$ Slope of B-H curve $= \tan\theta = \dfrac{\beta}{H} = \mu = \mu_0\mu_r$.

Clearly slope of B-H curve $\alpha\ \mu_r$

The portion AB of the curve is a straight line, μ_r of the material is constant. In the saturation region when the curve becomes almost horizontal the slope becomes almost zero. The B-H curve shows that the permeability μ_r of a magnetic material changes with the flux density B.

7.19 Magnetic Hysteresis

When a magnetic material is magnetise in one direction and then in other direction and finally it demagnetise, the material is said to go through one cycle of magnetisation. In this process, it is found that the flux density (B) in the material lags behind the applied magnetising force (H). This phenomenon is known as hystresis.

The phenomenon of lagging of flux density (B) behind the magnetising force (H) in a magnetic meterial subjected to cycles of magnetisation is known as magnetic hystresis.

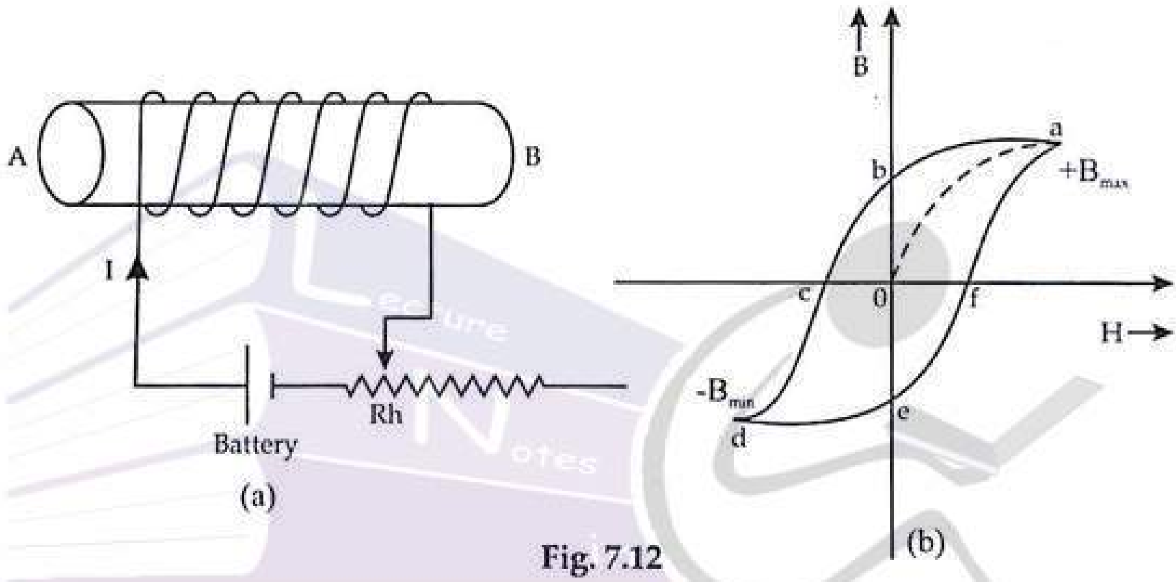


Fig. 7.12

Let us consider an unmagnetised iron bar AB wound with N turns which known as solenoid as shown in fig 7.12(a). When current flows through the coil then magnetising force (H) is produced.

(i) When current in the solenoid is zero then H= 0 and hence flux density (B) in the iron is zero. When H is increased by increasing current then B increases until the point of maximum flux density ($+B_{max}$) is reached. At this stage the material is saturated and beyond this point the flux density will not increase. The B-H curve of iron is along path 0a.

(ii) It now H is gradually reduced by reducing current then B does not decrease along path oa but follows path 'ab'. At point 'b' the magnetising force (H) is zero but flux density (B) in the material has some value. This flux density is called ***residual flux density***. In fig 7.12 (b) ob = residual flux density. This shows that B lags behind H. The greater the lag, the greater is residual magnetism. The power of retaining residual magnetism is called ***retentivity*** of the material.

(iii) To remove the residual magnetism 'ob', the magnetising force H is reversed by reversing the current through the coil. When H is gradually increased in reverse direction then B-H curve follows the path 'bc'. When H = oc then residual magnetism is zero. The value of magnetising force required to wipe out residual magnetism is called as ***coercive force*** or ***coercivity***. In fig 7.12 (b) oc = coercive force.

(iv) If H is further increased in the negative direction then the material again saturates (at point d) in the negative direction. Reducing H to zero and then increasing it in the positive direction which completes the curve "defa". Thus when an ironp iece is subjected to one cycle of magnetisation then the B-H curve traces a closed loop ***ab cdefa*** called as ***hysteresis loop.***

It is clear from hystresis loop that B lags behind H. The two never attain zero value simultaneously.

7.20 Hysteresis loss

When a magnetic material is subjected to a cycle of magnetisation then an energy loss takes place due to the molecular friction in the material. This loss is in the form of heat and is called ***hysteresis loss.*** The effect of hysteresis loss is the rise of temperature of the machine.

Hysteresis loss occurs in transformers, electric motors operate on a.c, d.c machines etc.

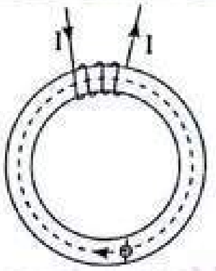
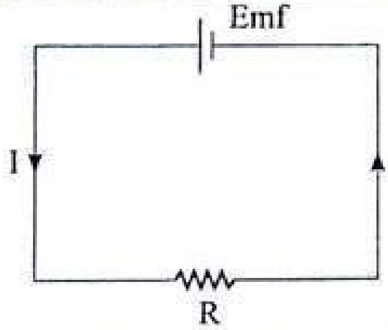
Hysteresis power loss, $P = \eta f B_{max}^{1.6} V$ watts.

Where η = hysteresis coefficient whose value depends on the nature of the material.

f = frequency of magnetisation.

V = volume of the material.

7.21 Comparison between magnetic circuit & Electric circuit

Magnetic circuit	Electric circuit
 	
1. Magneto motive force (mmf)in AT.	1. Electro-motive-force (emf) in volts.
2. Flux = $\frac{\text{mmf}}{\text{reluctance}}$	2. Current = $\frac{\text{Emf}}{\text{resistance}}$
3. Reluctance, $S = \ell / \mu_0 \mu_r a$	3. Resistnace, $R = \rho \frac{\ell}{a}$
4. Permeance = $\frac{1}{\text{reluctance}}$	4. Conductance = $\frac{1}{\text{resistance}}$
5. Flux density, $B = \phi / a$	5. Current density, $J = I / a$
6. mmf drop = ϕS	6. Voltage drop = IR
7. Magnetic intensity $H = \frac{NI}{\ell}$	7. Electric intensity $E = \frac{V}{d}$
8. Total mmf in series magnetic circuit $= \phi S_1 + \phi S_2 + \phi S_3 + ..$	8. Total emf in series electric circuit $= IR_1 + IR_2 + IR_3 + ...$
9. In series magnetic circuit, total reluctance $S = S_1 + S_2 + S_3 + ...$	9. In series electric circuit, total resistance $R = R_1 + R_2 + R_3 + ..$
10. In parallel magnetic circuit (for two path) the total reluctance is given by $S = \frac{S_1 S_2}{S_1 + S_2}$	10. In parallel electric circuit (for two path) total resistance is given by $R = \frac{R_1 R_2}{R_1 + R_2}$
11. In flux division rule, $\phi_1 = \phi\left(\frac{S_2}{S_1 + S_2}\right)$ and $\phi_2 = \phi\left(\frac{S_1}{S_1 + S_2}\right)$	11. In current division rule, $I_1 = I\left(\frac{R_2}{R_1 + R_2}\right)$ and $I_2 = I\left(\frac{R_1}{R_1 + R_2}\right)$

Example 7.1 : An air gap 1.1mm long and 40 sq cm in cross -section exists in a magnetic circuit.

Determine (i) reluctance of air gap

(ii) mmf required to create a flux of 10×10^{4} wb in the air gap.

Solution : (i) Reluctance (s) = $\dfrac{l}{\mu_0\mu_r A} = \dfrac{1.1\times10^{-3}}{4\pi\times10^{-7}\times40\times10^{-4}}$

$$=2.188\times10^5 \ AT/wb$$

(ii) mmf = *flux* × *reluctance*

$$=10\times10^{-4}\times2.188\times10^5$$

$$=218.85 \text{ AT}$$

Example 7.2 : An air cored coil has 500 turns. The mean length of magnetic flux path is 50 cm and the area of cross section is $5\times10^{-4}\,m^2$. If the exciting current is 5A, determine

(i) H (ii) Flux density (B) (iii) flux (ϕ)

Solution : (i) $H = \dfrac{NI}{l} = \dfrac{500\times5}{0.5} = 5000\dfrac{AT}{M}$

(ii) $B = \mu_0\mu_r H = \mu_0 H \ (\because \mu_r = 1)$

$$= 4\pi\times10^{-7}\times5000 = 6283\times10^{-3}\ T$$

(iii) $\phi = BA = 6283\times10^{-3}\times5\times10^{-4} = 3.14\times10^{-6}$ wb.

Example 7.3 : A mild steel ring having a cross-section area of 400 mm^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Given $\mu_r = 300$. Determine (i) reluctance of the ring. (ii) the current required to produce a flux of 800μ wb in the ring.

Solution : Flux density $B = \dfrac{\phi}{A} = \dfrac{800\times10^{-6}}{400\times10^{-6}} = 2\ wb\,m^2$

The reluctance of ring $S = \dfrac{l}{\mu_0\mu_r A} = \dfrac{0.4}{300\times4\pi\times10^{-7}\times400\times10^{-6}}$

$$=2.65\times10^6\ \frac{AT}{wb}$$

mmf = flux × reluctance

$$=800\times10^{-6}\times2.65\times10^6$$

$$=2.122\times10^3 \text{ AT}$$

Current required, $I = \dfrac{mmf}{No.of\ turns} = \dfrac{2.122\times10^3}{200} = 10.6$ A

Example 7.4 : What is the value of the net mmf acting in the magnetic circuit shown in fig 7.13

Solution : As flux in all coils are additive within core, hence mmfs are additive.

∴ Net mmf acting in magnetic circuit =

$(300\times 1)-(10\times 2)+(50\times 1)$

$=330AT$

(anticlockwise current produces +ve mmf & clockwise current produces - ve mmf)

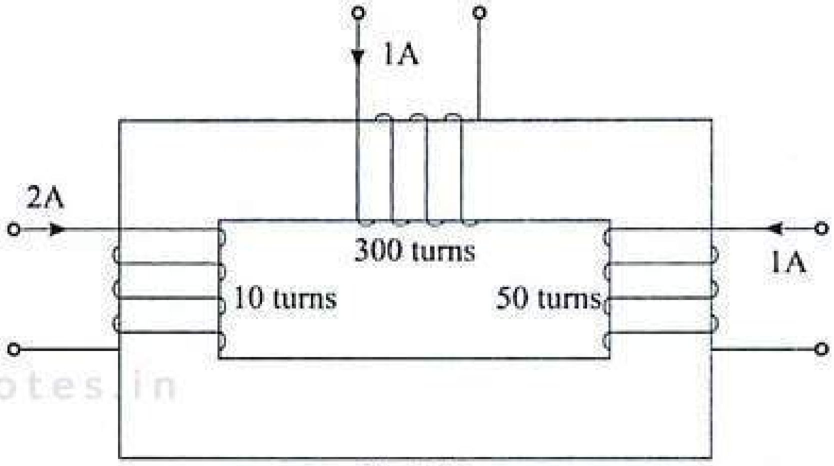


Fig 7.13

Example 7.5 : Calculate flux, flux density and field intensity on the magnetic structure shown in fig 7.14 Given $\mu_r = 1000$, N= 500 turns

i = 0.1 A, the cross-sectional area is A= $0.0001 m^2$

l = 0.1m, h = 0.1m, w = 0.01m

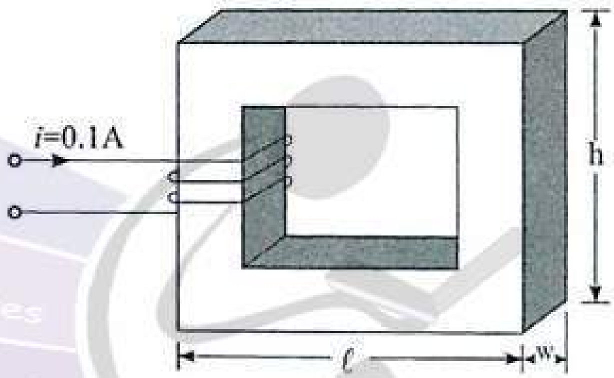


Fig 7.14

Solution :

mmf = Ni = (500)(0.1)= 50 AT

Mean path = $l_c = 4\,(0.1 - 0.01) = 0.36$ m

$$\text{Reluctance, } S = \frac{l_c}{\mu_0 \mu_r A}$$

$$= \frac{036}{4\pi \times 10^{-7} \times 1000 \times 0.0001}$$

$$= 2.865 \times 10^6 \quad AT/wb$$

$$\text{Magnetic flux } \phi = \frac{Ni}{S} = \frac{50}{2.865\times 10^6} = 1.75\times 10^{-5} \text{ wb}$$

$$\text{Flux density } B = \frac{\phi}{A} = \frac{1.75\times 10^{-5}}{0.0001} = 0.175\frac{wb}{m^2}$$

$$\text{Magnetic filed intensity} \quad H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{0.175}{1000\times 4\pi\times 10^{-7}} = 139\frac{AT}{M}$$

Example 7.6 : Fig 7.15 depicts the configuration of an electric motor. The electric motor consists of a stator and a rotor. Compute the air gap flux and flux density.

Given $\mu_r = \infty$, number of turns N = 1000,

coil current i = 10 A, $l_{gap} = 0.01\text{m}$, $A_{gap} = 0.1m^2$

Solution : $mmf = Ni = 1000 \times 10 = 10{,}000\,AT$

Reluctance at gap $= S_g = \dfrac{l_{gap}}{\mu_0 A} = \dfrac{0.01}{4\pi \times 10^{-7} \times 0.1} = 7\cdot96 \times 10^4\ \dfrac{AT}{wb}$

Total reluctance at gap $= S_g + S_g = 2S_g = 1.59 \times 10^5\ \dfrac{AT}{wb}$

Magnetic flux $\phi = \dfrac{mmf}{reluctance} = \dfrac{10{,}000}{1.59 \times 10^5} = 0\cdot0628 wb$

Magnetic flux density $B = \dfrac{\phi}{A} = \dfrac{0\cdot0628}{0.1} = 0\cdot628 \dfrac{wb}{m^2}$

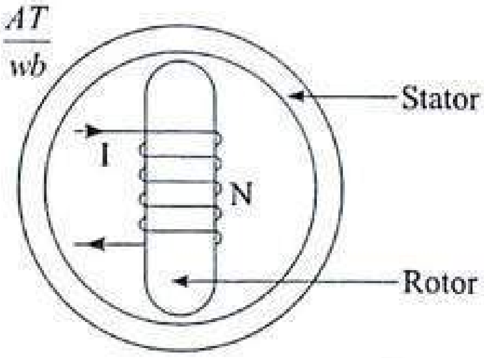
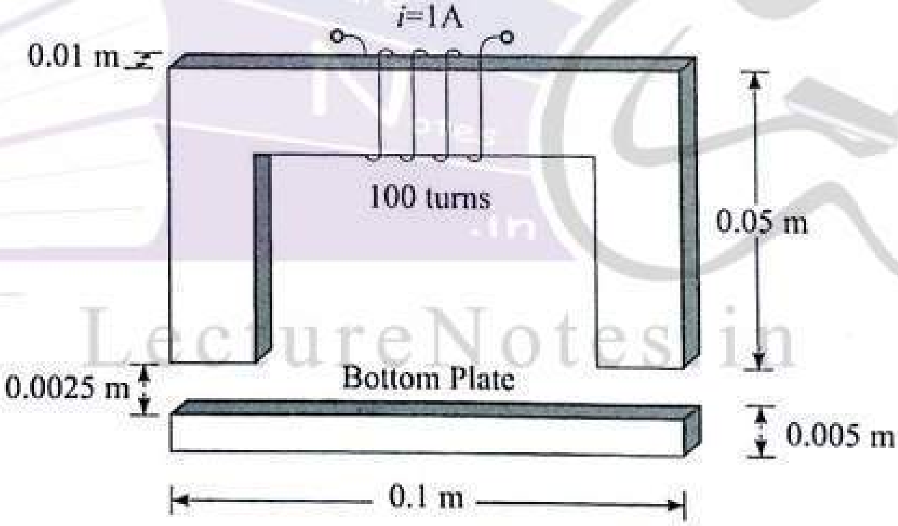


Fig 7.15

Example 7.7 : Calculate the equivalent reluctance of the magnetic circuit of fig 7.16 and the flux density established in the bottom bar of the structure

Given $\mu_r = 10{,}000$, N= 100 turns, i=1A



Electromagnetic structure with air gaps

Fig 7.16

Solution : Mmf = Ni = 100 × 1= 100 AT

The path broken into three legs as shown in fig 7.17 (i) U-shaped element (ii) air gap (iii) bar. Mean path length of U-shaped element $= l_\mu = l_1 + l_2 + l_3$

$= 0\cdot09 + 0\cdot045 + 0\cdot045 = 0\cdot18m$

Mean path of air gap,

$$l_g = 0\cdot0025 + 0\cdot0025 = 0\cdot005m$$

Mean path of bar,

$$l_b = l_4 + l_5 + l_6 = 0.09 + 0.0025 + 0.0025$$

$$= 0.095\text{m}$$

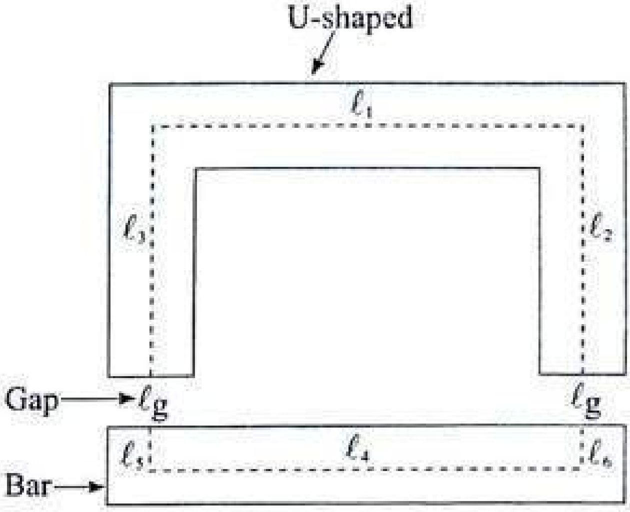


Fig 7.17

Neglect fringing effects in air gap;

area of cross-section at gap

$$A_{gap} = A_{bar} = A_u = (0.01)^2 = 0.001m^2$$

Reluctance of U-shaped element, $S_u = \dfrac{l_\mu}{\mu_0 \mu_r A}$

$$= \frac{0.18}{10000 \times 4\pi \times 10^{-7} \times 0.0001}$$

$$= 1.43 \times 10^5 \ \frac{AT}{wb}$$

Reluctance of air gap, $S_{gap} = \dfrac{l_g}{\mu_0 A} = \dfrac{0.05}{4\pi \times 10^{-7} \times 0.0001}$

$$= 3.98 \times 10^7 \ \frac{AT}{wb}$$

Reluctance of bar, $S_{bar} = \dfrac{l_{bar}}{\mu_0 \mu_r A} = \dfrac{0.095}{10000 \times 4\pi \times 10^{-7} \times 0.0001}$

$$= 0.715 \times 10^5 \ \frac{AT}{wb}$$

Total reluctance of the structure is

$$S = S_u + S_{gap} + S_{bar}$$

$$= 1.43 \times 10^5 + 3.98 \times 10^7 + 0.715 \times 10^5$$

$$= 400.145 \times 10^5 \ \frac{AT}{wb}$$

$$\text{Flux} = \frac{mmf}{reluctance} = \frac{100}{400\cdot145 \times 10^5} = 2\cdot499 \times 10^{-6} \ wb$$

$$\text{Flux density B} = \frac{\phi}{A} = \frac{2\cdot499 \times 10^{-6}}{0\cdot0001} = 24\cdot99 \times 10^{-3} \ \frac{wb}{m^2} \approx 25 \times 10^{-3} \ \frac{wb}{m^2}$$

Example 7.8 :

For the electromagnet of fig 7.18

a) Find the flux density in the core

b) Sketch the magnetic flux lines and indicate their direction

c) Indicate the north and south poles of the magnet.

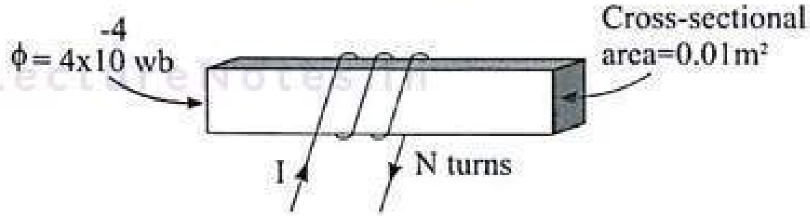


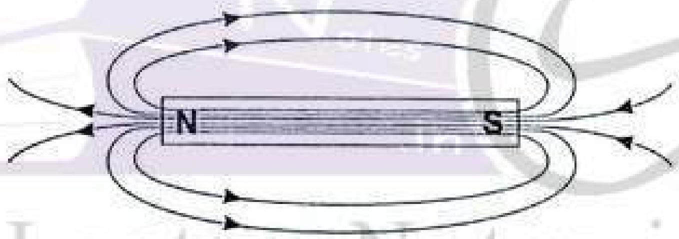
Fig 7.18

Solution : (a) Given $\phi = 4 \times 10^{-4}\ wb$

$A = 0.01m^2$

$\therefore$ Flux density $B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{0 \cdot 01} = 0 \cdot 04\,Tesla$

(b) Viewed from the top :



(c) See above

Example 7.9 : Determine the reluctance of the structure of fig 7.19 if the cross-sectional area is $0.01 \times 0.01m^2$ and $\mu_r = 2000$. Assume that each leg is o.1m in length and that the mean magnetic path runs through the exact center of the structure.

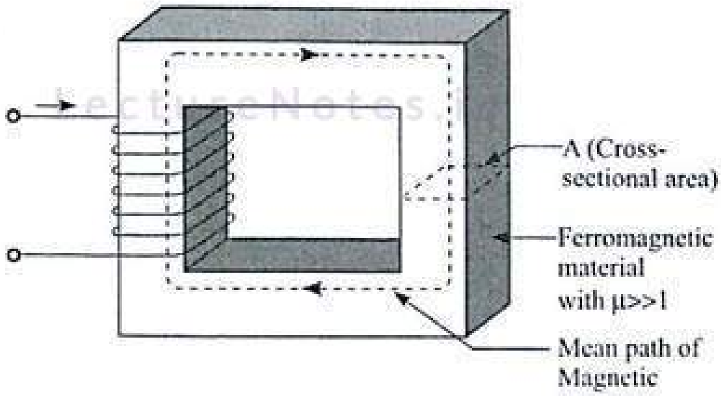


Fig 7.19

Solution : Given $A = 0 \cdot 01 \times 0 \cdot 01m^2$

$\mu_r = 2000$

Each leg is 0.1m in length

From fig 7.20 mean magnetic length is $l_c = 4\times0\cdot09 = 0\cdot36m$

$$\text{Reluctance S} = \frac{l_c}{\mu_0\mu_r A} = \frac{0\cdot36}{2000\times4\pi\times10^{-7}\times0\cdot01\times0\cdot01}$$

$$= 14\cdot33\times10^5\,\frac{AT}{wb}$$

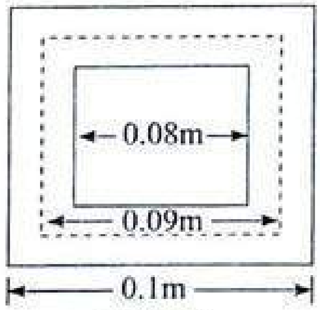


Fig 7.20

Example 7.10 : (i) Find the reluctance of a magnetic circuit if a magnetic flux $\phi = 42\times10^{-4}$ wb is established by an impressed mmf of 400 AT.

(ii) Find the magnetizing force H in SI units if the magnetic circuit is 6 inch long.

Solution : Given $\phi = 4.2\times10^{-4}$ wb, mmf = 400AT, length (i) = 6 inch= $15\cdot24\times10^{-2}\,m$

(I) $\text{Reluctance} = \frac{mmf}{flux} = \frac{400}{42\times10^{-4}} = 952\times10^5\,\frac{AT}{wb}$

(ii) $H = \frac{mmf}{length} = \frac{400}{15\cdot24\times10^{-2}} = 2624\cdot67\,\frac{AT}{m}$

Example 7.11: For the circuit shown in Fig 7.21

(i) Determine the reluctance values and show the magnetic circuit, assuming that $\mu = 3000\mu$.

(ii) Determine the inductance of the device

(iii) The inductance of the device can be modified by cutting an air gap in the magnetic structure. If a gap of 0.1 mm is cut in the arm of length l_3, what is the new value of inductance ?

(iv) As the gap is increased in size (length), what is the limiting value of inductance ? Neglect leakage flux and fringing effects.

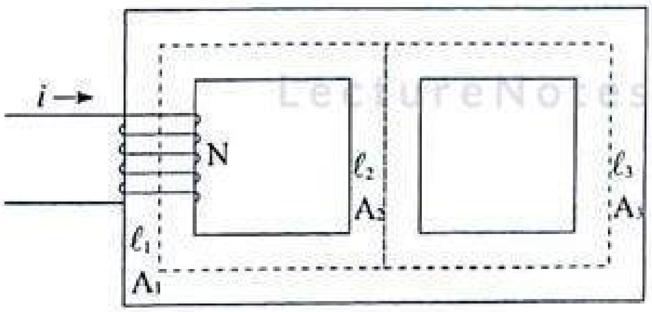


Fig 7.21

$N = 100\,turns, A_1 = 100cm^2, A_2 = 25cm^2, A_3 = 100cm^2$

$l_1 = 30cm, l_2 = 10cm\ and\ l_3 = 30cm$

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Solution : (i) $S_1 = \dfrac{l_1}{\mu_0\mu_r A_1} = \dfrac{30\times10^{-2}}{4\pi\times10^{-7}\times3000\times100\times10^{-4}} = 7961\cdot78\dfrac{AT}{wb}$

$$S_2 = \frac{l_2}{\mu_0\mu_r A_2} = \frac{10\times10^{-2}}{4\pi\times10^{-7}\times3000\times25\times10^{-4}} = 10615\cdot71\frac{AT}{wb}$$

$$S_3 = \frac{l_3}{\mu_0\mu_r A_3} = \frac{30\times10^{-2}}{4\pi\times10^{-7}\times3000\times100\times10^{-4}} = 7961\cdot78\frac{AT}{wb}$$

The circuit is shown in fig 7.22

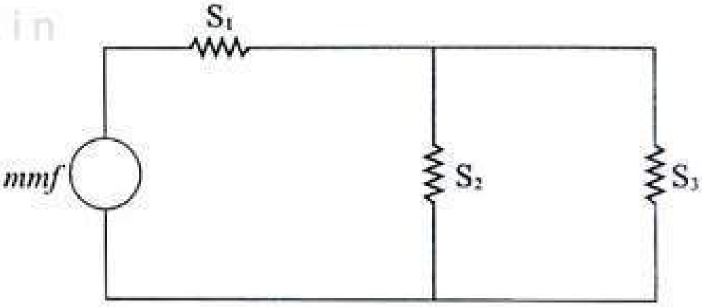

Fig 7.22

Total reluctance, $S = S_1 + \dfrac{S_2S_3}{S_2+S_3}$

$$\Rightarrow S = 1251\times10^3\ \frac{AT}{wb}$$

(ii) Inductance (L) $= \dfrac{N\phi}{I}$

$$= \frac{(N\phi)N}{NI} \qquad (\because \phi = \frac{NI}{S})$$

$$= \frac{N^2\phi}{NI} = \frac{N^2\left(NI/_S\right)}{NI}$$

$$= \frac{N^2}{S} = \frac{(100)^2}{1251\times10^3} = 0.7993\,H$$

(iii) Given $l_g = 0\cdot1\,mm = 0\cdot1\times10^{-3}\,m,\ A = 100\times10^{-4}\,m^2$

$$S_g = \frac{l_g}{\mu_0 A} = \frac{0\cdot1\times10^{-3}}{4\pi\times10^{-7}\times100\times10^{-4}} = 7961\cdot78\frac{AT}{wb}$$

S_g is in series with S_3

$\therefore$ Total reluctance, S $= S_1 + \dfrac{S_2(S_3+S_g)}{S_2+S_3+S_g} = 14\cdot33\times10^3\ \dfrac{AT}{wb}$

$\therefore$ Inductance (L), $= \dfrac{N^2}{S} = \dfrac{(100)^2}{1433\times10^3} = 0\cdot6978\,H$

(iv) As the gap gets longer, then S_g will also get larger and as an extreme case the circuit is made of S_1 and S_2 in series.

$\therefore$ Total reluctance $S = S_1 + S_2 = 18577\cdot43\dfrac{AT}{wb}$

Inductance (L) $= \dfrac{N^2}{S} = \dfrac{(100)^2}{18577\cdot43} = 0\cdot538\,H$

Example 7.12: The magnetic circuit shown in fig 7.23 has two parallel paths. Find the flux and flux density in each leg of the magnetic circuit. Neglect fringing at the air gap and any leakage fields. N = 1000 turns, I = 0.2 A, l_{g1}=0.02 cm and l_{g2}= 0.04cm. Assume the reluctance of the magnetic core to be negligible.

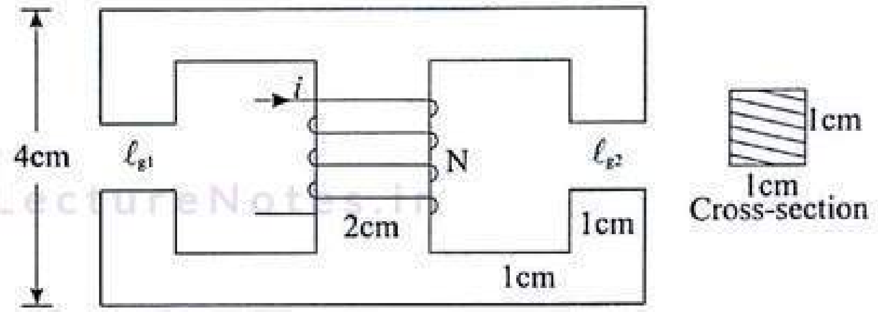

Fig 7.23

Solution : Given N = 1000 turns, i = 0.2A, $l_{g1} = 0\cdot02\times10^{-2}\,m$

$$l_{g2} = 0\cdot04\times10^{-2}\,m,\ A = (0\cdot01)^2\,m^2$$

Assume the reluctance of the material can be neglected. The analogous circuit is show in fig 7.24

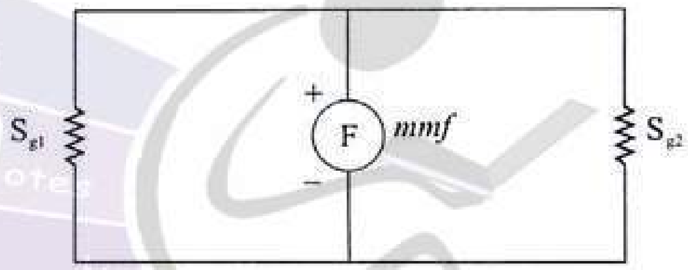

Fig 7.24

Reluctance of 1st air gap

$$= S_{g1} = \frac{l_{g1}}{\mu_0 A}$$

$$\Rightarrow S_{g1} = \frac{0\cdot02\times10^{-2}}{4\pi\times10^{-7}\times(0\cdot01)^2}$$

$$= 1\cdot59\times10^6\,\frac{AT}{wb}$$

$$S_{g2} = \frac{l_{g2}}{\mu_0 A} = \frac{0\cdot04\times10^{-2}}{4\pi\times10^{-7}\times(0\cdot01)^2} = 3\cdot1847\times10^6\,\frac{AT}{wb}$$

$$\phi_1 = \frac{Ni}{S_{g1}} = \frac{1000\times0\cdot2}{1\cdot59\times10^6} = 1.26\times10^{-4}\text{ wb}$$

$$\phi_2 = \frac{Ni}{S_{g2}} = \frac{1000\times0\cdot2}{3\cdot1847\times10^6} = 6.28\times10^{-5}\text{ wb}$$

$$B_1 = \frac{\phi_1}{A} = \frac{126\times10^{-4}}{(0.01)^2} = 1.26\,\frac{wb}{m^2}$$

$$B_2 = \frac{\phi_2}{A} = \frac{6\cdot28\times10^{-5}}{(0\cdot01)^2} = 0.63\,\frac{wb}{m^2}$$

Example 7.13 : Find the current necessary to establish a flux of $\phi = 3\times10^{-4}$ wb in series magnetic circuit of fig 7.25. Here $l_{iron} = l_{steel} = 0\cdot3m$, area (through out) = $5\times10^{-4}\,m^2$ and N=100 turns.

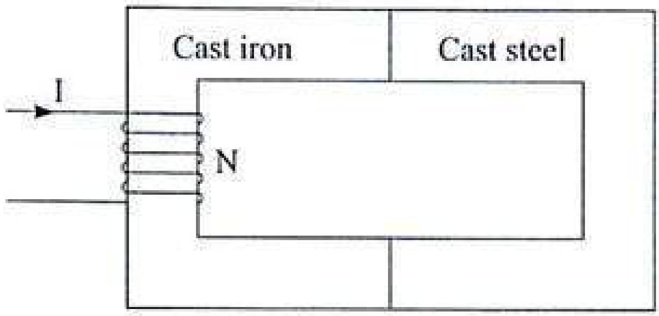

Fig 7.25

Solution : for cast iron $\mu_r = 519$

for cast steel $\mu_r = 1000$

Reluctance of cast iron

$$S_{iron} = \frac{l_{iron}}{\mu_0 \mu_r A} = \frac{03}{4\pi \times 10^{-7} \times 5195 \times 5 \times 10^{-4}}$$

$$= 91955.15 \frac{AT}{wb}$$

Reluctance of cast steel, $S_{steel} = \frac{l_{steel}}{\mu_0 \mu_r A} = \frac{0 \cdot 3}{4\pi \times 10^{-7} \times 1000 \times 5 \times 10^{-4}}$

$$= 477707 \frac{AT}{wb}$$

Total reluctance, S = $S_{iron} + S_{steel} = 91955 \cdot 15 + 477707$

$$= 56966215 \frac{AT}{wb}$$

But flux = $\frac{mmf}{relutance}$

$$\Rightarrow \quad \phi = \frac{NI}{S}$$

$$\Rightarrow I = \frac{\phi_s}{N} = \frac{3 \times 10^{-4} \times 569662 \cdot 15}{100} = 1.7 \text{ A}$$

Example 7.14 :- Find the magnetic flux φ established in the series magnetic circuit of fig 7.26

Solution : Given that current I = 2A, r = 0.8m

Area of cross - section A= $A = 0.009\, m^2$, $\mu_r = 1000$

$\therefore$ length $(l) = 2\pi r = 2\pi(0 \cdot 08) = 0 \cdot 5024 m$

Reluctance

$$(S) = \frac{l}{\mu_0 \mu_r A} = \frac{0 \cdot 5024}{4\pi \times 10^{-7} \times 1000 \times 0 \cdot 009} = 44444 \cdot 44 \frac{AT}{wb}$$

$$\text{Flux } (\phi) = \frac{mmf}{reluctance} = \frac{NI}{S} = \frac{100 \times 2}{44444 \cdot 44} = 4.5 \times 10^{-3} \text{ wb}$$

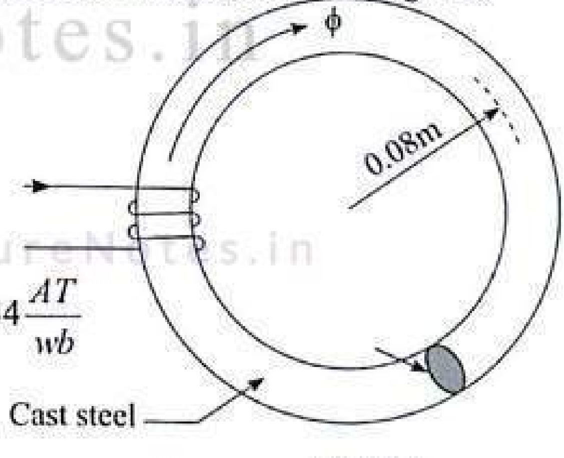

Fig 7.26

Example 5.15 : (a) Find the current I required to establish a flux $\phi = 2 \cdot 4 \times 10^{-4}\, wb$ in the magnetic circuit of fig 7.27. Here area (throughout) $A = 2 \times 10^{-4}\, m^2$, $l_{ab} = l_{ef} = 0.05m, l_{af} = l_{be} = 0.02m, l_{bc} = l_{dc}$ and the material is steel.

(b) Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results, using the value of μ for each material.

Solution : (a) Given the material is cast steel

So $\mu_r = 1000$

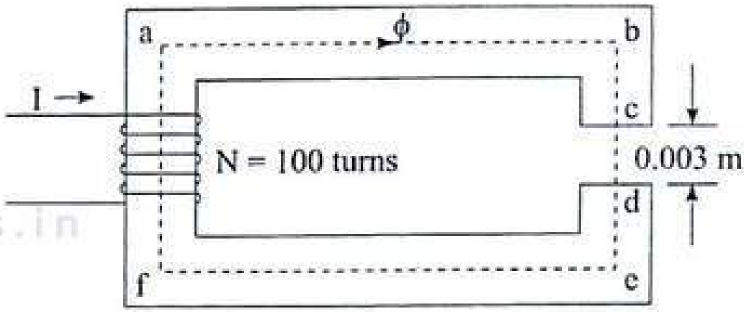

Fig 7.27

Flux density $B = \frac{\phi}{A} = \frac{2\cdot4\times10^{-4}}{2\times10^{-4}} = 1\cdot2\frac{wb}{m^2}$

Magnetic intensity of cost steel $(H_{steel}) = \frac{B}{\mu_0\mu_r}$

$= \frac{1\cdot2}{4\pi\times10^{-7}\times1000} = 955\cdot41\frac{AT}{m}$

Magnetic intensity of air $H_{gap} = \frac{B}{\mu_0} = \frac{12}{4\pi\times10^{-7}} = 955414\cdot01\frac{AT}{m}$

Total mmf = mmf of cast steel material + mmf of gap.

$\Rightarrow NI = H_{steel}\,(l_{af} + l_{ab} + l_{fe} + l_{bc} + l_{de}) + H_{gap}\,(l_{gap})$

$\Rightarrow 100I = 955\cdot41(0\cdot2 + 0\cdot05 + 0\cdot05 + 8\cdot5\times10^{-3} + 8\cdot5\times10^{-3}) + 955414\cdot01(0\cdot003)$

$\Rightarrow$ 2997.13 AT

$\Rightarrow$ I =29.97 A

(b) mmf of air gap = $H_{gap}\, l_{gap} = 2866\cdot24AT$

mmf of steel = $H_{steel}\, l_{steel} = 130\cdot89AT$

Example 7.16 : For the series-parallel magnetic circuit of fig 7.28 find the value of current I required to establish a flux in the gap of $\phi = 2\times10^{-4}$ wb. Here

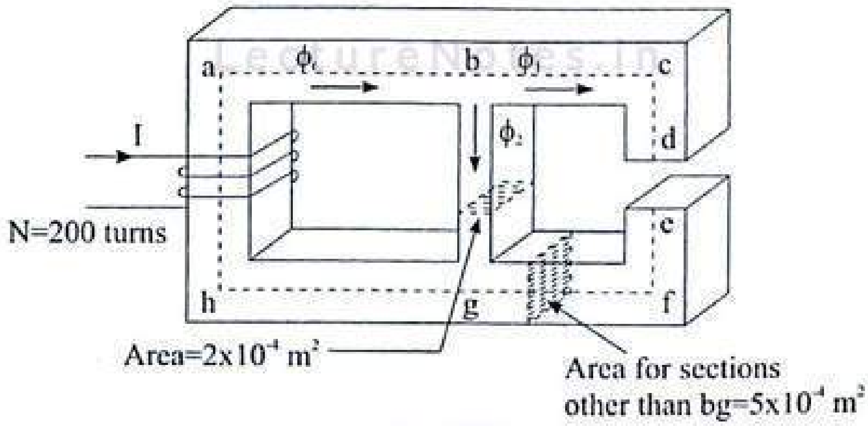

Fig 7.28

$l_{ab} = l_{bg} = l_{gh} = l_{ha} = 0.2m$, $l_{bc} = l_{fg} = 0.1m$, $l_{cd} = l_{ef} = 0.099m$ and the material in sheet steel

Solution : Let $\mu_r = 4000$

Given $A_1 = 2\times10^{-4}\, m^2$ *and* $A_2 = 5\times10^{-4}\, m^2$

Gap length $l_{gap} = l_{ha} - 2l_{cd} = 2\times10^{-3}\, m$

Reluctance at gap, $S_{gap} = \dfrac{l_{gap}}{\mu_0 A_2} = \dfrac{2\times10^{-3}}{4\pi\times10^{-7}\times5\times10^{-4}} = 3.18\times10^{6}\,\dfrac{AT}{wb}$

mmf at gap = $\phi_1 S_{gap} = 2\times10^{-4}\times3\cdot18\times10^{6} = 6\cdot36\times10^{2}$ AT

Reluctance at path ef = $S_{ef} = \dfrac{l_{ef}}{\mu_0\mu_r A_2} = \dfrac{0\cdot099}{4\pi\times10^{-7}\times4000\times5\times10^{-4}}$

$= 39410\cdot82\,\dfrac{AT}{wb}$

mmf of path 'ef' = $\phi_1 S_{ef} = 2\times10^{-4}\times39410.82 = 7.882$ AT

Reluctance of path 'bc' = $S_{bc} = \dfrac{l_{bc}}{\mu_0\mu_r A_2} = \dfrac{0.1}{4\pi\times10^{-7}\times4000\times5\times10^{-4}}$

$=39808.917\,\dfrac{AT}{wb}$

mmf of path 'bc'= $\phi_1 S_{bc}$ = 7.961 AT

mmf parallel = 2(mmfbc) + mmf gap +2 (mm ef)

$= 2(7.961) + 6.36\times10^{2} + 2(7.882)$

=667.686 AT

mmg of path 'bg' = mmf parallel = 667.686 AT

Reluctances of path 'bg' = $S_{bg} = \dfrac{l_{bg}}{\mu_0\mu_r A_1} = \dfrac{0.2}{4\pi\times10^{-7}\times4000\times2\times10^{-4}} = 199044\cdot58\,\dfrac{AT}{wb}$

Flux across path 'bg' = $\phi_2 = \dfrac{mmf\ \ of\ \ bg}{S_{bg}} = \dfrac{667\cdot686}{199044\cdot58} = 3.354\times10^{-3}\ wb$

Total flux, $\phi_T = \phi_1 + \phi_2 = 2\times10^{-4} + 3.354\times10^{-3} = 3.55\times10^{-3}$ wb

Reluctance of path 'ab' = $S_{ab} = \dfrac{l_{ab}}{\mu_0\mu_r A_2} = \dfrac{0.2}{4\pi\times10^{-7}\times4000\times5\times10^{-4}}$

$=79617.83\,\dfrac{AT}{wb}$

mmf of path 'ab' = $\phi_T S_{ab} = 3.55\times10^{-3}\times79617.83 = 282.9971$ AT

mmf series = mmf ab + mmf ah + mmf gh

$$= 282 \cdot 997 + 282 \cdot 997 + 282 \cdot 997$$

$$= 848 \cdot 99 \text{ AT}$$

Total mmf = mmf series + mmf parallel

$$= 848.99 + 667.686$$

$$= 1516.67 \text{ AT}$$

$$\therefore \quad \text{Current (I)} = \frac{mmf}{no\ of\ turns} = \frac{1516.67}{200} = 7.583A$$

Example 7.17 : A core is shown in fig 7.29 with $\mu_r = 2000$ and N=100. Find (a) the current needed to produce a flux density of $0.4 wb/m^2$ in the center leg. (b) the current needed to produce a flux density of $0.8 \frac{wb}{m^2}$ in the center leg.

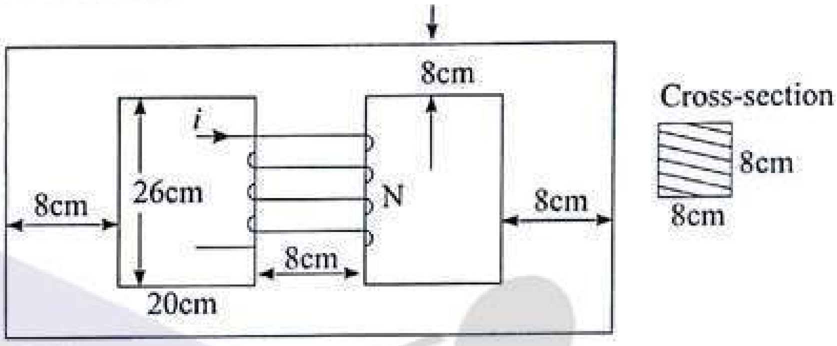

Fig 7.29

Solution. **:** The analogy circuit is shown in fig 7.30

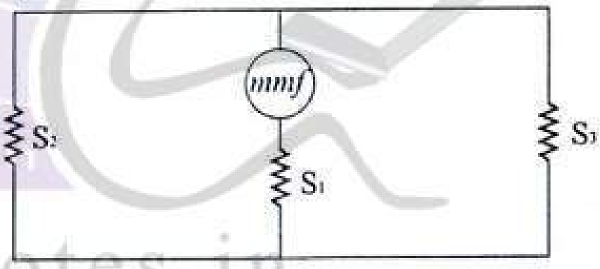

Fig 7.30

(a) $\quad S_1 = \frac{l_1}{\mu_0 \mu_r A}$

$$= \frac{0.34}{4\pi \times 10^{-7} \times 2000 \times (8 \times 10^{-2})^2}$$

$$= 2 \cdot 114 \times 10^4 \frac{AT}{wb}$$

$$S_2 = \frac{0.90}{4\pi \times 10^{-7} \times 2000 \times (8 \times 10^{-2})^2}$$

$$= 5 \cdot 598 \times 10^4 \frac{AT}{wb}$$

Similarly $S_3 = S_2 = 5.598 \times 10^4 \frac{AT}{wb}$

Total reluctance, $S = S_1 + \frac{S_2 S_3}{S_2 + S_3} = 4 \cdot 91 \times 10^4 \frac{AT}{wb}$

Flux (ϕ) $= \frac{mmf\ (NI)}{reluctance(s)}$

$$\Rightarrow I = \frac{\phi S}{N} = \frac{(BA)S}{N} = \frac{0 \cdot 4 \times (0 \cdot 08)^2 \times 4 \cdot 91 \times 10^4}{100}$$

$$\Rightarrow I = 1.256 \text{ A}$$

7.22 Transformer

A transformer is static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transferring electrical energy from one circuit to another circuit at the same frequency but with changed voltage (or current or both).

A transformer is essentially an a.c device. It can not work on d.c. When a.c voltage is raised n times, the corresponding alternating current reduces to 1/n times.

7.22.1 Principle of operation

A transformer is based on the principle of mutual induction i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is indued in the neighbouring coil.

When alternating voltage V_1 is applied to primary winding of a transformer then a current (I_0) flows through it. This current (I_0) is called exciting current which produces an alternating magnetic flux (ϕ) in the core. This flux links with both the windings (i.e. primary and secondary). According to Faraday's laws of electro-magnetic induction, emf E_1 is induced in primary and emf E_2 is induced in secondary winding. But according to lenz's law primary induced emf (E_1) will oppose the applied voltage and in magnitude this emf E_1 is almost equal to the applied voltage (V_1).

In brief we can say, emf induced in primary winding is equal and opposite to the applied voltage i.e. $E_1 = -V_1$

When a load is connected to the secondary side, then current will start flowing in the secondary winding. The voltage induced in the secondary winding is responsible to deliver power to the load connected to it. In this way electric power is transferred from primary winding to secondary winding through a magnetic circuit by electro magnetic induction. The induced emf in the secondary E_2 is in phase opposition to the applied voltage V_1 at primary. If the secondary is open circuited then terminal voltage V_2 at secondary is equal in magnitude and in phase with the induced emf at secondary. i.e. $E_2 = V_2$.

Fig 7.31

7.22.2. Construction of single phase transformers

A single phase transformer consists of primary and secondary windings placed on a magnetic core. The magnetic core is a stack of thin silicon steel laminations. The present trend is to use cold -rolled grain - oriented steel (i.e. CRGOS) which shows excellent magnetic properties. The thin laminations reduce eddy current loss and silicon steel reduces hysteresis loss. There are two basic types of transformer constructions, the core type and shell type.

$= R_2 + K^2 R_1$

Similarly X_{02} = effective reactance of whole transformer referred to secondary.

$= X_2 + K^2 X_1$

Z_{02} = effective impedance of whole transformer referred to secondary.

= secondary impedance + primary impedance referred to secondary.

$= Z_2 + Z_1^1$

$= Z_2 + K^2 Z_1$

Also $Z_{02} = R_{02} + jX_{02}$

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$

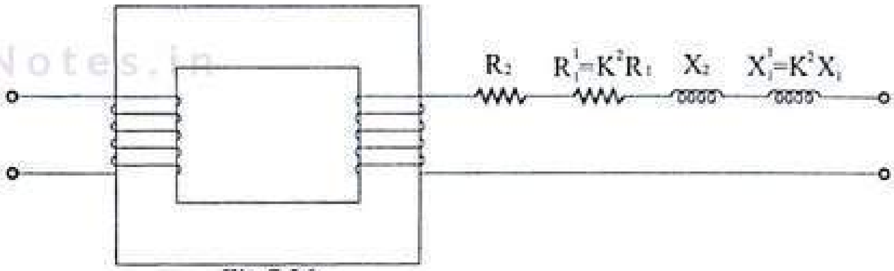

Fig 7.36

Note : It is important to remember that;

(1) When transfering resistance or rectance from primary to secondary, multiply it by K^2.

(2) When transferring resistance or reactance from secondary to primary, divide it by K^2.

(3) When transfering voltage or current from one winding to other, only K is used.

(a) any voltage V_1 in primary becomes KV_1 in secondary.

(b) any voltage V_2 in secondary becomes $\frac{V_2}{K}$ in primary.

(c) any currnet I_1 in primary becomes $\frac{I_1}{K}$ in secondary.

(d) any current I_2 in secondary becomes KI_2 in primary.

(4) A resistance R_1 in primary bcomes $K^2 R_1$ when transferred to secondary.

(5) A resistance R_2 in secondary becomes $\frac{R_2}{K^2}$ when transferred to primary.

(6) A reactance X_1 in primary becomes $K^2 X_1$ when transferred to secondary.

(7) A reactance X_2 in secondary becomes $\frac{X_2}{K^2}$ when transferred to primary.

7.22.8 Practical Transformer on No Load

A transformer is said to be on no load, if its primary winding is connected to an ac supply source and the secondary is open. The secondary current is thus zero. When an alternating voltage is applied to the primary, a small current I_0 flows in primary. The current I_0 is called no-load current of the transformer. It resolves into two rectangular components I_m and I_w. The component I_m is called

Example 7.21 :- If the transformer shown in fig 7.45 is ideal, find the turns ratio that will provide maximum power transfer to the load.

Solution : Given that

$R_1 = 1800$ ohm $\qquad R_1^1 = R_1 K^2$

$R_2 = 8$ ohm

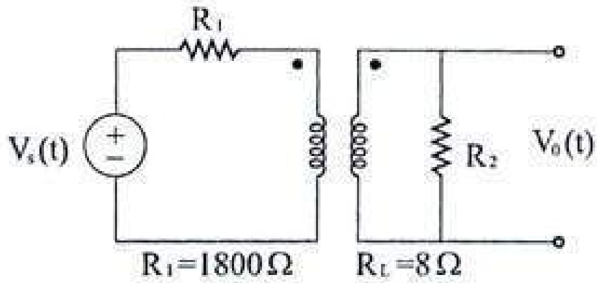

Fig 7.45

The equivalent circuit of the transformer referred to secondary side can be shown as in fig 7.46.

According to maximum power transfer theorem, maximum power can be delivered when

$$R_1^1 = R_2$$

$$\Rightarrow R_1 K^2 = R_2$$

$$\Rightarrow K = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{8}{1800}} = 0.067$$

$$\therefore \quad \text{Turn ratio} = \frac{N_1}{N_2} = \frac{1}{K} = \frac{1}{0.067} = 15$$

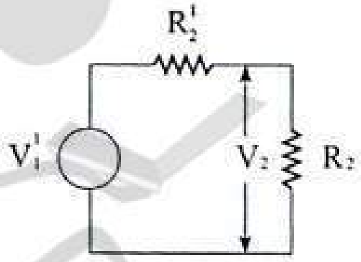

Fig 7.46

Example 7.22 : For the transformer shown in fig 7.47, N = 1000 turns, l_1 = 16cm, $A_1 = 4cm^2$,

l_2 = 22cm, $A_2 = 4cm^2$, l_3 = 5cm and $A_3 = 2cm^2$.

The relative permeability of the material μ_r = 1500.

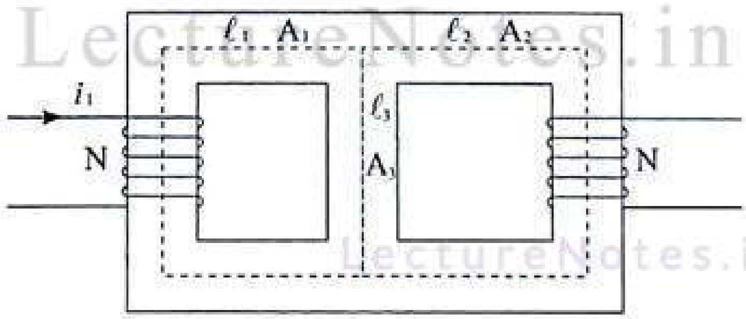

Fig 7.47

(a) Construct the equivalent magnetic circuit and find the reluctance associated with each part of the circuit.

(b) Determine the mutual inductance for the pair of coils.

In core type transformers, the windings surround the steel core. The core consists of two vertical legs or limbs with two horizontal sections, called yokes. To keep the leakage flux to a minimum, half of each winding is placed on each leg of the core as shown in fig 7.32(a). The low voltage winding is placed adjacent to the steel core and high voltage is placed outside to reduce the insulating material required.

In the shell type transformers the steel core surrounds the windings as shown in fig. 7.32 (b). The low voltage (LV) and high voltage (HV) windings are wound over the central limb.

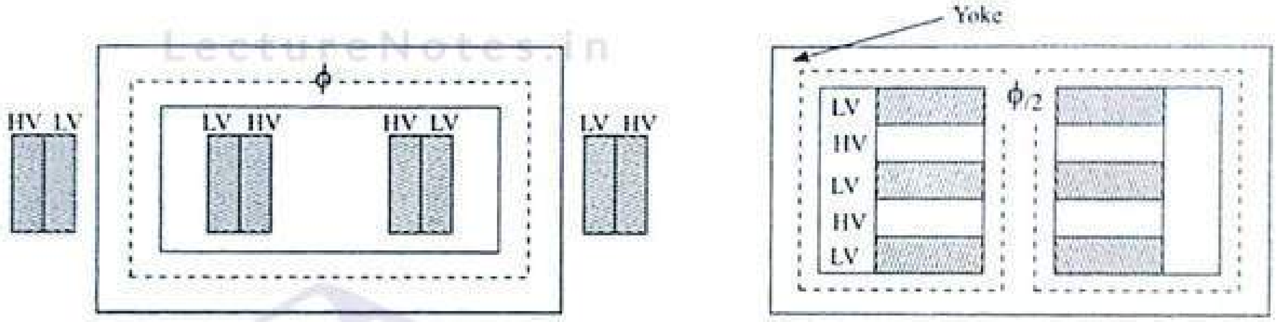

Fig 7.32

The shell type transformer requires more conductor material as compared to core type transfomer. In core type transformers the flux has a single path around the legs or limbs where as in shell type transformers the flux in the central limb devides equally and returns through the outer two legs.

7.22.3 EMF Equation

Since the applied voltage is sinusoidal at primary, the flux produced by exciting current (I_0) is also sinusoidal i.e. $\phi = \phi_m \sin\omega t$. According to Faradays laws of electromagnetic induction, the induced emf is,

$e = -N.\frac{d\phi}{dt}$, where N is the turns of the coil

$$\Rightarrow e = -N.\frac{d}{dt}(\phi_m \sin\omega t)$$

$$\Rightarrow e = -\omega\phi_m N\cos\omega t$$

$$\Rightarrow e = -2\pi f\,\phi_m\, N\cos\omega t \qquad \because \omega = 2\pi f$$

$$\Rightarrow e = -2\pi f\,\phi_m\, N\sin(90-\omega t)$$

$$\Rightarrow e = 2\pi f\,\phi_m\, N\sin(\omega t-90)$$

$$\Rightarrow e = E_m\, Sin\,(\omega t-90)$$

Where $E_m = 2\pi f\,\phi_m\, N$ = maximum value of induced emf 'e'.

For a sine wave, the r.m.s value of emf is given by,

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magnetising current. It magnetizes the core. It sets a flux in the core and therefore I_m is in phase with flux (ϕ_m). The current I_m is called reactive or wattless component of no load current.

The other component I_w produces eddy current and hystresis losses in the core and ~~mall~~ very small copper loss in primary. It is called active component or wattful component of no load current. It is in phase with applied voltage (V_1) at primary.

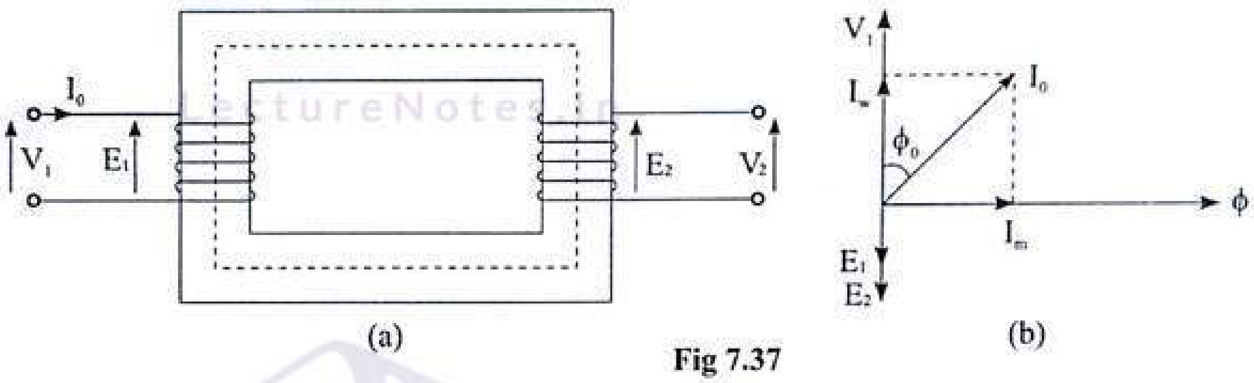


Fig 7.37

The no load current I_0 is not behind 90^0 with the applied voltage (V_1) but lags it an angle ϕ_0 as shown in fig 7.37 (b).

Note : (i) The no load current I_0 is small. So the drops in R_1 and X_1 on primary side are very small. We can say that at no load $V_1 = E_1$ (but V_1 and E_1 are 180^0 out of phase due to lenz's law).

(ii) The no load primary copper loss ($I_0^2 R_1$) is very small. So the no load primary input power is equal to iron loss .

(iii) Since a current is being drawn from the primary (I_0) with secondary open, a magnetising branch is available across the supply. This consists of a parallel combination of reactance (X_0) and resistance (R_0). The no load current component (I_0) is resolved into two components, namely I_w and I_m. The component I_w is responsible for no load loss and flows through R_0 and the component I_m is responsible for magnetising the core and flows through X_0 as shown in fig 7.38.

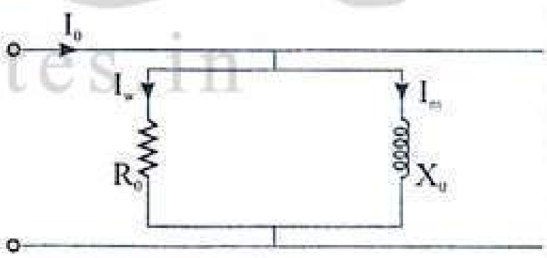


Fig 7.38

$$\therefore \quad I_w = I_0 \cos\phi_0$$

$$\text{and } I_m = I_0 \sin\phi_0$$

$$I_0 = \sqrt{I_m^2 + I_w^2}$$

$$\text{No load power factor, } \cos\phi_0 = \frac{I_w}{I_0}$$

Solution : Given that, N= 1000, $l_1 = 16\times10^{-2}\,m$

$$A_1 = 4\times10^{-4}\,m^2, \qquad l_2 = 22\times10^{-2}\,m, \quad A_2 = 4\times10^{-4}\,m^2$$

$$l_3 = 5\times10^{-2}\,m, \quad A_3 = 2\times10^{-4}\,m^2, \qquad \mu_r = 1500$$

Reluctance of the 1st path, $S_1 = \dfrac{l_1}{\mu_0\mu_r A_1}$

$$= \frac{16\times10^{-2}}{4\pi\times10^{-7}\times1500\times4\times10^{-4}}$$

$$= 2.12\times10^5\ \frac{AT}{wb}$$

Similarly $S_2 = \dfrac{l_2}{\mu_0\mu_r A_2} = \dfrac{22\times10^{-2}}{4\pi\times10^{-7}\times1500\times4\times10^{-4}} = 2.92\times10^5\ \dfrac{AT}{wb}$

and $S_3 = \dfrac{l_3}{\mu_0\mu_r A_3} = \dfrac{5\times10^{-2}}{4\pi\times10^{-7}\times1500\times2\times10^{-4}} = 1.33\times10^5\ \dfrac{AT}{wb}$

(a) Equivalent circuit bs shown in fig 7.48

(b) Self inductance of each part is given by,

$$L_1 = \frac{N^2}{S_1} = \frac{(1000)^2}{2.12\times10^5} = 4.72 \text{ Henry}$$

$$L_2 = \frac{N^2}{S_2} = \frac{(1000)^2}{2.92\times10^5} = 3.43 \text{ Henry}$$

and $$L_3 = \frac{N^2}{S_3} = \frac{(1000)^2}{1.33\times10^5} = 7.54 \text{ Henry}$$

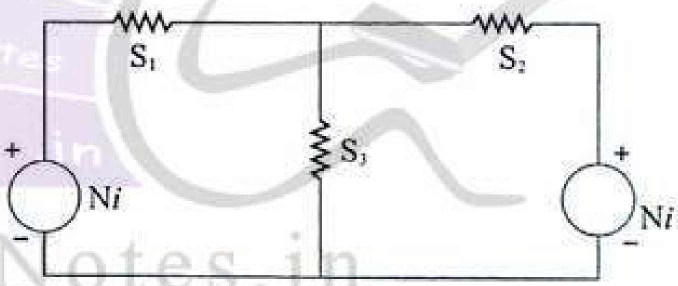


Fig 7.48

$\therefore$ Mutual inductance, $M = L_{12} = L_{21} = \dfrac{L_1 L_2}{L_1 + L_2 + L_3}$

$$= \frac{16.1896}{15.69} = 1.031$$

Example 7.23 : A transformer is delivering power to a 300Ω resistive load. To achieve the desired power transfer, the turns ratio is chosen so that the resistive load referred to the primary is 7500Ω. The parameter values referred to the secondary winding are.

$$R_1 = 20\Omega, \qquad L_1 = 1mH, \ L_M = 25 \text{ mH}$$

$$R_2 = 20\Omega, \qquad L_2 = 1 \text{ mH}$$

Core losses are negligible.

$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{2\pi f \phi_m N}{\sqrt{2}} = 4 \cdot 44\phi_m f \ N \text{ volt}$$

This is the emf equation of the transformer. If N_1 be the primary number of truns, then the rms values of induced voltage at primary is given by,

$$E_1 = 4.44\phi_m f \ N_1 \text{volt}$$

Similarly, the rms value of the induced emf at secondary is obtained as,

$$E_2 = 4.44 \ \phi_m f \ N_2 \text{volt}$$

7.22.4 Voltage Transformation Ratio

Emf induced at primary is $E_1 = 4 \cdot 44\phi_m \ f \ N_1$ volts

and emf induced at secondary is $E_2 = 4 \cdot 44\phi_m \ f \ N_2$ volts

$$\therefore \ \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

The ratio N_2 / N_1 is known as *voltage transformation ratio*. It is represented by K. **Thus voltage transformation ratio is defined as the ratio of secondary winding turns to primary winding turns and is the same as the ratio of secondary voltage to primary voltage.**

$$\therefore \ K = \frac{N_2}{N_1} = \frac{E_2}{E_1}$$

(i) If $N_2 > N_1$ i.e. K>1 then the transformer is called ***step-up transformer.***

(ii) If $N_2 < N_1$ i.e. K<1 then the transformer is called ***step-down transformer.***

7.22.5 Ideal Transformer

It is an imaginary transformer which has the following properties.

(i) The primary and secondary winding resistances are negligible, hence there is no resistive voltage drop and no resistive heat loss.

(ii) The leakage flux and leakge inductances are zero, there is no reactive voltage drop in the windings.

(iii) The power transfer efficiency is 100% i.e. there are no hystresis losses, eddy-current losses and heat losses due to resistance.

(iv) The permeability of the core is infinite so that it requires zero mmf to create flux in the core.

If I_1 and I_2 are the currents in the primary and secondary windings of an ideal transformer (i.e. having no losses), we should have

Power (volt -ampere) in primary = power (volt-ampere) in secondary.

$$\Rightarrow E_1 I_1 = E_2 I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K = \frac{V_2}{V_1}$$

No load input power (i.e. active power) = $V_1 I_0 \cos\phi_0$

No load reactive power = $V_1 I_0 \sin\phi_0$

7.22.9 Practical Transformer on Load

Practical transformer has winding resistance and leakage reactance. When a.c. supply is fed to primary then there is voltage drop in resistance (R_1) and reactance (X_1). So the primary emf E_1 is less than the applied voltage(V_1). Similarly there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary emf E_2.

When the secondary is loaded then secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f($N_2 I_2$) and hence its own flux ϕ_2. This flux ϕ_2 opposes the main primary flux (ϕ) which is due to I_0.The secondary ampere turns ($N_2 I_2$) are called demagnetising ampere turns. The opposing secondary flux ϕ_2 weakens the primary main flux ϕ momentarily. Hence primary back e.m.f E_1 reduces. For a moment V_1 increases over E_1 and hence causes more current to flow primary. Let the additional primary current be I_2^1. This current is anti phase with I_2. The additional primary mmf $N_1 I_2^1$ sets up its own flux ϕ_2^1. The flux ϕ_2^1 is equal and opposite to flux ϕ_2. Hence the two cancel each other. As a result magnetic effects of secondary current I_2 are immediately neutralized by additional primary current I_2^1. So we can write $N_2 I_2 = N_1 I_2^1$

$$\Rightarrow \quad I_2^1 = \frac{N_2}{N_1} I_2$$

$$\Rightarrow \quad I_2^1 = K I_2 \qquad \because K = \frac{N_2}{N_1}$$

Hence when transformer is on load, the primary winding has two currents, one is I_0 and other is I_2^1 which is antiphase with I_2 and K times in magnitude. The total primary current (I_1) is the vector sum of I_2^1 and I_0

$\therefore \; I_1 = I_2^1 + I_0$

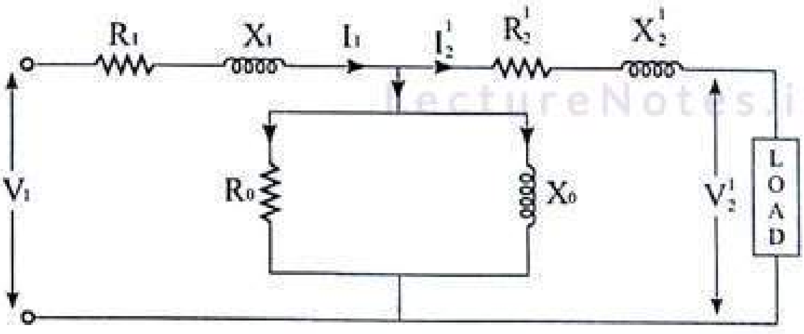


Fig 7.39

The fig7.39 shows the equivalent circuit of the transformer referred to the primary side. From the fig 7.39 it is clear that the primary current on load is the vector sum of no load current (I_0)and secondary

(a) Determine the turns ratio

(b) Determine the input voltage, Current and power and the efficiency when this transformer is delivering 12W to the 300Ω load at a frequency f = 10,000/2π HZ.

Solution : Load resistance = R_L = 300 ohms

Load resistance referred to primary side = $R_L^1 = \frac{R_L}{K^2} = 7500$ ohms

$R_1^1 = 20$ *ohms*, $R_2 = 20$ ohms, $L_1^1 = 1$ mH, $L_2 = 1.0$ mH

Magnetising inductance $L_m^1 = 25$ mH

Out put power = 12 watts

Supply frequency $f = \frac{10000}{2\pi}$ HZ

$\therefore$ $W = 2\pi f = 2\pi(\frac{10000}{2\pi}) = 10000$ rad/sec

$X_1^1 = wL_1^1 = 10000 \times 10^{-3} = 10$ ohms

$X_2 = wL_2 = 10000 \times 10^{-3} = 10$ ohms and $X_m^1 = wL_m^1 = 250$ ohms

The equivalent circuit of the transformer referred to secondary side is show in fig 7.49

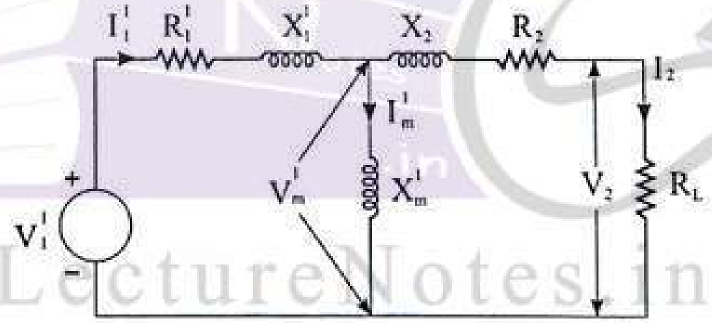


Fig 7.49

(a) $R_L^1 = \frac{R_L}{K^2}$

$\Rightarrow K = \sqrt{\frac{R_L}{R_L^1}} = 0.2$

Turn ration = $\frac{N_1}{N_2} = \frac{1}{K} = 5$

(b) Assuming output voltage V_2 to be the reference vector.

$V_2 = \sqrt{R_L P} = \sqrt{300 \times 12} = 600 = 60\angle 0^0$ volts

Thus we find that the current is transformed in the reverse ratio of the voltage. If a transformer steps up the voltage then it steps down the current. Similarly if a transformer steps down the voltage then it steps up the current.

7.22.6. Practical Transformer

Practical transformers are far different from ideal transformers. The practical transformer has (i) winding resistances, (ii) leakage reactances, (iii) iron losses

(i) Winding resistance :

In actual transformer the winding, (both primary and secondary) consist of copper conductors, so it posses resistance. The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown fig 7.33

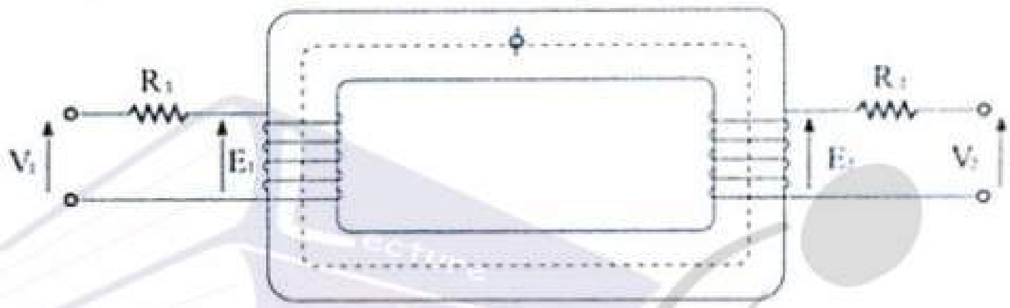


Fig 7.33

(ii) Leakage reactance

In the ideal transformer, we assumed that all the magnetic flux produced by primary windining links both the primary and secondary windings. But in actual transformer a portion of this flux is diverted to the surroundings. It is because the surrounding medium (air or oil) has a definite permeability. This small portion of the flux which traverses an external path is known as primary leakage flux ϕ_{L1} as shown in fig 7.34 (a) This flux ϕ_{L1} Links only the primary and induces an emf E_{L1} in primary.

The secondary current I_2 produces a flux ϕ_2 which opposes the main flux ϕ_m. A portion of this flux ϕ_2 is also diverted to the surrounding. This leakage flux is called secondary leakage flux (ϕ_{L2}). It only links the secondary turns and induces an emf E_{L2} in the secondary. The flux which passes completely through the core links both in windings is called mutual flux and is shown as ϕ_m in fig 7.34 (a)

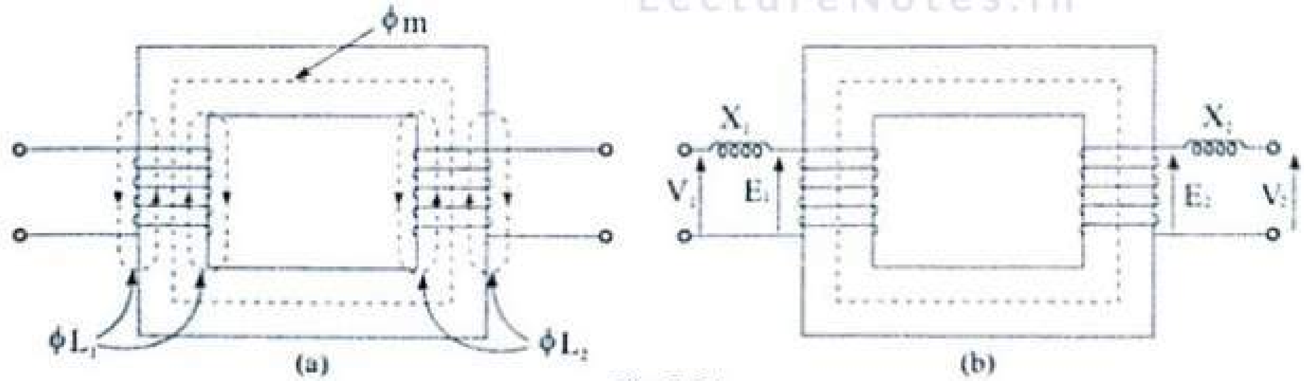
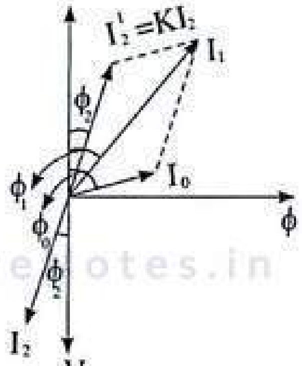
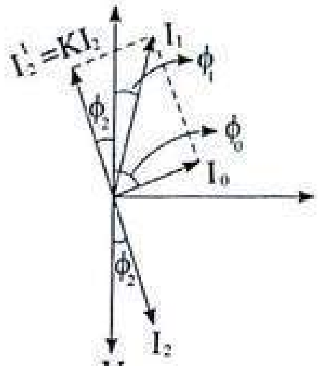


Fig 7.34

current referred to primary side (I_2^1). The vector diagram showing I_0, I_2, I_2^1 and I_1 for different power factor loads is shown in fig 7.40.



(a) for lagging load



(b) for leading load

Fig 7.40

Example 7.18 : For the circuit shown in fig 7.41, assume that V_g = 120 volt rms. Find (a) the total resistance seen by the voltage source. (b) The primary current (c) The primary power.

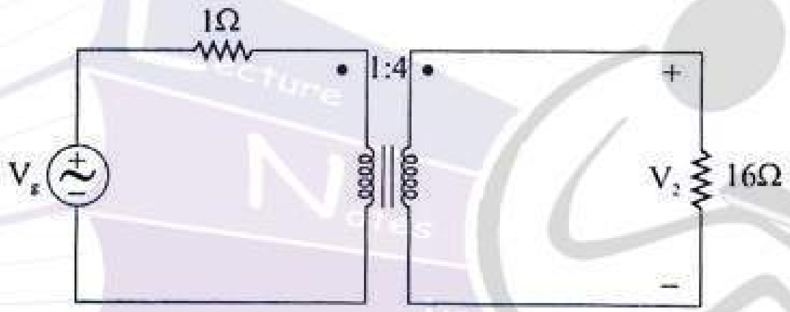


Fig 7.41

Solution : Given primary voltage V_1 = 120 volts

R_1=1 ohm, R_2= 16 ohm

$$\text{Turn ratio} = \frac{N_1}{N_2} = \frac{1}{4}$$

$$\therefore K = \frac{N_2}{N_1} = 4$$

$$R_2^1 = \frac{R_2}{K^2} = \frac{16}{16} = 1 \text{ 0hm}$$

Fig 7.42

(a) ∴ The total resistance seen by the supply side (primary) = $R_1 + R_2^1$ = 2 ohms

(b) Primary current $I_1 = \dfrac{V}{R_1 + R_2^1} = 60$ A

$$I_2 = \frac{V_2}{R_L} = \frac{60\angle 0^0}{300} = 0\cdot 2\ \angle 0^0\ A$$

$$V_m^{i} = V_2 + I_2\left(R_2 + jx_2\right) = 60\angle 0^0 + 0\cdot 2\angle 0^0\ \ (20 + j10)$$

$$= 60 + 4 + j2 = 64 + j2 = 64\cdot 031\angle 1\cdot 8^0 \quad \text{volt}$$

$$I_m^1 = \frac{V_m^1}{jX_m^1} = \frac{64\cdot 031\angle 1\cdot 8^0}{250\angle 90^0} = 0\cdot 256\ \angle -88\cdot 2^0\ A$$

$$\therefore\ I_1^1 = I_2 + I_m^1 = 0\cdot 2\angle 0^0 + 0\cdot 256\ \angle -88\cdot 2^0$$

$$= 0\cdot 2 + 8\cdot 041\times 10^{-3} - j0\cdot 255\times 10^{-3}$$

$$= 0\cdot 33\angle -50\cdot 9\ \ A$$

$$V_1^1 = I_1\left(R_1^1 + jx_1^1\right) + V_m^1$$

$$= 0\cdot 33\angle -50\cdot 9\times (20 + j10) + 64 + j2$$

$$= 70\cdot 72\ \angle -0\cdot 84^0$$

Taking magnitude only input voltage $V_1 = V_1^1 / k$

$$= \frac{70\cdot 72}{0\cdot 2} = 353\cdot 6\angle -0\cdot 84\ \text{volts}$$

$$\therefore I_1 = I_1^1 K = 033\times 02 = 0.066\ \angle -50.9\ A$$

Angle between V_1 and I_1 = 50.9 - 0.84 = 50.06^0

Input power to the transformer $= V_1\ I_1\ \cos\phi$

$$= 353\cdot 6\times 0\cdot 066\times \cos 50.56$$

$$= 14\cdot 98\ \textit{watts}$$

$$\therefore\ \textit{Efficiency} = \frac{\textit{output}}{\textit{input}} = \frac{12}{14\cdot 98} = 0\cdot 80 = 80\%$$

Example - 7.24 : The high voltage side of a transformer has 750 turns, and the low voltage side has 50 turns. When the high side is connected to a rated voltage o 120 volt, 60 Hz, a rated load of 40A is connected to the low side.

Calculate : (a) The turns ratio

(b) The secondary voltage (assuming no internal transformer impedance voltage drops).

(c) The resistance of the load.

Solution : Given that, $N_1 = 750$, $N_2 = 50$, $V_1 = 120\ \textit{volts}$

$$f = 60\,Hz,\ I_2 = 40A$$

It should be noted that the induced voltages E_{L1} and E_{L2} due to leakage flux ϕ_{L1} and ϕ_{L2} are different from induced voltages E_1 and E_2. As the leakage flux linking with each winding produces a self-induced emf in that winding, hence the effect of leakage flux is equivalent to an inductance in series with each winding. So a practical transformer has inductive reactances X_1 and X_2 connected in series with primary and secondary windings respectively as shown in fig 7.34 (b). The reactances X_1 and X_2 are called as primary and secondary leakage reactances.

(iii) Iron losses

As core of the transformer subjected to alternating flux, there occurs eddy current and hystersis loss in it. These two losses are known as ***iron loss (or core loss).*** The iron losses depnd upon supply frequency, maximum flux density in the core and volume of the core. Its value is less in practical transformer.

7.22.7 Impedance Reflection and Power Transform

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance either to the primary or to the secondary. In that case, we would have to work in one winding only which is more convenient.

(i) Equivalent values reflected to primary :

Let R_2 is the resistance of secondary winding. R_2^1 is the resistance of secondary winding reflected to primary. This reflected resistance R_2^1 should produce the same effect in primary as R_2 produces in secondary. Therefore power consumed by R_2^1 when carrying the primary current is equal to the power consumed by R_2 due to secondary current.

$$\therefore \quad \left(I_2^1\right)^2 R_2^1 = I_2^2 R_2$$

$$\Rightarrow \quad R_2^1 = \left(\frac{I_2}{I_2^1}\right)^2 R_2 \text{ ....(1)}$$

Where I_2^1 is the current of secondary winding reflected to primary winding. This current I_2^1 is used to counteract the demagnetizing effect of secondary current I_2.

Therefore $N_1 I_2^1 = N_2 I_2$

$$\Rightarrow \frac{I_2^1}{I_2} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{I_2^1}{I_2} = \text{K} \qquad \because \quad \frac{N_2}{N_1} = \text{K}$$

$$\Rightarrow I_2^1 = KI_2$$

Putting this value in equation (1) we get,

$$R_2^1 = \frac{1}{K^2} R_2 = \frac{R_2}{K^2}$$

(c) The primary power is $= I_1^2 \ (R_1 + R_2^1)$

$= (60)^2 (2) = 7200 = 7.2$ Kw

Example 7.19 : An ideal transformer is rated to deliver 460 KVA at 380V to acustomer as shown in fig 7.43.

(a) How much current can the transformer supply to the customer ?

(b) If the customer's load is purely resistive (i.e. p.f=1), what is the maximum power that the customer can receive ?

(c) If the customer's power factor is 0.8 (lag), what is the maximum usable power the customer can receive ?

(d) What is the maximum power if the pf is 0.7 (lag) ?

(e) If the customer requires 300Kw to operate, what is the minimum power factor with the given size transformer ?

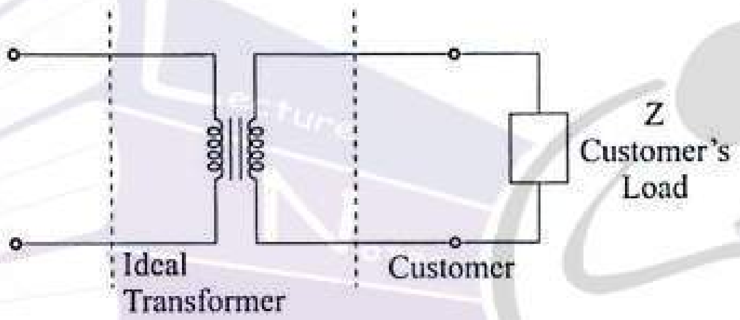


Fig 7.43

Solution : Given that $V_2 = 380$ volts

Maximum apparent power output of the transformer is = 460 KVA.

Since the transformer is ideal, $V_1 I_1 = V_2 I_2 = 460 \times 10^3 \, VA$

(a) Supply current to the Customer $(I_2) = \dfrac{460 \times 10^3}{V_2} = \dfrac{460 \times 10^3}{380} = 1.21 \times 10^3 \, A$

(b) For resistive load $\cos\phi = 1$

$\therefore$ Power output $= V_2 I_2 \cos\theta = 460 \times 10^3 \times 1 = 460 \times 10^3$ *watts*

(c) When $\cos\phi = 0.8$ the maximum out put power $= 460 \times 10^3 \times 0.8 = 368 Kw$

(d) When $\cos\phi = 0.7$ the maimum out put power $= 460 \times 10^3 \times 0.7 = 322 Kw$

(e) Required out put power = 300Kw

$\therefore$ Minimum power factor required for the load

$$Cos\phi = \frac{V_2 I_2 \cos\phi}{V_2 I_2} \phi = \frac{300 \times 10^3}{460 \times 10^3} = 0.65$$

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and delivery.

(a) Turn ratio = $\frac{N_1}{N_2} = \frac{750}{15} = 15$

(b) $\frac{N_2}{N_1} = \frac{V_2}{V_1}$

$\Rightarrow V_2 = \frac{N_2}{N_1} . V_1 = \frac{50}{750} \times 120 = 8 \; volts$

(c) Load resistance, $R_L = \frac{V_2}{I_2} = \frac{8}{40} = 02 \; ohms$

Example - 7.25 : The high voltage side of a step-down transformer has 800 turns and the low voltage side has 100 turns. A voltage of 240 v AC is applied to the high side and the load impedance is 3 Ω (low side). Find

(a) The secondary voltage and current.

(b) The primary current

(c) The primary input impedance from the ratio of primary voltage to current.

(d) The primary input impedance.

Solution : Given that, $N_1 = 800, \; N_2 = 100, \; V_1 = 240 \, volts, \; R_L = 3 \; ohms$

(a) $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

$\Rightarrow V_2 = V_1 . \frac{N_2}{N_1} = 240 \times \frac{100}{800} = 30 \; volts$

$I_2 = \frac{V_2}{R_L} = \frac{30}{3} = 10 \; A$

(b) Total input current $I_1 = K \, I_2 = \frac{1}{8} \times 10 = \frac{10}{8} \; A$

(c) *Input impedance seen from the primary side* $= \frac{V_1}{I_1} = \frac{240}{10/8} = 192 \; ohms$

(d) Considering the transformer to be ideal i.e. R_1, R_2, X_1 *and* X_2 being neglected, only impedance available on the secondary is $R_L = 3 \; ohms$

∴ Im*pedance of the transformer seen from primary side*

$$Z_{in} = \frac{R_L}{K^2} = \frac{3}{(1/8)^2} = 192 \; ohms.$$

Example 7.26 : A 2300 / 240 volt, 60 Hz, 4.6 KVA transformer is designed to have an induced emf of 2.5 *v / turn*. Assuming an ideal transformer, find

Let X_2 is the reactance of secondary winding

X_2^1 is the reactance of secondary winding reflected to primary as shown in fig 7.35.

This reflected reactance X_2^1 should produce the same effect in primary as X_2 produces in secondary. Therefore reactive power absorbed by X_2^1 when carrying the primary current is equal to the reactive power absorbed by X_2 due to secondary current.

$$\therefore \quad (I_2^1)^2 X_2^1 = I_2^2 X_2 \qquad \because \text{ Reactive power} = \text{V I Sin}\,\phi = \text{IZ} \cdot \text{I} \cdot \frac{X}{Z} = I^2 X$$

$$\Rightarrow X_2^1 = \left(\frac{I_2}{I_2^1}\right)^2 X_2$$

$$\Rightarrow X_2^1 = \frac{1}{K^2} X_2 \qquad \because \frac{I_2^1}{I_2} = K$$

$$\Rightarrow X_2^1 = \frac{X_2}{K^2}$$

Let R_{01} = effective resistance of the whole transformer referred to primary.

=primary resistance + secondary resistance referred to primary

$= R_1 + R_2^1$

$= R_1 + \frac{R_2}{K^2}$

Similarly X_{01} = effective reactance of the whole transformer reffered to primary.

$= X_1 + X_2^1$

$= X_1 + \frac{X_2}{K^2}$

Z_{01} = effective impedance of whole transformer referred to primary.

= primary impedance + secondary impedance referred to primary.

$= Z_1 + Z_2^1$

$= Z_1 + \frac{Z_2}{K^2}$

Also $Z_{01} = R_{01} + jX_{01}$

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

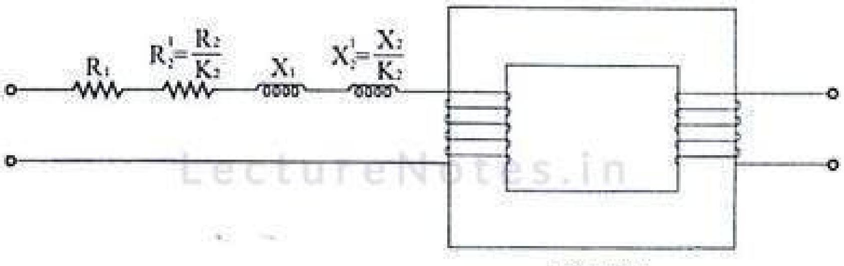

Fig 7.35

(ii) Equivalent values reflected to secondary

The equivalent values reflected to secondary can also be found in same manner.

R_{02} = effective resistance of the whole transformer referred to secondary.

= secondary resistance + primary resistance referred to secondary.

$= R_2 + R_1^1 \qquad \because R_1^1 = K^2 R_1$

Example 7.20 : For the ideal transformer shown in fig 7.44, consider that $V_s(t)$= 294 cos (377t) volt. Find (a) Primary current, (b) $V_0(t)$ (c) Secondary power (d) The installation efficincy P_{load} / P_{source} .

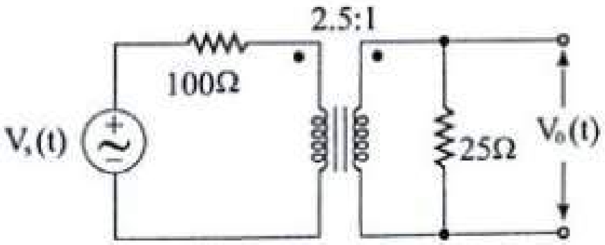

Fig 7.44

Solution : Given that supply voltage $V_1(t)$ = 294 cos 377t volts.

$$\text{Turn ration} = \frac{N_1}{N_2} = 2.5$$

R_1 = 100 ohms, R_2 = 25 ohms

(a) $$R_2^1 = \frac{R_2}{K^2} = \frac{25}{(1/25)^2} = 15625 \text{ ohms} \quad (\text{where } K = \frac{N_2}{N_1} = \frac{1}{25})$$

$$\therefore \text{ Maximum value of primary current} = \frac{V_{1\,max}}{R_1 + R_2^1} = \frac{294}{100+15625} = 1.147A$$

Instantaneous value of primary current is $i_1(t)$ = 1.147 cos 377t A

(b) $$i_2(t) = \frac{i_1(t)}{K} = \frac{1\cdot147\cos 377t}{1/2\cdot5} = 2\cdot86\cos 377t\,A$$

$\therefore V_0(t) = i_2(t)R_2$ = 71.68 cos 377t volts.

(c) Secondary power $P_2 = I_2 \; V_0 \; \cos\phi$

$$I_2 = \text{rums value of secondary current} = \frac{2.86}{\sqrt{2}} = 2.02 \text{ A}$$

$$V_0 = \text{rums value of out put voltage} = \frac{71.68}{\sqrt{2}} = 50.68 \text{ volts}$$

Cos ϕ = cos 0 = 1

$$\therefore \; P_2 = I_2 \; V_0 \; \cos\phi = 2.02 \times 50.68 \times 1 = 102.37 \text{ watts.}$$

(d) $$\text{Input power } (P_1) = V_1 I_1 = \frac{294}{\sqrt{2}} \times \frac{1.147}{\sqrt{2}} = 168.61 \text{ watt}$$

Cos ϕ = 1 for resistive load and ideal transformer.

$$\therefore \text{ Installation efficiency} = \frac{P_2}{P_1} = \frac{102.37}{168.61}$$

=0.607

= 60.7%

(a) The numbers of high-side turns N_1 and low side turns N_2

(b) The rated current of the high voltage side I_1.

(c) The transformer ratio when the device is used as a step-up transformer.

Solution : Given that $V_1 = 2300\ volts$

$V_2 = 240\ volts \qquad f = 60Hz$

Rated Power = 4.6 KVA

Induced emf per turn = 2.5 volts

(a) Primary turns, $N_1 = \dfrac{V_1}{25} = \dfrac{2300}{25} = 920$

Secondary turns, $N_2 = \dfrac{240}{25} = 96$

(b) Primary Current $I_1 = \dfrac{rated\ KVA}{V_1} = \dfrac{4.6 \times 10^3}{2300} = 2\ A$

(c) Transformation ratio $= K = \dfrac{V_2}{V_1} = \dfrac{240}{2300} = 0.1043$

Example - 7.27 : A transformer is used to match an 8 ohm loudspeaker 500 Ω audio line. What is the turns ratio of the transformer and what are the voltages at the primary and secondary terminals when 10 watt of audio power delivered to the speaker ? Assume that the speaker is a resistive load and that the transformer is ideal.

Solution : In this problem a tansformer is used for impedance matching whose turn ratio is N_1 / N_2.

Given that, $R_1 = R_s = 500\ ohms$

$R_2 = R_L = 8\ ohms$

Assuming the transformer is to be ideal.

To match R_L with R_s we must have

$R_s = R_L^1$

$\Rightarrow \quad R_1 = R_2^1$

$\Rightarrow \quad 500 = \dfrac{R_2}{K^2}$

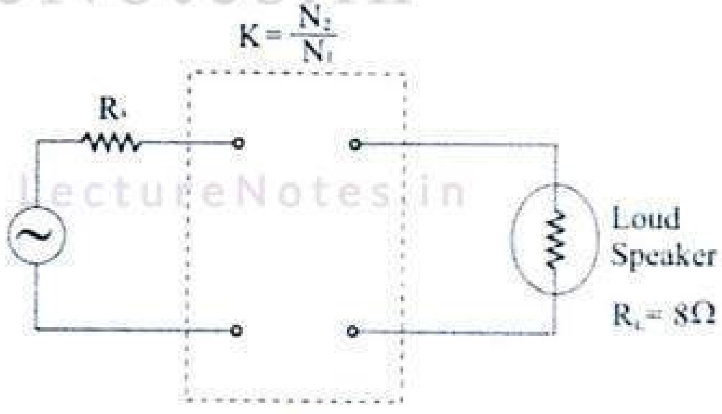

Fig 7.50

$\Rightarrow \quad K = \sqrt{\dfrac{R_2}{500}} = \sqrt{\dfrac{8}{500}} = 0.126$

$\therefore\ Turn\ ratio = \dfrac{1}{k} = 8$

Problem Solution Incomplete

7.23.Electro mechanical Energy Conversion

Electro- magneto mechanical devices are those which converts mechanical forces and displacements to electro-magnetic energy and vive versa. In this section we shall discuss the basic principle of energy conversion in electro- magneto mechanical system in applying the devices like, energy transducers..

A transducer is a device which converts mechanical energy to electrical engergy or vice versa. Accurately the transducer which converts electrical to mechanical energy known as actuator and those which converts mechanical energy to electrical energy known a sensors. The transducer has wide applications in controlling large systems such as electrical power systems, robotics, processing industry etc,

Conversion of electrical to mechanical energy and back is based on the following effects.

(i) Piezo- electric effect.

(ii) Electro-striction

(iii) Magneto - Striction

Piezo- electric effect is the process in which the change in electric field takes place with presence of strain in certain crystals such as quartz.This change in electric field is utilised for generating an electric signal.

The electrostriction is an effect in which there is a change in the dimensions of certain materials will lead to change in the electrical properties. Similarly magneto- striction is an effect in which change in dimension of certain material changes its magnetic properties,

These effects lead to intresting applications such as current sensor voltage sensor, speed sensor etc.

7.24.Forces in magnetic structure

Mechanical forces can be converted to electrical signals by means of the coupling provided by the energy stored in magnetic field similarly electrical signals can be converted to mechanical forces by means of the coupling provided by the energy stored in magnetic field,. How ever some losses takes place during this conversion such as resistive losses and friction losses . The fig 7.52 shows the flow of energy in both cases.

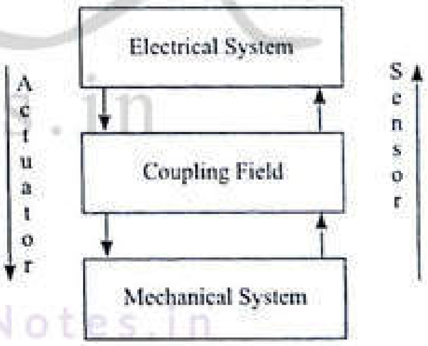

Fig 7.51

Fig 7.52

Now let us discuss some of the applications in electro - mechanical energy conversion'.

7.25 Moving iron transducer

Moving iron transducer is an electromagnet which consists of a U-shaped element and a bar. The U-shaped element is fixed and bar is movable.

The Principle of M.I transducer is that for a mass to be displaced, some work needs to be done, this work corresponds to a change in the energy stored in the electromagnetic field, which causes the mass to be displaced.

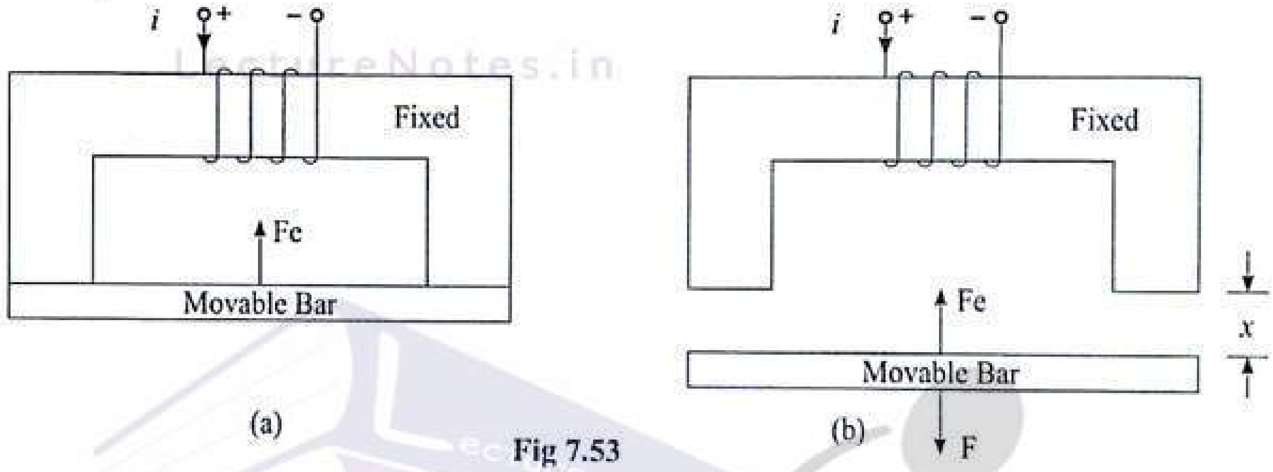

Fig 7.53

When current i flows through the coil, then U-shaped element behaves as a magnet which attracts the movable bar as shown in fig 7.53(a). Let Fe represents the magnetic force acting on the bar. If the bar moves through a small distance x against the magnetic force Fe then some mechanical work is to be done, It is shown in fig 7.53 (b)

The interaction between the electric and mechanical terminals & i.e the electro- mechanical energy conversion, occurs through the medium of the magnetic stored energy . According to Law of Conservation of energy,

Electrical energy input = increase in stored energy + mechanical work done by the system.

Let dx = Small displacement of the bar in small time dt .

Amount of mechanical work done in time dt is = $F_e \cdot dx$

Amount of electric work done in time dt is = ei.dt

Amount of energy stored in time dt is = dW_m (say)

Putting these above values in equation (1) we get ,

$$ei \cdot dt = dW_m + F_e \cdot dx$$

$$\Rightarrow F_e \cdot dx = ei \cdot dt - dW_m$$

$$\Rightarrow F_e \cdot dx = \left(\frac{d\lambda}{dt}\right) i \cdot dt - dW_m \qquad \because e = \frac{d\lambda}{dt}$$

$$\Rightarrow F_e \cdot dx = i \cdot d\lambda - dW_m$$

$$\Rightarrow F_e = i \cdot \frac{d\lambda}{dx} - \frac{dW_m}{dx}$$

$$\Rightarrow F_e = \frac{d}{dx}\left(i\lambda - W_m\right) = \frac{d}{dx} W_m^1 \text{................ (2)}$$

Where $W_m^1 = i\tau - W_m =$ ***co-energy*** (a fictitious quantity but sometimes useful)

The relationship between flux linkage $\lambda = (N\phi)$ and current i is non -linear, and might be described by a curve as shown in fig 7.54

Let $OP = \lambda$

and OR = i

Area of OPQR= length × breadth

$= OP \times OR$

$= \lambda i$

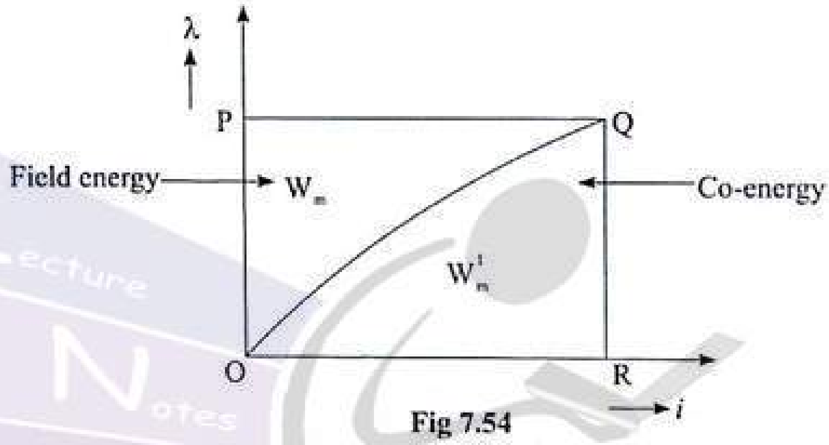

Fig 7.54

Area under the curve = area of OQR
$= W_m^1 = co - energy$

Area above the curve = area of OPQ $= W_m = field\ energy\ stored.$

∴ Area of OQR = area of OPQR - area of OPQ

$\Rightarrow \quad W_m^1 = \lambda i - W_m$

If λ and i relationship is linear then, area of OPQ = area of OQR

$$\Rightarrow W_m = W_m^1$$

Equation (2) becomes,

$$F_e = \frac{d}{dx} W_m^1 = \frac{d}{dx} W_m \text{....... (3)}$$

Equation (3) stating that the magnetic force acting on the moving iron is proportional to the rate of change of stored energy with displacement, applies only for linear magnetic structure.

But energy stored in magnetic structure is given by,

$$W_m = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{N\phi}{i}\right)\left(i^2\right) \qquad \because L = \frac{N\phi}{i}$$

$$= \frac{1}{2}(N\phi)\,i$$

$$= \frac{1}{2}(Ni)\phi$$

$$= \frac{1}{2}(\phi S)(\phi) \qquad \because Flux(\phi) = \frac{mmf\ (NI)}{reluctance(s)} \Rightarrow NI = \phi S$$

$$= \frac{1}{2}\phi^2 S \ \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equation (4) stating that stored energy related to reluctance of the structure.

Putting this value in equation (3) we get,

$$F_e = \frac{d}{dx}\left[\frac{1}{2}\phi^2 S\right] = \frac{\phi^2}{2}\frac{ds}{dx}$$

This is the expression for magnetic force acting on movable iron bar.

The force acting to pull the bar is ,

$$F = -F_e = \frac{-\phi^2}{2}\frac{ds}{dx}$$

Note : For a magnetically - linear system , the field energy and co energy are numerically equal; $\frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2}Li^2$. For a non- linear system in which λ and i (or B and H) are not linearly proportional , the two functions are not even numerically equal. A graphical interpretation of the energy and co energy for a non -linear system is shown in fig 7.54. The area between the λ curve and vertical axis (area of OPQ), equal to the integral of i.dλ, is the field energy. The area to the horizontal axis (area of OQR) given by the integral of $\lambda.di$ is the co energy . For this system field energy + co energy = λi

7.26 Moving Coil Transducers

A moving coil transducer is a classical example of eletro mechanical transducer. The example of such class of transducers are microphones, loud speakers (small power application), motors, Generators (medium and large power application) . To understand the principal of operation of such transducer , consider the figure 7.55 shown below.

The arrangement consists of a sliding conducting bar of length 'l' in an uniform magnetic field of B $wb\ m^2$ with a pair of Support bars. If a current i will be allowed to flow in a conducting bar then a force will act on the conductor whose magnitude in given by F =Bil Sinθ.

$\sin\theta = 1$, as θ is the angle between magnetic field and direction of current which is in this case 90^o.The direction of the force is given by Fleming's left hand rule.

In the above example if the current in the conducting bar flows bottom to top than the force acting on the conducting bar will move from right to left.Similarly if the direction of current is from top to bottom then the sliding bar will move from left to right. The same principle is applied in the operation of a motor.

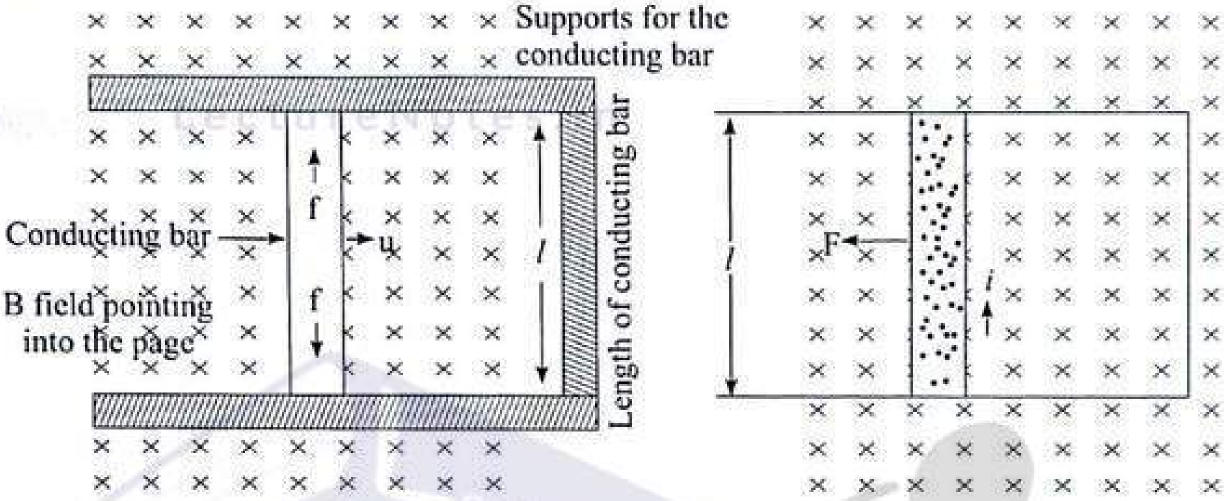

Fig 7.55

Similarly this arrangement can be used as a sensor i.e by applying a force on the sliding bar will generate a current in the same bar where magnitude of the current is given by $i = \frac{e}{r}$

Where r is the resistance of the current carrying path and e is the induced emf in the bar given by Faradays law of electro-magnetic induction. Magnitude of induced emf $e = BlV$. Where V is the velocity of the sliding bar produced due to applied external force.The direction of e is given by Flemings right hand rule.

In the above arrangement if the sliding bar is moved with a velocity V from left to right then the direction of current in the sliding bar will be from bottom to top the direction of current will reverse when the bar moves from right to left.

(1) What is he relation between permeability and reluctance in a magnetic circuit ?

(1st Semestar 2003)

Solution : Reluctance is inversly proportional to the permeability.

$$\text{Reluctance (s)} = \frac{l}{\mu_0 \mu_r A} \quad (\because\ l = \text{length of magnetic circuit.}\ A = \text{Area of cross-section of the circuit.})$$

(2) What is the Secondary to Primary turns ratio of a I-phase transformer excited by 230 volts ac supply with 12v output voltage at no load ?

(1st Semestar 2003)

Solution : $\frac{N_2}{N_1} = \frac{12}{230} = 0.0521$

(3) An iron ring of mean length 50 cm has an air gap of 1 mm and a winding of 200 turns. A current of 1 A flows through the coil. Find the flux density. The relative permeability of the iron is 300.

(1st Semester 2003)

Solution : Let a = area of cross-section of the iron ring.

$l_g = \text{length of air gap} = 10^{-3}\ m$

$l_i = \text{length of iron path} = 50 \times 10^{-2}\ m$

N = 200

I = 1 A

$$\text{Reluctance of iron path, } S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 300 \times a}$$

$$= \frac{1326.963}{a}\ \frac{AT}{wb}$$

$$\text{Reluctance of air gap, } S_g = \frac{l_g}{\mu_0 a} = \frac{10^{-3}}{4\pi \times 10^{-7}\ a} = \frac{796.17}{a}\ \frac{AT}{wb}$$

$$\text{Total reluctance } S = S_i + S_g = \frac{2123.141}{a}\ \frac{AT}{wb}$$

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Given, $\frac{V_2}{V_1} = \frac{50}{100}$

$\Rightarrow \quad V_2 = \frac{50}{100} \quad V_1 = \frac{50}{100} \times 100 = 50V$

Also, $\frac{V_2}{V_3} = \frac{10}{200}$

$\Rightarrow V_3 = \frac{200}{10} V_2 = \frac{200}{10} \times 50 = 1000\,V$

14. Justify the use of high permeabilities magnetic materials in electrical equipments.

(1st semester 2005)

Solution : Reluctance, $S = \frac{l}{\mu a}$

Flux $\phi = \frac{NI}{S} = \frac{NI}{\frac{l}{\mu a}} = \frac{NIa\mu}{l}$

clearly, $\phi \propto \mu$

Therefore higher permabilities will produce more flux for same ampere turns.

15. A Core of cast steel has a cross -sectional area of $10Cm^2$ and an average length of 35 Cm with a 1mm air gap. It is wound with 200 turns of wire carrying a current of 3A. determine the total magnetic flux in the air gap if the relative permeability of cast steel is 1000.

(1st semester 2005)

Solution : Area of cross-section of core, $a = 10\times10^{-4}\ m^2$

Length of core. $l_c = 35\times10^{-2}\ m$

Length of air gap, $l_g = 10^{-3}\ m$

N= 200

I = 3A

Reluctance of core, $S_c = \frac{l_c}{\mu_0\mu_r a} = \frac{35\times10^{-2}}{4\pi\times10^{-7}\times1000\times10\times10^{-4}}$

$= 27.866\times10^4\ \frac{AT}{wb}$

Reluctance of air gap, $S_g = \frac{l_g}{\mu_0 a} = \frac{10^{-3}}{4\pi\times10^{-7}\times10\times10^{-4}}$

f = frequency of flux in Hz.

v = volume of material in m^3.

(ii) Hysteresis power loss, $P = \eta f \ B_m^{1.6} \ V$ *watts.*

Where V = volume of the material in m^3

f = frequency of magnetization in Hz.

η = hysteresis coefficient whose value depends upon nature of material.

10. An electro magnet has an air gap of length 2mm and an iron path of length 30 cm. Find the number of ampere - turns necessary to produce a flux density of 0.8 $\frac{wb}{m^2}$ in the gap. take μ_r = 1500 for the magnetic material of the electromagnet. ***(2nd Semester 2004)***

Solution : Reluctance of air gap, $S_g = \frac{l_g}{\mu_o a} = \frac{2\times10^{-3}}{4\pi\times10^{-7} a}$

$$= \frac{0.159 \times 10^4}{a} \frac{AT}{wb}$$

Reluctance of iron path, $S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{30\times 10^{-2}}{4\pi \times 10^{-7} \times 1500 \times a}$

$$= \frac{15921}{a} \frac{AT}{wb}$$

Total reluctance, $S = S_i + S_g = 174923 / a \ \frac{AT}{wb}$

$$\phi = \frac{NI}{S}$$

$$\Rightarrow NI = \phi S = B a S$$

$$= (0.8)(a)\left(\frac{174923}{a}\right)$$

$$= 1400 \text{ AT.}$$

11. The eddy current loss in a cold rolled grain oriented silicon steel sheet is 100 w when the supply frequency is 50 c/s. Find the eddy current loss when the frequency is 40 c/s the flux density remaining same. ***(2nd Semester 2004)***

Solution : Eddy current loss, $P \ \alpha \ f^2$

$$\therefore \frac{P_1}{P_2} = \frac{f_1^2}{f_2^2}$$

$$\Rightarrow P_2 = \frac{P_1 f_2^2}{f_1^2} = \frac{(100)(40)^2}{(50)^2} = 64 \ watt.$$

Solution : $E_1 = 220\,V,\ f = 50\,Hz$

$N_2 = 2000,\ \phi_m = 0.003\,wb$

(i) $E_1 = 4.44\,f\,\phi_m\,N_1$

$$\Rightarrow N_1 = \frac{E_1}{4.44\,f\,\phi_m} = \frac{220}{4.44\,(50)(0.003)} = 330$$

(ii) $E_2 = 4.44\,f\,\phi_m\,N_2 = 4.44\,(50)\,(0.003)(2000) = 1332$

23. Write down and explain the expression for reluctance in a magnetic circuit.

(2nd semester 2006)

Work done in moving a unit magnetic pole once round the magnetic circuit is equal ot the ampere-turns enclosed by the magnetic circuit.

Mathematically, $Hl = NI$

$$\Rightarrow \frac{Bl}{\mu} = NI$$

$$\Rightarrow \frac{\phi l}{a\mu_0\,\mu_r} = NI \qquad \because \mu = \frac{B}{H} \text{ and } B = \frac{\phi}{a}$$

$$\Rightarrow \phi = \frac{NI}{l/a\mu_0\,\mu_r} = \frac{NI}{S}$$

$$\Rightarrow Flux = \frac{mmf}{reluctance}$$

The quantity $l/a\mu_0\,\mu_r$ is called the reluctance of the magnetic circuit, 'l' and 'a' represent length and area of cross-section of the magnetic circuit.

$$\therefore \text{ Reluctance } (s) = \frac{l}{a\mu_0\,\mu_r}$$

Reluctance is the obstruction possed by the magnetic material to the flow of magnetic flux. Its S.I unit is $AT.\,wb^{-1}$

24. What is he expression for induced emf in the high voltage side of a single phase transformer ?

(2nd semester 2006)

Solution : $E_2 = 4.44 f \phi_m\,N_2$

Where, E_2 = *emf across high voltage side.*

N_2 = *number of turns of hight voltage side.*

But $$\phi = \frac{NI}{S} = \frac{200 \times 1}{\frac{2123.141}{a}} = 0.0942\ a\ wb$$

Flux density, $$B = \frac{\phi}{a} = \frac{0.0942\ a}{a} = 0.0942\ \frac{wb}{m^2}$$

4. The hysteresis loss in cold rolled grain oriented silicon sheet steel is 40 w when the supply frequency is 50 c/s. Find the hysteresis loss when the frequency is 60 c/s, the flux density remains same. *(1st semester 2003)*

Solution : Hysteresis loss, $P \propto f$

$$\therefore \quad \frac{P_1}{P_2} = \frac{f_1}{f_2}$$

$$\Rightarrow \quad P_2 = \frac{P_1\ f_2}{f_1} = \frac{(40)\ (60)}{50} = 48\ watt$$

5. A single phase transformer has 40 primary and 1100 secondary turns. The net cross-sectioned area of the core is 500 cm^2. If the primary winding be connected to 50 Hz, 400 volts supply, calculate the value of maximum flux density in the core and the emf induced in the secondary. *(1st Semester 2003)*

Solution : $N_1 = 40,\ N_2 = 1100,\ f = 50Hz$

$E_1 = 400\ v,\ E_2 = ?$

We know that emf induced in primary

$$E_1 = 4.44 f\ N_1 \phi_m$$

$$\Rightarrow \quad \phi_m = \frac{E_1}{4.44 f\ N_1} = \frac{400}{4.44\ (50)\ (40)} = 0.045\ wb$$

Cross-sectional area of core, A = $500 \times 10^{-4}\ m^2$

$$\therefore \text{ Maximum flux desity, } B_m = \frac{\phi_m}{A} = \frac{0.045}{500 \times 10^{-4}} = 0.9\ \frac{wb}{m^2}$$

We know that $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$$\Rightarrow \quad E_2 = \frac{N_2}{N_1}\ E_1 = \frac{1100}{40}(400) = 11000\ volt.$$

6. An ideal transformer has 90 turns in the primary and 2250 turns on the Secondary and is connected to a 200 v, 50 Hz supply. The load ~~a cross~~ across the secondary draws a current of 2A at a p.f. 80 % lagging.

$$=79.617\times10^4\ \frac{AT}{wb}$$

Total reluctance, s= $S_c + S_g$ = $107.483\times10^4\ \frac{AT}{wb}$

$$\phi=\frac{NI}{S}=\frac{(200)(3)}{107.483\times10^4}=5.582\times10^{-4}\ wb$$

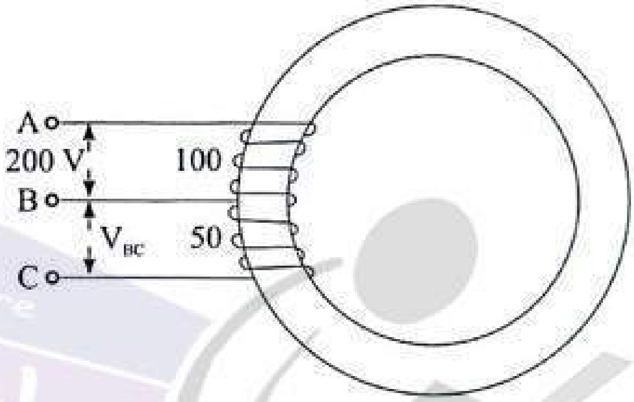
∴ Flux ϕ= 0.5582 m wb.

16. On a closed iorn core a winding A B C is wound AB= 100 turns, BC = 50 turns. An a.c voltage of 200 v is applied a cross AB. what is the voltage across AC ? ***(2nd semester 2005)***

Solution : From figure

$$\frac{200}{V_{BC}}=\frac{100}{50}$$

$\Rightarrow\quad V_{BC}=100volt$



$\therefore\quad V_{AC}=V_{AB}+V_{BC}$

= 200+ 100

= 300 volts.

17. A magnetic core has a length of 50 cm and cross-sectional area of 16 sq cm, relative permiability of core material in 1000. If applied mmf is 1000 AT, what is the flux ? By what percentage the flux will reduce if a saw cut of 1mm long is made in the core section ?

(2nd Semester 2005)

Solution : Length of core, $l_c = 50\times10^{-2}\ m$

Area of cross-section of core, $a = 16\times10^{-4}\ m^2$

NI = mmf = 1000 AT

(i) Flux $\phi=\frac{NI}{S}=\frac{NI}{l/\mu_0\mu_r a}=\frac{NIa\mu_0\mu_r}{l_c}$

$$=\frac{1000(16\times10^{-4})(4\pi\times10^{-7})(1000)}{50\times10^{-2}}=4.019\times10^{-3}\ wb$$

(ii) Let S_1 = total reluctance when a saw cut of 10^{-3} m long is made in the core section.

$\therefore\qquad S_1=S_c+S_g$

$$=\frac{50\times10^{-2}-10^{-3}}{\mu_0\mu_r A}+\frac{10^{-3}}{\mu_0 A}$$

12. The emf per turn of a single phase 6600 v/440v, 50Hz transformer is 12v. Calculate, (i) the number of turns in the primary and secondary windings, (ii) the net cross-sectional area of the core for a maximum flux density of 1.5 wb/m^2.

(2nd semester 2004)

Solution : $f = 50\ Hz$

$$E_1 = 4.44\ f\ N_1\ \phi_m$$

$$\Rightarrow \frac{E_1}{N_1} = 4.44\ f\ \phi_m$$

$$\Rightarrow 12 = 4.44 f\ \phi_m$$

$$\Rightarrow \phi_m = \frac{12}{4.44\ f} = \frac{12}{4.44(50)} = 0.054\ wb$$

(i) $K = \frac{440}{6600} = 0.0667$

given, $\frac{E_1}{N_1} = 12$

$$\Rightarrow N_1 = \frac{E_1}{12} = \frac{6600}{12} = 550$$

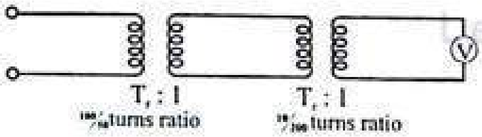
Also, $\frac{N_2}{N_1} = K$

$$\Rightarrow N_2 = N_1 K = 550\ (0.0667) = 37$$

(ii) Cross-sectional area of the core in

$$a = \frac{\phi}{B} = \frac{0.054}{15} = 0.036\ m^2$$

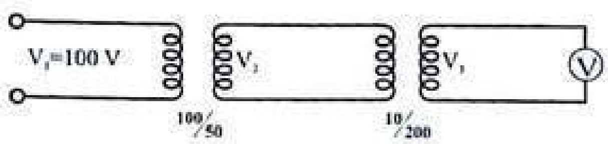
13.



What is voltmeter reading.

(supplementary 2004)

Solution :



25. An iron ring has a circular cross-section of 3 cm diameter and a mean circumference of 75 cm. Calculate the reluctance offered by the iron ring assuming its relative permeability to be equal to 500. *(1st semester 2007)*

Solution : Area of cross-section of the ring

$$a = \frac{\pi d^2}{4} = \frac{\pi(3\times10^{-2})}{4} = 7.065\times10^{-4}\ m^2$$

length of ring, $l = 75\times10^{-2}\ m$

Reluctance of ring, $S = \dfrac{l}{a\mu_0\mu_r} = \dfrac{75\times10^{-2}}{7.065\times10^{-4}\times4\pi\times10^{-7}\times500}$

$$= 1.69\times10^{6}\ \frac{AT}{wb}$$

26. A magnetic circuit comprises three parts in series, each of uniform corss-sectional area (C.S.a). They are,

(i) iron of length 75 mm and C.S.a 60 mm^2

(ii) iron of length 50 mm and C.S.a 80 mm^2.

(iii) an airgap of length of 0.6 mm and C.S.a 140 mm^2.

A coil of 3500 turns is wound on part (ii), and the flux density in the air gap is 0.25 T. Estimate the coil current assuming all the flux to pass through the given magnetic circuit.

(1st Semester 2007)

Solution : (i) Reluctance of iron of length 75 mm is

$$S_1 = \frac{l}{a\mu_0\mu_r} = \frac{75\times10^{-3}}{60\times10^{-6}\times4\pi\times10^{-7}\times1500}$$

$$= 6634\times10^{4}\ \frac{AT}{wb}$$

(ii) Reluctance of iron of length 50 mm is

$$S_2 = \frac{l}{a\mu_0\mu_r} = \frac{50\times10^{-3}}{80\times10^{-6}\times4\pi\times10^{-7}\times1500}$$

$$= 33.17\times10^{4}\ \frac{AT}{wb}$$

(iii) Reluctance of air gap is,

$$S_g = \frac{l}{a\mu_0} = \frac{0.6\times10^{-3}}{140\times10^{-6}\times4\pi\times10^{-7}}$$

(i) Find the instantaneous current in the primary when the instantaneous current in the secondary is 100 mA.

(ii) Find the primary current and peak flux linked in the secondary ?

(iii) The power delivered to the primary.

(1st semester 2004)

Solution : $N_1 = 90,\ N_2 = 2250,\ f = 50\ Hz$

$$E_1 = 200\ volts,\ I_2 = 2A,\ K = \frac{N_2}{N_1} = 25$$

(i) Given instantaneous current in the secondary, $i_2 = 100 \times 10^{-3}\ A$

$\therefore$ Instantaneous current in the primary

is $i_1 = -Ki_2 = -25 \times 100 \times 10^{-3} = -2.5A$

(Negative sign indicates that i_1 and i_2 both are opposite to each other)

(ii) Primary current $I_1 = KI_2 = 25(2) = 50\ A$

$E_1 = 4.44\ f\ N_1\ \phi_m$

$$\Rightarrow \quad \phi_m = \frac{E_1}{4.44\ f\ N_1} = \frac{200}{4.44\ (50)(90)} = 0.01\ wb$$

(iii) Power delivered to the primary is,

$P = V_1\ I_1\ \cos\phi = E_1\ I_1\ \cos\phi = (200)\ (50)\ (0.08) = 8\ KW$

7. What is he relative permeability of a non-magnetic material ?

(2nd semester 2004)

Solution : Relative permeability of a non-magnetic material is 1.

8. An iron rod 1.8 cm diameter is bent in the form of a ring of mean diameter 25cm and wound with 250 turns wire. A gap of 1 mm exists in between the end faces. Relative permeability of iron = 1200. Find (i) the current required to produce a flux of 0.6 m wb. (ii) energy stored in the coil. *(1st semester 2004)*

Solution : Area of cross section of iron rod

$$\text{is } a = \frac{\pi d^2}{4} = \frac{\pi (1.8 \times 10^{-2})^2}{4} = 2.5434 \times 10^{-4}\ m^2$$

l_i = Length iron riong $= \pi(25 \times 10^{-2}) = 0.785\ m$

l_g = Length of gap = 10^{-3} m

$$= \frac{50\times 10^{-2} - 10^{-3}}{4\pi\times 10^{-7}\times 1000\times 16\times 10^{-4}} + \frac{10^{-3}}{4\pi\times 10^{-7}\times 16\times 10^{-4}}$$

$$= 7.459\times 10^{5}\ \frac{AT}{wb}$$

$$\therefore\ \text{Flux}\ \phi_1 = \frac{NI}{S_1} = \frac{1000}{7.459\times 10^{5}} = 134\times 10^{-3}\ wb$$

$$\therefore\ \text{Change in flux} = \frac{\phi - \phi^1}{\phi} = 0.666$$

Percentage change in flux = $0.666\times 100 = 66.66\ \%$

18. The Primary of a single phase transformer is connected to a 222 V, 50 Hz supply. If the peak flux in the core is 10mwb, what is the no. of turns in the primary ? How may number of turns are required in the secondary to obtain a voltage of 110 V. ***(2nd semester 2005)***

Solution : $E_1 = 222\ V,\ f = 50\ Hz,\ \phi_m = 10\times 10^{-3}\ wb$

$$E_1 = 4.44\ f\ N_1\ \phi_m$$

$$\Rightarrow\ N_1 = \frac{E_1}{4.44 f\ \phi_m} = \frac{222}{4.44(50)(10\times 10^{-3})} = 100$$

Similarly, $E_2 = 4.44\ f\ \phi_m\ N_2$

$$\Rightarrow\ N_2 = \frac{E_2}{4.44 f\ \phi_m} = \frac{110}{4.44(50)(10\times 10^{-3})} = 49.54 = 50$$

19. Calculate the currents in the two windings of a single phase fully loaded transformer of 1000/200 V, 25 KVA rating. Neglect the no-load current.

(1st semester 2006)

Solution : Current in Primary side in

$$I_1 = \frac{25\times 10^{3}}{1000} = 25\ A$$

Current in secondary side is, $I_2 = \dfrac{25\times 10^{3}}{200} = 125\ A$

20. A coil is wound uniformly over a steel ring of relative permeability 900, having a mean circumference of 50 mm and cross sectional area of 100 mm^2, calculate the reluctance offered by the steel ring. ***(1st semester 2006)***

Solution : $\mu_r = 900,\ l = 50\times 10^{-3}\ m,\ a = 100\times 10^{-6} m^2$

Reluctance offered by the steel ring

Given, $\frac{V_2}{V_1} = \frac{50}{100}$

$$\Rightarrow \quad V_2 = \frac{50}{100} \quad V_1 = \frac{50}{100} \times 100 = 50V$$

Also, $\frac{V_2}{V_3} = \frac{10}{200}$

$$\Rightarrow V_3 = \frac{200}{10} V_2 = \frac{200}{10} \times 50 = 1000\,V$$

14. Justify the use of high permeabilities magnetic materials in electrical equipments.

(1st semester 2005)

Solution : Reluctance, $S = \frac{l}{\mu a}$

$$\text{Flux } \phi = \frac{NI}{S} = \frac{NI}{\frac{l}{\mu a}} = \frac{NIa\mu}{l}$$

clearly, $\phi \propto \mu$

Therefore higher permabilities will produce more flux for same ampere turns.

15. A Core of cast steel has a cross -sectional area of $10Cm^2$ and an average length of 35 Cm with a 1mm air gap. It is wound with 200 turns of wire carrying a current of 3A. determine the total magnetic flux in the air gap if the relative permeability of cast steel is 1000.

(1st semester 2005)

Solution : Area of cross-section of core, $a = 10\times10^{-4}\ m^2$

Length of core. $l_c = 35\times10^{-2}\,m$

Length of air gap, $l_g = 10^{-3}\,m$

N= 200

I = 3A

$$\text{Reluctance of core, } S_c = \frac{l_c}{\mu_0\mu_r a} = \frac{35\times10^{-2}}{4\pi\times10^{-7}\times1000\times10\times10^{-4}}$$

$$= 27.866\times10^4 \frac{AT}{wb}$$

$$\text{Reluctance of air gap, } S_g = \frac{l_g}{\mu_0 a} = \frac{10^{-3}}{4\pi\times10^{-7}\times10\times10^{-4}}$$

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Solution : (i) Since secondary voltage $V_2 = KV_1$, where K = transformation ration = $\frac{115}{230} = 0.5$, it does not depend on supply frequency. However frequency on both side remains same. Therefore V_2 for V_1 = 230 volts is given by $V_2 = 0.5 \times 230 = 115$ volts.

Frequency in secondary side is 40 HZ.

(ii) For $V_1 = 115$ volts, $V_2 = 0.5 \times 115 = 57.5$ volts .

Frequency is 25 HZ.

(iii) Transformer does not operate on dc and hence the secondary voltage is Zero.

40. The emf per turn for a single phase, 2200/220V , 50 HZ transformer is approximately 15 volts. Calculate

(i) The number of primary and secondary turns.

(ii) The net cross sectioned area of the core, for a maximum flux density of 1.25 wb/m^2 in the core. *(2nd semester 2009)*

Solution : $E_1 = 2200$ volts, $E_2 = 220$ volts, f= 50 HZ

(i) $\frac{E_1}{N_1} = 15$

$$\Rightarrow N_1 = \frac{E_1}{15} = \frac{2200}{15} = 146.67 = 147$$

Similarly $N_2 = \frac{E_2}{15} = \frac{220}{15} = 14.66 = 15$

(ii) $E_1 = 4.44 f \phi_M N_1$

$$\Rightarrow \quad \phi_M = \frac{E_1}{4.44 f\ N_1} = \frac{2200}{4.44 \times 50 \times 147}$$

$$\Rightarrow \phi_M = 0.0674 \text{ wb} \qquad B_M = \frac{\phi_M}{a}$$

$$\Rightarrow a = \frac{\phi_M}{B_M} = \frac{0.0677}{125} = 0.0539\ m^2$$

(i) $N = 250,\ \phi = 0.6 \times 10^{-3}\ wb$

Reluctance of iron path, $S_i = \dfrac{l_i}{\mu_0 \mu_r a}$

$$= \frac{0.785}{4\pi \times 10^{-7} \times 1200 \times 25434 \times 10^{-4}}$$

$$= 2.0477 \times 10^6\ \frac{AT}{wb}$$

Reluctance of gap, $S_g = \dfrac{l_g}{\mu_0 a} = \dfrac{10^{-3}}{4\pi \times 10^{-7} \times 25434 \times 10^{-4}}$

$$= 3.13 \times 10^6\ \frac{AT}{wb}$$

Total reluctance, $S = S_i + S_g = 5.177 \times 10^6\ \dfrac{AT}{wb}$

But $\phi = \dfrac{NI}{S}$

$$\Rightarrow I = \frac{\phi S}{N} = \frac{0.6 \times 10^{-3} \times 5.177 \times 10^6}{250} = 12.42\ A$$

(ii) Inductance, $L = \dfrac{N\phi}{I} = \dfrac{(250)(0.6 \times 10^{-3})}{12.42} = 0.012\ H$

Energy stored $= \dfrac{1}{2} LI^2$

$$= \frac{1}{2}(0.012)(12.42)^2$$

$$= 0.9255\ joule.$$

9. Write down formulas for eddy current and hystresis loss in a magnetic material, when it is subjected to an alternating magnetic field. *(1st Semester 2004)*

Solution : (i) It is difficult to determine the eddy current power loss because the current and resistance values can not be determined directly. Experimentally the eddy current power loss in a magnetic material can be expressed as,

$$P = K_e B_m^2\ t^2\ f^2\ V\ watts.$$

Where K_e = eddy- current coefficient and its value depends upon the nature of the material.

B_m = maximum flux density in wb / m^2

t = thickness of Lamination in m.

$$\text{is } S = \frac{l}{\mu_0 \mu_r a} = \frac{50\times 10^{-3}}{4\pi\times 10^{-7} \times 900\times 100\times10^{-6}}$$

$$= 4423 \times 10^4 \; \frac{AT}{wb}$$

21. A Ring shaped electromagnet has an airgap 6mm long and 20 cm^2 in area, the mean length of the core being 50 cm, and its cross section 10 cm^2. Calculate the ampere- turns required to produce a flux density of 0.5 $wb\ m^2$ in the air gap. The relative permeability of iron in 1800.

(1st Semester 2006)

Solution : Area of cross-section of air gap $a_g = 20 \times 10^{-4}\ m^2$

length of air gap, $l_g = 6\times 10^{-3}\ m$

length of core, $l_c = 50 \times 10^{-2}\ m$

Area of cross section of core, $a_c = 10 \times 10^{-4}\ m^2$

μ_r of iron = 1800

Reluctance of air gap, $S_g = \frac{l_g}{\mu_0 a_g} = \frac{6\times 10^{-3}}{4\pi\times10^{-7}\times 20\times10^{-4}}$

$$= 0.023 \times 10^8 \; \frac{AT}{wb}$$

Reluctance of core, $S_c = \frac{l_c}{\mu_0 \mu_r a_c} = \frac{50\times10^{-2}}{4\pi\times10^{-7}\times1800\times10\times10^{-4}} = 221160\cdot6\frac{AT}{wb}$

Total reluctance, S = $S_g + S_c$ = 2521160.651 $\frac{AT}{wb}$

Flux at air gap = *Flux density × area of air gap*

$$= 05 \times 20 \times 10^{-4} = 10^{-3}\ wb$$

$$\phi = \frac{NI}{S}$$

$$\Rightarrow NI = \phi S = 10^{-3} \times 4423 \times 10^4 = 4423\ AT$$

22. The primary winding of a single phase transformer is connected to a 220 V, 50 Hz supply. The Secondary winding has 2000 turns. If the maximum value of the core flux is 0.003 wb, determine (i) the number of turns on the primary winding (ii) the secondary induced voltage.

(1st semester 2006)

$$=79.617\times10^4\ \frac{AT}{wb}$$

Total reluctance, s= $S_c + S_g$= $107.483\times10^4\ \frac{AT}{wb}$

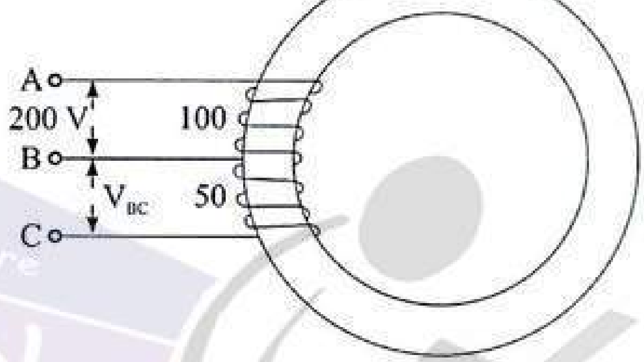
$$\phi=\frac{NI}{S}=\frac{(200)(3)}{107.483\times10^4}=5.582\times10^{-4}\ wb$$

∴ Flux ϕ= 0.5582 m wb.

16. On a closed iorn core a winding A B C is wound AB= 100 turns, BC = 50 turns. An a.c voltage of 200 v is applied a cross AB. what is the voltage across AC ? *(2nd semester 2005)*

Solution : From figure

$$\frac{200}{V_{BC}}=\frac{100}{50}$$



$\Rightarrow\quad V_{BC}=100volt$

$\therefore\quad V_{AC}=V_{AB}+V_{BC}$

= 200+ 100

= 300 volts.

17. A magnetic core has a length of 50 cm and cross-sectional area of 16 sq cm, relative permiability of core material in 1000. If applied mmf is 1000 AT, what is the flux ? By what percentage the flux will reduce if a saw cut of 1mm long is made in the core section ?

(2nd Semester 2005)

Solution : Length of core, $l_c = 50\times10^{-2}\ m$

Area of cross-section of core, $a = 16\times10^{-4}\ m^2$

NI = mmf = 1000 AT

(i) Flux $\phi=\frac{NI}{S}=\frac{NI}{l/\mu_0\mu_r a}=\frac{NIa\mu_0\mu_r}{l_c}$

$$=\frac{1000(16\times10^{-4})(4\pi\times10^{-7})(1000)}{50\times10^{-2}}=4.019\times10^{-3}\ wb$$

(ii) Let S_1 = total reluctance when a saw cut of 10^{-3} m long is made in the core section.

$\therefore\quad S_1=S_c+S_g$

$$=\frac{50\times10^{-2}-10^{-3}}{\mu_0\mu_r A}+\frac{10^{-3}}{\mu_0 A}$$

f = frequency of flux in Hz.

v = volume of material in m^3.

(ii) Hysteresis power loss, $P = \eta f\ B_m^{1.6}\ V$ *watts.*

Where V = volume of the material in m^3

f = frequency of magnetization in Hz.

η = hysteresis coefficient whose value depends upon nature of material.

10. An electro magnet has an air gap of length 2mm and an iron path of length 30 cm. Find the number of ampere - turns necessary to produce a flux density of 0.8 $\frac{wb}{m^2}$ in the gap. take μ_r = 1500 for the magnetic material of the electromagnet . ***(2nd Semester 2004)***

Solution : Reluctance of air gap, $S_g = \frac{l_g}{\mu_o a} = \frac{2\times10^{-3}}{4\pi\times10^{-7} a}$

$$= \frac{0.159\times10^4}{a}\ \frac{AT}{wb}$$

Reluctance of iron path, $S_i = \frac{l_i}{\mu_0 \mu_r\ a} = \frac{30\times 10^{-2}}{4\pi\times10^{-7}\times1500\times a}$

$$= \frac{15921}{a}\ \frac{AT}{wb}$$

Total reluctance, $S = S_i + S_g = 174923 / a\ \frac{AT}{wb}$

$$\phi = \frac{NI}{S}$$

$$\Rightarrow\ NI = \phi S = B\,a\,S$$

$$= (0.8)(a)\left(\frac{174923}{a}\right)$$

$$= 1400\ \text{AT}.$$

11. The eddy current loss in a cold rolled grain oriented silicon steel sheet is 100 w when the supply frequency is 50 c/s. Find the eddy current loss when the frequency is 40 c/s the flux density remaining same. ***(2nd Semester 2004)***

Solution : Eddy current loss, $P \,\alpha\, f^2$

$$\therefore\ \frac{P_1}{P_2} = \frac{f_1^2}{f_2^2}$$

$$\Rightarrow\ P_2 = \frac{P_1\ f_2^2}{f_1^2} = \frac{(100)(40)^2}{(50)^2} = 64\ watt.$$

Solution : $E_1 = 220\,V,\ f = 50\ Hz$

$N_2 = 2000,\ \phi_m = 0.003\ wb$

(i) $E_1 = 4.44\, f\, \phi_m\, N_1$

$$\Rightarrow N_1 = \frac{E_1}{4.44\, f\, \phi_m} = \frac{220}{4.44\,(50)(0.003)} = 330$$

(ii) $E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\,(50)\,(0.003)(2000) = 1332$

23. Write down and explain the expression for reluctance in a magnetic circuit.

(2nd semester 2006)

Work done in moving a unit magnetic pole once round the magnetic circuit is equal ot the ampere-turns enclosed by the magnetic circuit.

Mathematically, $Hl = NI$

$$\Rightarrow \frac{Bl}{\mu} = NI$$

$$\Rightarrow \frac{\phi l}{a\mu_0\,\mu_r} = NI \qquad \because \mu = \frac{B}{H} \text{ and } B = \frac{\phi}{a}$$

$$\Rightarrow \phi = \frac{NI}{l/a\mu_0\,\mu_r} = \frac{NI}{S}$$

$$\Rightarrow Flux = \frac{mmf}{reluctance}$$

The quantity $l/a\mu_0\,\mu_r$ is called the reluctance of the magnetic circuit, 'l' and 'a' represent length and area of cross-section of the magnetic circuit.

$$\therefore \text{Reluctance } (s) = \frac{l}{a\mu_0\,\mu_r}$$

Reluctance is the obstruction possed by the magnetic material to the flow of magnetic flux. Its S.I unit is $AT.\,wb^{-1}$

24. What is he expression for induced emf in the high voltage side of a single phase transformer ?

(2nd semester 2006)

Solution : $E_2 = 4.44 f\phi_m\, N_2$

Where, E_2 = *emf across high voltage side.*

N_2 = *number of turns of hight voltage side.*

$$= \frac{50\times 10^{-2} - 10^{-3}}{4\pi\times 10^{-7}\times 1000\times 16\times 10^{-4}} + \frac{10^{-3}}{4\pi\times 10^{-7}\times 16\times 10^{-4}}$$

$$= 7.459\times 10^{5}\ \frac{AT}{wb}$$

$$\therefore \text{ Flux } \phi_1 = \frac{NI}{S_1} = \frac{1000}{7.459\times 10^{5}} = 134\times 10^{-3}\ wb$$

$$\therefore \text{ Change in flux } = \frac{\phi - \phi^1}{\phi} = 0.666$$

Percentage change in flux = $0.666\times 100 = 66.66\ \%$

18. The Primary of a single phase transformer is connected to a 222 V, 50 Hz supply. If the peak flux in the core is 10mwb, what is the no. of turns in the primary ? How may number of turns are required in the secondary to obtain a voltage of 110 V. ***(2nd semester 2005)***

Solution : $E_1 = 222\ V,\ f = 50\ Hz,\ \phi_m = 10\times 10^{-3}\ wb$

$$E_1 = 4.44\ f\ N_1\ \phi_m$$

$$\Rightarrow N_1 = \frac{E_1}{4.44 f\ \phi_m} = \frac{222}{4.44(50)(10\times 10^{-3})} = 100$$

Similarly, $E_2 = 4.44\ f\ \phi_m\ N_2$

$$\Rightarrow N_2 = \frac{E_2}{4.44 f\ \phi_m} = \frac{110}{4.44(50)(10\times 10^{-3})} = 4954 = 50$$

19. Calculate the currents in the two windings of a single phase fully loaded transformer of 1000/200 V, 25 KVA rating. Neglect the no-load current.

(1st semester 2006)

Solution : Current in Primary side in

$$I_1 = \frac{25\times 10^3}{1000} = 25\ A$$

Current in secondary side is, $I_2 = \frac{25\times 10^3}{200} = 125\ A$

20. A coil is wound uniformly over a steel ring of relative permeability 900, having a mean circumference of 50 mm and cross sectional area of 100 mm^2, calculate the reluctance offered by the steel ring. ***(1st semester 2006)***

Solution : $\mu_r = 900,\ l = 50\times 10^{-3}\ m,\ a = 100\times 10^{-6} m^2$

Reluctance offered by the steel ring

12. The emf per turn of a single phase 6600 v/440v, 50Hz transformer is 12v. Calculate, (i) the number of turns in the primary and secondary windings, (ii) the net cross-sectional area of the core for a maximum flux density of 1.5 wb/m^2.

(2nd semester 2004)

Solution : $f = 50\ Hz$

$$E_1 = 4.44\ f\ N_1\ \phi_m$$

$$\Rightarrow \frac{E_1}{N_1} = 4.44\ f\ \phi_m$$

$$\Rightarrow 12 = 4.44\ f\ \phi_m$$

$$\Rightarrow \phi_m = \frac{12}{4.44\ f} = \frac{12}{4.44(50)} = 0.054\ wb$$

(i) $$K = \frac{440}{6600} = 0.0667$$

given, $$\frac{E_1}{N_1} = 12$$

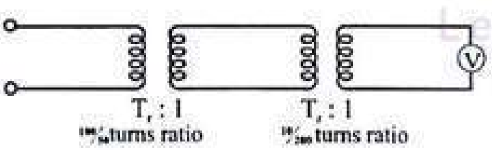
$$\Rightarrow N_1 = \frac{E_1}{12} = \frac{6600}{12} = 550$$

Also, $$\frac{N_2}{N_1} = K$$

$$\Rightarrow N_2 = N_1 K = 550\ (0.0667) = 37$$

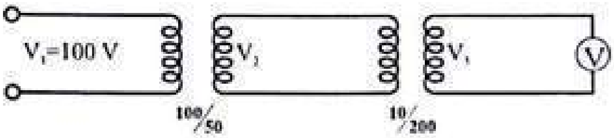
(ii) Cross-sectional area of the core in

$$a = \frac{\phi}{B} = \frac{0.054}{1.5} = 0.036\ m^2$$

13. 

What is voltmeter reading. *(supplementary 2004)*

Solution :



25. An iron ring has a circular cross-section of 3 cm diameter and a mean circumference of 75 cm. Calculate the reluctance offered by the iron ring assuming its relative permeability to be equal to 500. *(1st semester 2007)*

Solution : Area of cross-section of the ring

$$a = \frac{\pi d^2}{4} = \frac{\pi(3\times10^{-2})}{4} = 7.065\times10^{-4}\ m^2$$

length of ring, $l = 75\times10^{-2}\ m$

Reluctance of ring, $S = \dfrac{l}{a\mu_0\mu_r} = \dfrac{75\times10^{-2}}{7.065\times10^{-4}\times4\pi\times10^{-7}\times500}$

$$= 1.69\times10^6\ \frac{AT}{wb}$$

26. A magnetic circuit comprises three parts in series, each of uniform corss-sectional area (C.S.a). They are,

(i) iron of length 75 mm and C.S.a 60 mm^2

(ii) iron of length 50 mm and C.S.a 80 mm^2.

(iii) an airgap of length of 0.6 mm and C.S.a 140 mm^2.

A coil of 3500 turns is wound on part (ii), and the flux density in the air gap is 0.25 T. Estimate the coil current assuming all the flux to pass through the given magnetic circuit.

(1st Semester 2007)

Solution : (i) Reluctance of iron of length 75 mm is

$$S_1 = \frac{l}{a\mu_0\mu_r} = \frac{75\times10^{-3}}{60\times10^{-6}\times4\pi\times10^{-7}\times1500}$$

$$= 6634\times10^4\ \frac{AT}{wb}$$

(ii) Reluctance of iron of length 50 mm is

$$S_2 = \frac{l}{a\mu_0\mu_r} = \frac{50\times10^{-3}}{80\times10^{-6}\times4\pi\times10^{-7}\times1500}$$

$$= 33.17\times10^4\ \frac{AT}{wb}$$

(iii) Reluctance of air gap is,

$$S_g = \frac{l}{a\mu_0} = \frac{0.6\times10^{-3}}{140\times10^{-6}\times4\pi\times10^{-7}}$$

$$\text{is } S = \frac{l}{\mu_0 \mu_r a} = \frac{50\times 10^{-3}}{4\pi\times 10^{-7} \times 900\times 100\times10^{-6}}$$

$$= 4423 \times 10^4 \ \frac{AT}{wb}$$

21. A Ring shaped electromagnet has an airgap 6mm long and 20 cm^2 in area, the mean length of the core being 50 cm, and its cross section 10 cm^2. Calculate the ampere- turns required to produce a flux density of 0.5 $wb\ m^2$ in the air gap. The relative permeability of iron in 1800.

(1st Semester 2006)

Solution : Area of cross-section of air gap $a_g = 20 \times 10^{-4}\ m^2$

length of air gap, $l_g = 6\times 10^{-3}\ m$

length of core, $l_c = 50 \times 10^{-2}\ m$

Area of cross section of core, $a_c = 10 \times 10^{-4}\ m^2$

μ_r of iron = 1800

$$\text{Reluctance of air gap, } S_g = \frac{l_g}{\mu_0\, a_g} = \frac{6\times 10^{-3}}{4\pi\times10^{-7} \times 20\times10^{-4}}$$

$$= 0.023 \times 10^8 \ \frac{AT}{wb}$$

$$\text{Reluctance of core, } S_c = \frac{l_c}{\mu_0 \mu_r a_c} = \frac{50\times10^{-2}}{4\pi\times10^{-7}\times1800\times10\times10^{-4}} = 221160\cdot6\frac{AT}{wb}$$

$$\text{Total reluctance, } S = S_g + S_c = 2521160.651 \ \frac{AT}{wb}$$

Flux at air gap $= \textit{Flux density} \times \textit{area of air gap}$

$$= 0.5 \times 20 \times 10^{-4} = 10^{-3}\ wb$$

$$\phi = \frac{NI}{S}$$

$$\Rightarrow NI = \phi S = 10^{-3} \times 4423 \times 10^4 = 4423\ AT$$

22. The primary winding of a single phase transformer is connected to a 220 V, 50 Hz supply. The Secondary winding has 2000 turns. If the maximum value of the core flux is 0.003 wb, determine (i) the number of turns on the primary winding (ii) the secondary induced voltage.

(1st semester 2006)

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Do Your Self

T7.1 What is the flux density in a magnetic field of cross sectional area 20cm by 6 cm, if the flux density in a magnetic field of crosss-sectional area 20cm 2having a flux of 3mWb? **[1.5T]**

T 7.2 Determine the total flux emerging fr4om a magnetic pole face havihng dimensions 5cm, if the flux density is 0.9 T. **[2.7m Wb]**

T.7.3 The maximum working flux denxity of a lifti8ng electromagnet is 1.9T and the effective area of a pole face is circular in cross-section.If the total magnete flux produced is 6.11 mWb determine the radius of the pole face. **[32 cm]**

T 7.4 A current of 5A is passed through a 1000-turn coil wound on a circular magnetic circuit magnetic circuit of radius 120 mm.Calculate(a) the magnetomotive force,and (b) the magnetic field strength. **[(a) 5000A (b) 6631 A/m]**

T 7.5 Find the magnetic field strength and the magnetomotive force needed to produce a flux density of 0.33T in an air gap of lenth 15mm **[(a)262600A/M (b)3939A]**

T 7.6 An air gap between two ple pieces is 20mm in length and the area of the flux path across the gap is 5 cm 2.If the flux required in the air gap is 0.75 mWb.find the mmf necessary. **[23 870 A]**

T7.7 Find the magnetic field strength applied to a magnetic circuit of mean length 50cm when a coil of 400 turns is applied to it carrying a current of 1.2 A. **[960 A/m]**

T7.8 A solenoid 20 cm long is wound with turns of wire.Find the current required to establish a magnetizing force of 2500 A. **[1A]**

T7.9 A magnetic field strength of 5000 A/mis applied to a circular magnetic circuit of mean diameter 250 mm.If the coil has 500 turns find the current in the coil. **[7.85A]**

T 7.10 Find the relative permeability of a piece of silicon iron if a flux density of 1.3 T is produced by a magnetic field strength of 700A/m **[1478]**

T 7.11 Part of a magnetic circuit is made from steel of length 120mm, cross sectional area 15cm2 and relative permeability 800.Calculate (A) the reluctance and (b) the absolute permeability of the steel. **[(a) 79580/H(b) 1mH/m]**

7.12A A steel ring of mean diameter 120mmis uniformly would with 1500 turns of wire .When a current of 0.30 A is passed through the coil a flux density of 1.5 T IS set up in the steel.Find the relative permeability of the steel under these conditions. **[1000]**

T7.13 A mild steel closed magnetic circuit has a mean length of 75 mm and a cross-sectional area of 320 mm2.A current of 0.40 A flows in a coil wound uniformly around the circuit and the flux produced is 200μ Wb.If the relative permeability of the steel at this value of current is 400 find (a) the reluctance of the material and (b) the number of turns of the coil (a) 466000/H (b) the number of turns of the coil. **[(a) 466000/H (b) 233]**

T 7.14 A uniform ring of cast steel has a cross ce4stional area of 5 cm2 and as mean circumference of 15 cm.Find the current required in a coil of 1200 turns wound on the ring to produce a flux of 0.8 mWb(Use the magnetization curve for cast steel shown of Fig T7.1

[0.60A]

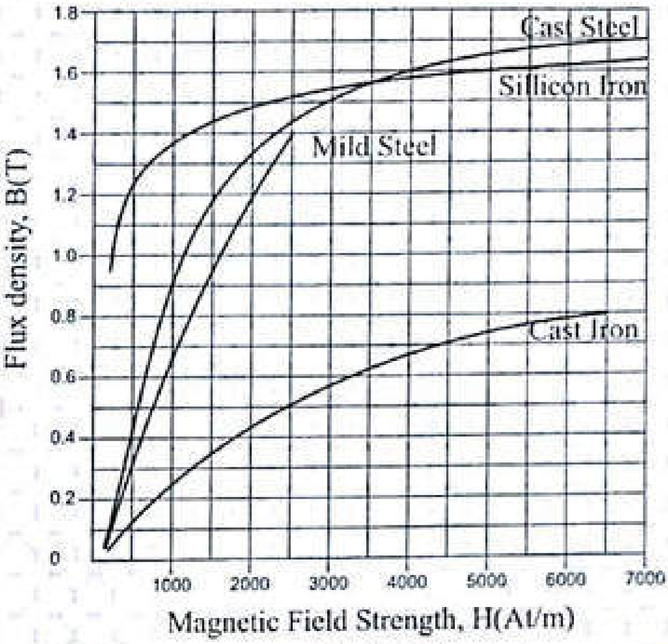

Fig T 7.1

T 7.15 (a) A uniform mild steel ring has a diameter of 50mm and a cross -sectional area of 1cm2.Determine the mmf necessary to produce a flux of 50 μ Wbin the ring .(Use the B-H curve for mild steel shown on Fig T7.1.(b) If a coil of 440 turns is wound uniformly around the ring in part (a) What current would be required to produce the flux?
[(a) 110 A (b) 0.25A]

T 7.16 A magnetic circuit of cross-sectional are $0.cm^2$ consists of one part 3cm long of material having relative permeability 750.With a 100 turn coil carryin 2A,find the value of flux existing in the circuit **[.0.195 Wb]**

T 7.17 (a) A cast steel ring has a cross sectional area of 600mm2 and a radius of 25 mm.Determine the in the ring .Use the B-H curve for cast steel shown in Fig T7.1. (b) If a radial air gap 1.5mm wide is cut in the ring of part(a) find the mmf now necessary to maintain the same flux in the ring. **[(a) 270 A (b) 1860 A]**

T 7.18 For the magnetic circuit shown in Figure T7.2 find the current I in the coil needed to produce a flux of 0.45 Wb in the air gap.The silicon iron magnetic circuit has a uniform cross-sectional area of 3 cm2 and its magnetization curve is as shown in Fig T7.1.
[0.83 A]

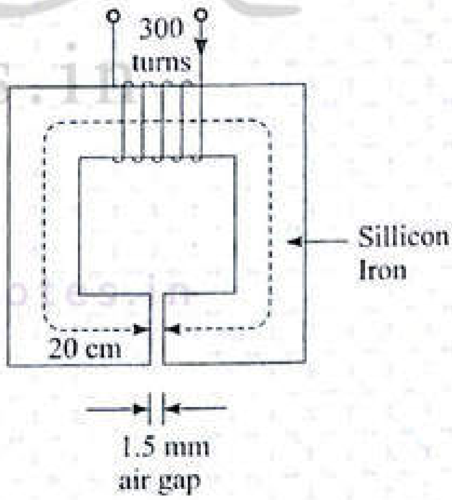

Fig T 7.2

T 7.19 A ring forming a magnetic circuit is made from two materials; one part is mild steel of mean length 25 cm and cross-sectional area 4cm2 and the remainder is cast iron of mean length 20cm and cross-sectional area 7.5 cm 2.Use a tabular approach to determine is cast iron of mean length 20cm and cross-sectional area 7.5 cm2.Use a tabular approach to determine the total mmf required to cause a flux of 0.30mWb in the magnetic circuit .Find also the total reluctance of the circuit.Use the magnetization curves shown of Figure T7.1.

[550 A, 1.83 x 106/H]

T 7.20 Figure T7.3 shows the magnetic circuit of a realy .When each of the air gaps are 1.5 mm wide find the mmf required to produce a flux density of 0.75T in the air gaps .Use the B-H curves shown in fig. T7.1. **[2970 A]**

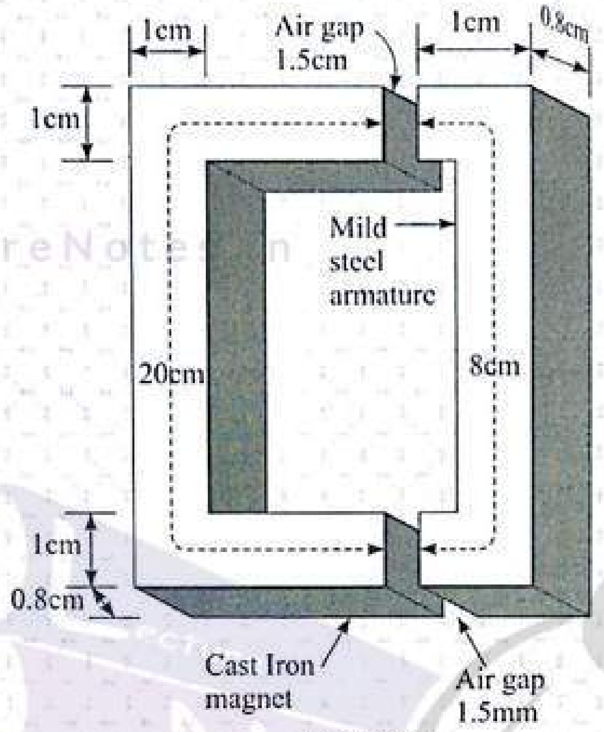

Fig T 7.3

T 7.21 In the magnetic circuit shown in Figure T 7.4 the coil of 500 turns carries a current of 4A..The air -gap lengths are gl=g2=0.25cm and g3=0.4cm.The cross-sectional areas are related such that Al=A2=0.5A3.The permeability of iron may be assumed to be infinite. Determine the flux densities B1,B2 and B3 in the gaps gl,g2and g3, respectively.Neglect leackage and fringing.

[B1=B2=B3=0.387]

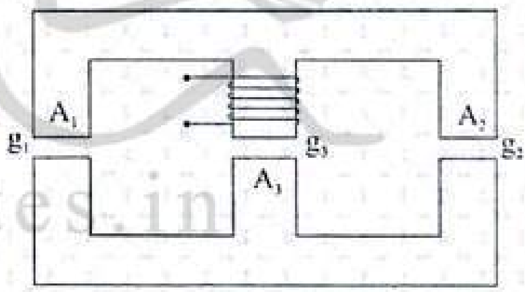

Fig T 7.4

T 7.22 In the magnetic circuit detailed in Figure T7.5 with all dimensions in mm, calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb in the air gpa .Neglect firinging effect and leakage flux,.The B-H curve of the material is given in Figure T7.6 Permeability of air may be taken as $\mu_0 = 4\pi \times 10^{-7}$ H/m.

[3.61 A]

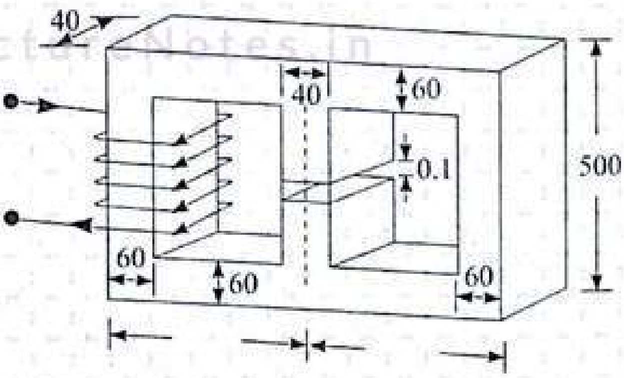

Fig T 7.5

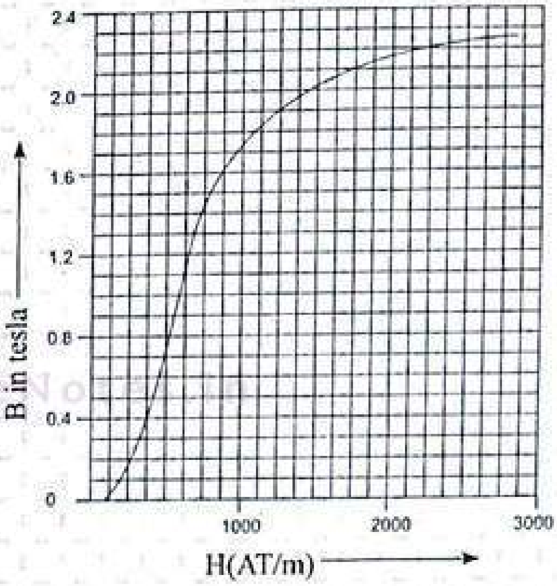

Fig T 7.6

T7.23 A solenoid of cylindri8cal geometry is shown in Fig T 7.7. If the exciting coil carries a dc steady current I derive an expression for the force on the plunger, (b)For the numerical values/=10A,N=500 turns, g=5mm,a= 20mm,b=2mm,and l=40mm,what is the magnitude of the force?Assume-$\mu_{core} = \infty$ and neglect leakage. **[600N]**

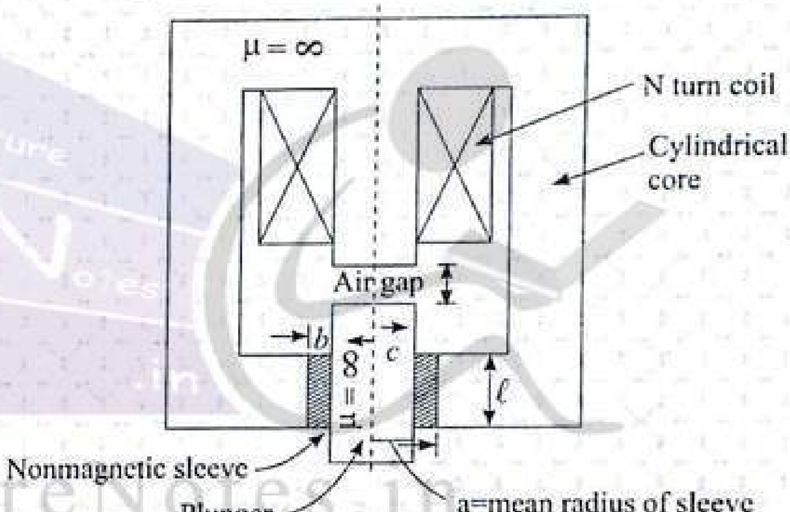

Fig T 7.7

T 7.24 An electromagnet,shown in Fig T7.8 is required to exert a 500-N force on the iron at an air gap of 1 mm,while the exciting coil is carrying 25 A dc. The core section at the air $600mm^2$ in area.Calculate the required number of turns of the exciting coil. **[65 turns]**

T7.25 (a) how many turns must the exciting coil of the electro magnet of Fig T7.8 have in order to produce a 500-N(average) force if the coil is excited by a 60-Hz altering current having a maximum value of 35.35 A? (b) Is the average force frequency-dependent ?

[(a) 65 turns, (b) no]

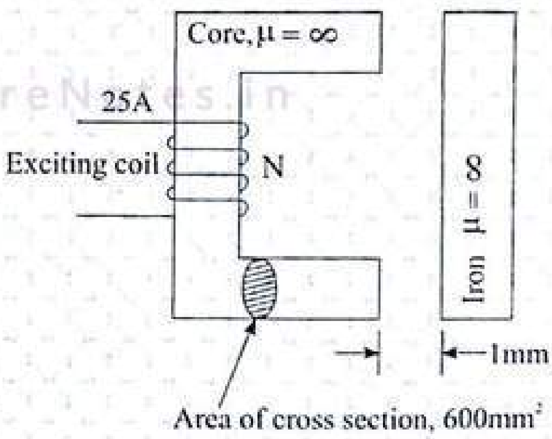

Fig T 7.8

T 26. A magnetic circuit with a uniform cross-sectional area of $6cm^2$ consists of a steel ring with a mean magnetic legth of 80 cm and an air gap of 2mm.The magnetising winding has 540 ampere-turns Estimate the magnetic flux produced in the gap .The relevant points on yhr magnetization curve of cast steel are:

$B(Wb/m^2)$:	0.12	0.14	0.16	0.18	0.20
$H(AT/m)$:	200	230	260	290	320

[0.1128m Wb]

T 7.27 A transformer having a turns ratio of 8:1 l supplies a load of resistance 50Ω . Determine the equivalent input resistance of the transformer. **[3.2 k Ω]**

T 7.28 What ratio of transformer turns is required to make a loaad of resistance 30Ω appear to have a resistance of 270Ω ? **[3.1]**

T 7.29 A single- phase, 240 V/2880V ideal transfore is supplied from a 240V source through a cable of resistance 3Ω. If the load across the secondary winding is 720 Ω determine (a) the primary current flowing and (b) the power dissipated in the load resistance.

[(a) 30 A (b) 4.5 k W]

T 7.30 A load of resistance 768 Ω is to be matched to an amplifier which has an effective output resistance of 12Ω. Determine the turns ratio of the coupling transformer. **[1.8]**

T 7.31 An a.c source of 20V and internal resistance 20Ω is matched to a load by a 16: single- phase transformer . Determine (a) the value of the load resistance and (b) the power dissipated in the load. **[(a) 78.13 Ω (b) 5m W]**

T 7.32 A transformer has 1200 primary turns and 200 secondary turns .The primary and secondary resistances are 0.2Ω and 0.02Ω repectively and the corresponding leakage reactances are1.2 Ω and0.05Ω respectively. Calculate (a) the equivalent resistance, reactance and impedence referred to the primary winding , and (b) the phase angle of the impedance .

[(a) 0.92Ω,3.0Ω,3.14Ω(β)72.95 °]

T 7.33 A60 kVA,1600 V/100 V,50 Hz, single-phase transformer has 50 secondary windings Calculate(a) the primary and secondary current, (b) the number of primary turns , and (c) the maximum value of the flux. **[(a) 37.5 A, 600A (b) 800 (c) 9.0 mwb]**

T 7.34 A single -phase ,50 Hz transformer has 40 primary turns and secondary turns. The cross-sectional area of the core is 270 cm2.When the primary winding is connected to a 300volt supply, determine (a) the maximum value of the flux density in the core,and (b) the voltage induced in the secondary winding. **[(a) 1.25T (b) 3.99kV]**

T 7.35 A single -phase 800V/100V,50 Hz transformer has a maximum core flux density of -1.294T and an effective cross-sectional area of 60cm2. Calculate the number of turns on the primary and secondary windings. **[464,58]**

T 7.36 A 3.3 kV/110 V, 50 Hz,single -phase transformer is to have an approximate e.m.f per turn of 22V and operate with a maximum flux of 1.25T.Calculate (a) the number of primary and secondary turns,and (b) the cross-sectional area of the core. **[(a) 150,5(b)792.8 cm2]**

T 7.37 A single-phase transformer has 2400 turns on the primary and 600 turns on the secondary.Its no load current is 4A at a power factor of 0.25 lagging.Assuming the volt drop in the windings is negligible ; calculate the primary current and factor when the secondary current is 80 A at a power factor of ~~0.80 of~~0.8 lagging. **[23.26A, 0.73]**

T 7.38 An ideal transformer is rated 2400 v/240 V.A certain load of 50 A, unity power factor , is to be connected to the low- voltage winding . This load must have exactly 200 V across it . With 2400 V applied to the high- voltage winding, what resistance must be added in series with the transformer , if located (a) in the low voltage winding, (b) in the high- voltage winding ?

[(a) 0.8 0.8Ω;(80Ω]

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AC Network Analysis

Chapter - 3

In previous chapters the voltage and current sources were unidirectional (DC). In contrast, many electrical networks of practical importance are excited by AC source. The most common example of an AC network is electric power network containing power generating stations, transmission lines, substations etc. Where as AC is an abbreviation for alternating current, implying a periodic reversal of directions(of current or voltage). In this chapter we will restrict our attention to quantities varying sinusoidally i.e. the voltage $V = V_m \sin \omega t$. Sometime a cosine function might be used instead of sine function.

3.1 Energy Storage Elements

The components of electric circuits are called circuit elements i.e. resistance, inductance and capacitance. Resistance directly opposes the current and the energy dissipated in over coming this opposition appears as heat. But capacitance and inductance, both of which lead to the storage of energy in an electromagnetic field.

3.2 Ideal Capacitor

Ideal capacitors exist only in text books not on real circuit boards. An ideal capacitor is characterized by a single constant value, capacitance (c) measured in farads. This is the ratio of the electric charge (Q) on each conductor to the potential difference (V) between them i.e. C = Q/V. In an ideal capacitor, the charge (Q) varies only in response to current flowing externally.

An ideal capacitor would not dissipate any power. Real capacitors dissipate a small amount of power when ever current flows through them due to ohmic losses. An ideal capacitor will never completely discharge. It will gradually approach zero volts but never quite reach it.

3.2.1 Energy Storage in Capacitors

When a d.c source (battery) is connected to uncharged capacitor then voltage across the capacitor increases. As a result the capacitor starts to store energy. During this process capacitor draws a current from d.c source which is called charging current. This current changes with time. When voltage across capacitor equals to voltage of d.c source then we say that the capacitor is fully charged. In this condition charging current is zero.

During charging period, i = instantaneous value of charging current.

dv = voltage between the plates of capacitor increases in time dt seconds.

dq = Stored charge in time dt seconds.

dq = C.dv

But dq = i.dt

$\therefore$ i.dt = C.dv

$\Rightarrow i = C.\frac{dv}{dt}$

Power supplied in time dt is, $P = vi = v.C\frac{dv}{dt}$

Energy supplied in time dt is, $W = p.dt = v \,.\, C\frac{dv}{dt}.dt = vcdv$

The total energy supplied to the capacitor when the voltage increases from zero to V volts is,

$W = \int_0^V v\times c\times dv = \frac{1}{2}cv^2$ joules.

3.3 Ideal Inductor

An ideal inductor has inductance, but no resistance or capacitance and does not dissipate energy. A real inductor may be partially modeled by a combination of inductance, resistance and capacitance. As ideal inductor has no resistance so a constant current flows through it without causing a voltage drop. In other words the ideal inductor acts as a short circuit in the presence of DC. When AC flows through ideal inductor then it produces a resistance (called reactance) which opposes a.c and causing a voltage drop.

3.3.1 Energy Storage in Inductor

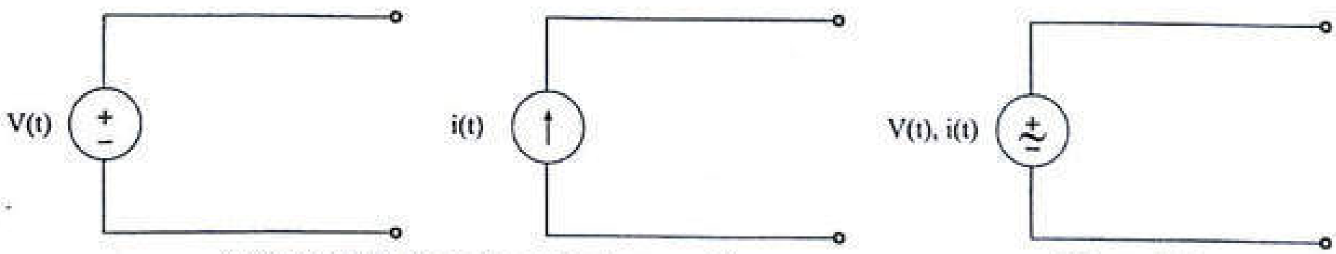
Instantaneous power entering the inductor at any instant is given by $P = Vi = L\frac{di}{dt}\cdot i = Li\frac{di}{dt}$

Energy Stored in inductor is given by, $W = \int P.dt = \int L.i.\frac{di}{dt}.dt = \int Li.di = \frac{1}{2}Li^2$ joule.

The energy is stored in the inductor in a magnetic field. This energy depends only on current magnitude. When the current increases, the stored energy in the magnetic field also increases. When the current reduces to zero, the energy stored in the inductor is returned to the source from which it receives the energy.

3.4 Time - dependent Signal Sources

Generally sources can be classified as out put with constant magnitude and out put with varying magnitude (with same polarity or opposite polarity) with respect to time. The Later type is termed as time - varying sources (voltages and currents). Fig 3.1 illustrates the convention that will be employed to denote time - dependent signal sources.



(Generalised time-dependent sources) (Sinusoidal source)

Fig 3.1

One of the most important classes of time - dependent signals is that of periodic signals. These signals appear frequently in practical applications and are a useful approximation of many physical phenomena. Fig3.2 illustrates a number of the periodic wave forms that are typically encountered in the study of electrical circuits. Wave forms such as the sine, triangle, square, pulse and sawtooth waves are provided in the form of voltages (or less frequently currents) by commercially available signal (or wave from) generators. Such instruments allow for selection of the wave form, peak amplitude, and of its period.

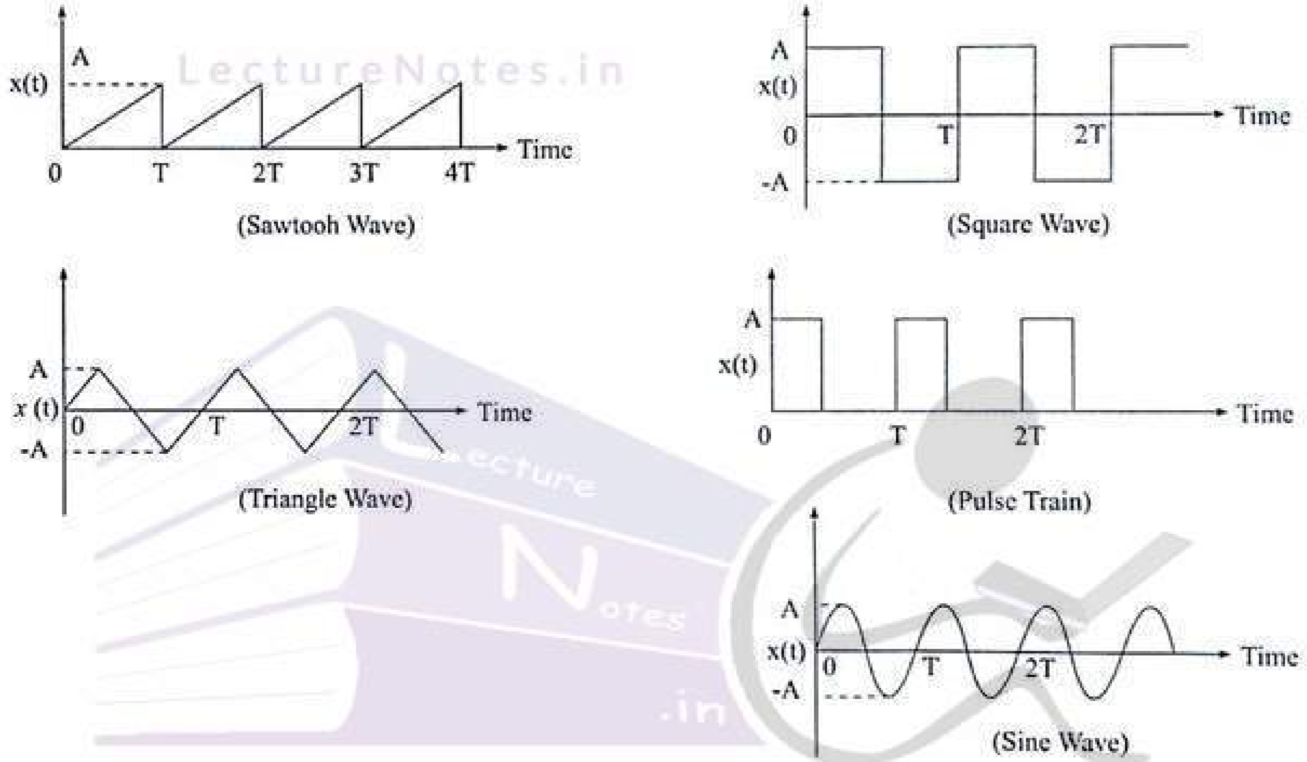


Fig 3.2

3.4.1 Sinusoids

An alternating current or voltage may have any wave shape. But it can be proved that any wave shape be made up of various combinations of sine waves. Thus sine wave is basic to all alternating currents and voltages. The quantities obeying sine law are called sinusoids.

Sinusoidal wave forms constitute by far the most important class of time - dependent signals as that the electric power used for industrial and house hold applications world wide is generated and delivered in this form of either 50 Hz or 60 Hz sinusoidal voltages or currents. A generalized sinusoid is defined as follows.

$$x(t) = A \sin \omega t = A \sin 2\pi ft.$$

Where, A is amplitude, ω is radian frequency (angular frequency)

f = natural frequency (cycles/sec or Hz)

ω = radian frequency = $2\pi f$ (radian/sec)

Sinusoidal waves have more advantages than other waves. In transformers and a.c. machines, sinusoidal waves (sinusoidal voltages and currents) produce least iron and copper loss for a given out put. These waves produce least disturbance in electric circuit. Also these waves produce less interference (noise) in telephone lines.

3.4.2 Generation of Alternating Voltages and currents

Consider a rectangular coil having N number of turns and Am^2 cross - sectional area which is rotating in a uniform magnetic field with a constant angular velocity $\omega = rad.sec^{-1}$ When coil rotates through an angle θ in time t from x-axis (as shown in fig 3.3) the magnetic flux (ϕ)linkages through the coil changes. According to Faraday's Laws of electromagnetic induction a voltage is generated in the coil. It is given by,

$$V = -N.\frac{d\phi}{dt} = -N.\frac{d}{dt}(BA\cos\theta)$$

$$= -N\frac{d}{dt}BA\cos\omega t\,(\because \phi = BA\cos\theta \; and \; \omega = \theta/t)$$

$$= NBA\,\omega\, sin\,\omega t \; .... (1)$$

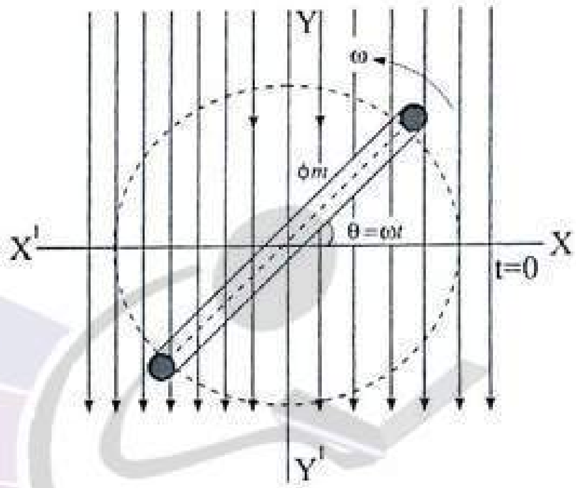


Fig. 3.3

The generated voltage V = maximum = V_m, when sin ωt = maximum = 1.

$$\therefore \quad V_m = NBA\,\omega(1) = NBA\,\omega$$

Putting this value in equation (1) we get,

$$V = V_m \; \sin\omega t$$

$$\text{or, } V = V_m \; \sin\theta \; .... (2)$$

From equation (2) it is clear that the voltage generated in the coil is sinusoidal. If this voltage is applied across a load, alternating current flows through the circuit which would also vary sinusoidally.

The equation of alternating current is given by, i = I_m sin ωt. provided the load is resistive. Alternating voltage and alternating current can be plotted against time as shown in fig 3.4

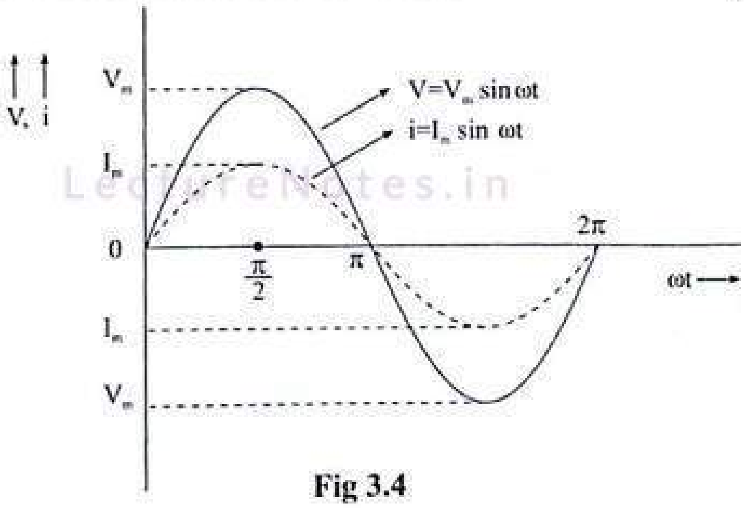


Fig 3.4

The value of an alternating quantity (alternating voltage or current) changes continuously and direction changes in every half of a cycle.

3.4.3 Definitions Relating to Alternating quantity

(i) ***Instantaneous Value :*** The value of alternating quantity at a given instant (time) is called the instantaneous value. It is represented by small letters i.e. **i** (for instantaneous current) and v (for instantaneous voltage)

(ii) ***Amplitude (Peak value) :*** It is the maximum value of the alternating quantity attained by it in a cycle. It is represented by capital letters i.e. I_m (for current) and V_m (for voltage)

(iii) ***Cycle :*** One complete set of positive and negative values of an alternating quantity is known as cycle.

(iv) ***Time Period :*** It is the time taken by an alternating quantity to complete one cycle. It is represented by T.

(v) ***Frequency :*** The number of cycles per second is called the frequency of the alternating quantity. It is represented by f.

(vi) ***Phase :*** Phase of an alternating quantity is the fraction of the time period that has elapsed since the quantity last passed through the chosen zero position of reference.

(vii) ***Phase difference :*** Phase difference between two alternating quantities is the fractional part of a period by which one has advanced over or lags behind the other. To measure phase difference the frequency of the alternating quantities should be same.

3.4.4 Average value

The average value of an alternating quantity over a given interval is the sum of all instantaneous values divided by the number of values taken over that interval. If the alternating quantity (alternating voltage or current) represented by a curve then average value is equal to the height of the curve. The average height of the curve can be calculated by dividing the area under the curve by the length of the base of the curve.

$$Average\ value = \frac{Area\ under\ the\ curve}{length\ of\ the\ base\ of\ the\ curve}$$

(i) In case of symmetrical wave (i.e. positive half cycle exactly equal to the negative half cycle), the average value is the ratio of area of one alternation (one half cycle) to the base length of one alternation.

$$Average\ value\ of\ symmetrical\ wave = \frac{Area\ of\ one\ alternation}{Base\ length\ of\ one\ alternation}$$

(ii) In case of unsymmetrical waves, the average value is the ratio of area of one cycle to the base length of one cycle.

$$Average\ value\ of\ un\ symmetrical\ wave = \frac{Area\ of\ one\ cycle}{Base\ length\ of\ one\ cycle}$$

Note : ***Examples of symmetrical waves are sinusoidal voltage or current, full wave rectified a.c Example of unsymmetrical waves are half wave rectified a.c,***

Also we can calculate average value of alternating quantity by method of integration. The average value of current i over a period T is given by, $I_{av} = \frac{1}{T}\int_o^T i \cdot dt$

The average value of alternating voltage V over a period T is given by $V_{av} = \frac{1}{T}\int_0^T v \cdot dt$

3.4.5 R.M.S value or Effective value of A.C

The effective value of a.c is that value of d.c which when flowing through a given resistance for a given time produces the same amount of heat as produced by a.c when flowing through the same resistance for same time.

It is represented by I_{rms} or I_{eff}. To calculate it, suppose an alternating current is represented by $I = I_m \sin \omega t$.

The effective value of a.c is that value of d.c which when flowing through a given resistance for a given time produces the same amount of heat as produced by a.c when flowing through the same resistance for same time.

$$H = \int_O^T I^2 R.dt$$

$$= \int_O^T (I_m \sin \omega t)^2 R \cdot dt$$

$$= \int_O^T I_m^2 \sin^2 \omega t \cdot R \cdot dt$$

$$= I_m^2 R \int_O^T (\frac{1-\cos 2\omega t}{2}) dt$$

$$= \frac{I_m^2 RT}{2} \text{.. (1)}$$

If r.m.s value is represented by I_{rms} then the amount of heat produced in the same resistance R, in the same time T would be

$$H = I_{rms}^2 \, R \, T \text{ .... (2)}$$

From equation (1) and (2) we get,

$$I_{rms}^2 \, R \, T = \frac{I_m^2 \, RT}{2}$$

$$\Rightarrow I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

Hence the r.m.s value of a.c is 0.707 times the peak value of a.c i.e 70.7% of the peak value of a.c

Note : ***Similarly r.m.s value of alternating emf,*** $E_{rms} = E_m / \sqrt{2}$ ***where*** E_m ***is the peak value of alternating emf.***

(i) The r.m.s value of alternating current can be calculated by mid ordinate method,

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2}{n}}$$

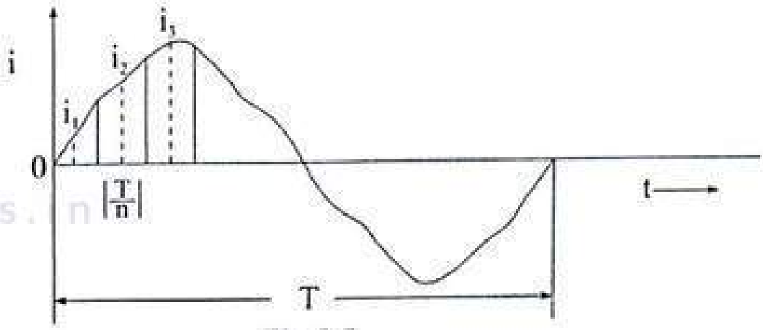


Fig 3.5

Where $i_1, i_2, i_3 \dots\dots i_n$ are mid ordinate current and n is the no of strips as shown in fig 3.5

Similarly the r.m.s value of alternating voltage can be expressed as,

$$E_{rms} = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2 + \dots + e_n^2}{n}}$$

(ii) The r.m.s value of a wave can also be expressed as,

$$R\ M\ S\ value = \sqrt{\frac{Area\ of\ half\ cycle\ wave\ squared}{Half\ cycle\ base}}$$

(iii) For symmetrical waves, the rms value can be found by considering half cycle or full cycle. How ever, for un symmetrical waves, full cycle should be considered.

3.4.6 Average value of Sinusoidal current

Let us consider an elementary strip of width dθ in the first half cycle of current wave as shown in fig3.6

Let i = mid-ordinate of the strip

Area of the strip = $i.d\theta$.

Area of half cycle = $\int_0^{\pi} i.d\theta = \int_0^{\pi} I_m \sin\theta.d\theta = 2I_m$

$$\therefore Average\ value\ I_{av} = \frac{Area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}$$

$$= \frac{2I_m}{\pi} = 0.637\, I_m$$

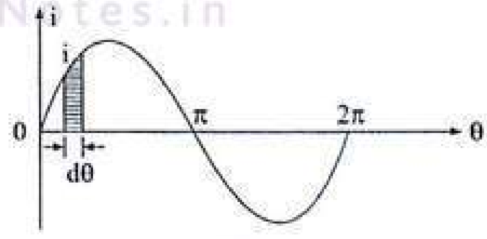


Fig 3.6

Note : ***Similarly average value of Sinusoidal emf*** $E_{av} = 0.637\, E_m$

3.4.7 R .M .S value of Sinusoidal current

Let us consider an elementary strip of width $d\theta$ in the first half cycle of squared current wave as shown in fig 3.7

Let i^2 = mid ordinate of the strip. Area of the strip = $i^2 \cdot d\theta$

Area of half cycle of the squared wave

$$= \int_0^{\pi} i^2 . d\theta = \int_0^{\pi} I_m^2 \sin^2\theta \cdot d\theta = \frac{\pi I_m^2}{2}$$

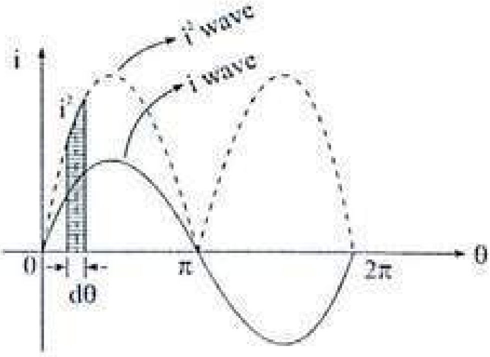


Fig 3.7

$$\therefore R\,M\,S \text{ value } I_{rms} = \sqrt{\frac{Area\ of\ half\ cycle\ squared\ wave}{Base\ length\ of\ half\ cycle}}$$

$$= \sqrt{\frac{\pi I_m^2/2}{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

3.4.8 Form factor and Peak factor

Form factor : ***The ratio of r.m.s value to average value of an alternating quantity is known as form factor. I is represented by K_f.***

$$\therefore K_f = \frac{R.M.S\ value}{Average\ value}$$

Peak factor : ***The ratio of maximum value to the r.m.s value of an alternating quantity is known as peak factor or amplitude factor. It is represented by K_a.***

$$\therefore K_a = \frac{Maximum\ value}{R.M.S\ value}$$

Example 3.1: Find the r.m.s value, average value and form factor of (i) half wave rectified a.c (ii) full wave rectified a.c.

solution (i) In half wave rectified a.c as shown in fig 3.8, complete cycle is to be considered. It is because the wave is not symmetrical.

$$Average\ value\ I_{av} = \frac{Area\ of\ one\ cycle}{Base\ length\ of\ one\ cycle} = \frac{2I_m + o}{2\pi} = \frac{I_m}{\pi}$$

$$R.M.S\ value\ I_{rms} = \left[\frac{Area\ of\ squared\ wave\ over\ one\ cycle}{Base\ length\ of\ one\ cycle}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\frac{\pi I_m^2}{2} + 0}{2\pi}\right]^{\frac{1}{2}} = \frac{I_m}{2}$$

$$Form\ factor\ \ K_f = \frac{rms\ value}{average\ value} = \frac{I_m/2}{I_m/\pi} = 1.57$$

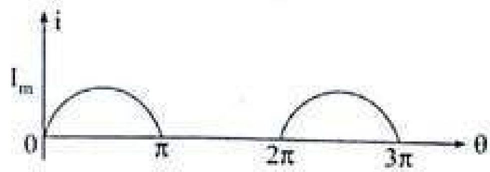


Fig 3.8

(ii) In full wave rectified a.c as shown in fig 3.9, half cycle is to be considered. It is because the wave is symmetrical.

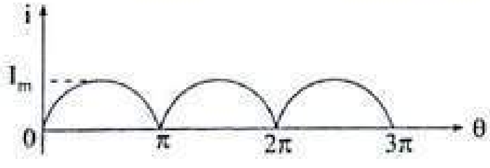


Fig 3.9

$$Average\ value\ I_{av} = \frac{Area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}$$

$$= \frac{2I_m}{\pi}$$

$$R.M.S\ value\ I_{rms} = \left[\frac{Area\ of\ squared\ wave\ over\ half\ cycle}{Base\ length\ of\ half\ cycle}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\pi I_m^2}{2} \Big/ \pi\right]^{\frac{1}{2}} = \frac{I_m}{\sqrt{2}}$$

$$Form\ factor\ K_f = \frac{I_{rms}}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

Example 3.2 Find the r.m.s and average value of the wave form shown in fig 3.10

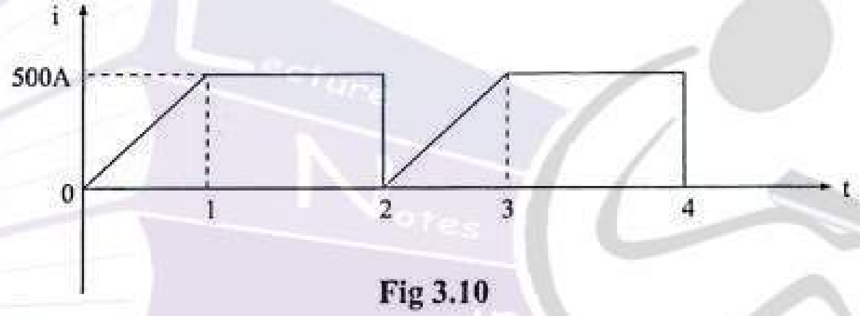


Fig 3.10

Solution : The given wave is symmetrical. So half wave is to be considered.

In fig 3.11 AB = 500 A
OB = 1 sec. BC = 1 sec.
OC = 2 sec.

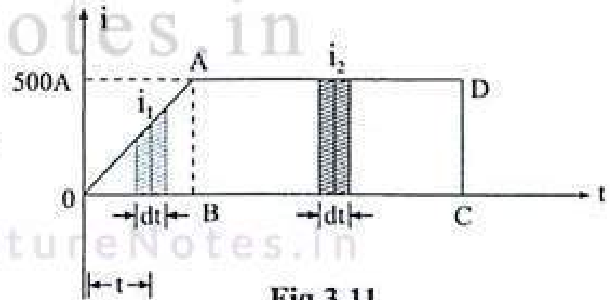


Fig 3.11

Consider a strip of thickness dt and i_1 is the mid-ordinate current at any time 't'.

$$\therefore Area\ of\ the\ strip = length \times breadth = i_1.dt$$

$$Area\ of\ OAB = \int_0^1 i_1.dt$$

$$It\ is\ seen\ that, \quad \frac{i_1}{t} = \frac{AB}{OB}$$

$$\Rightarrow \frac{i_1}{t} = \frac{500}{1}$$

$$\Rightarrow i_1 = 500t$$

$$\therefore Area\ of\ OAB = \int_0^1 500t.dt$$

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and delivery.

$$Also\ Area\ of\ ABCD = \int_1^2 i_2.dt = \int_1^2 500.dt$$

$$\therefore Total\ area\ of\ half\ cycle = \int_o^1 500t.dt + \int_1^2 500.dt$$

$$\therefore Average\ value\ I_{av} = \frac{Area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}$$

$$= \frac{\int_o^1 500t.dt + \int_1^2 500.dt}{2} = 375A$$

$$R\,M\,S\ value\ I_{rms} = \left[\frac{squared\ area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\int_0^1 (500t)^2 \cdot dt + \int_1^2 (500)^2 \cdot dt}{2}\right]^{\frac{1}{2}}$$

$$= 408.248A$$

Example 3.3 : Determine the r.m.s and average value of the wave form shown in fig 3.12

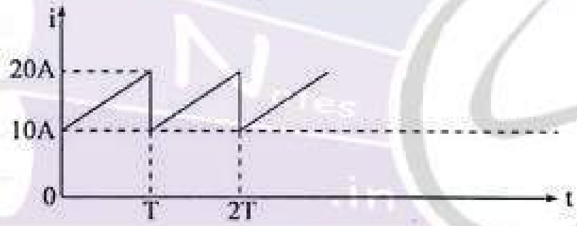


Fig 3.12

Solution : The given wave is symmetrical, so half cycle is to be considered. Consider current i at any time t. It is seen that,

$$\frac{QR}{PR} = \frac{SM}{PM}$$

$$\Rightarrow \frac{10}{T} = \frac{i-10}{t}$$

$$\Rightarrow i = 10 + \frac{10}{T}t$$

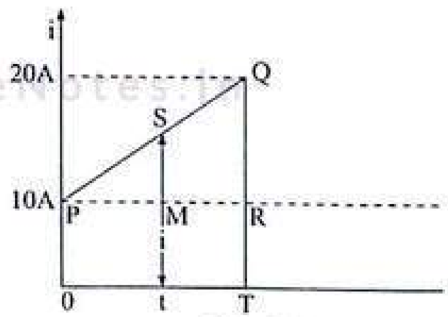


Fig 3.13

$$Area\ of\ half\ cycle = \int_o^T i.dt = \int_o^T (10 + \frac{10}{T}t).dt$$

$$= 15T$$

$$Average\ value\ I_{av} = \frac{Area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}$$

$$= \frac{15T}{T} = 15A$$

$$R.M.S\ value\ I_{rms} = \left[\frac{Squared\ area\ of\ half\ cycle}{Base\ length\ of\ half\ cycle}\right]^{\frac{1}{2}}$$

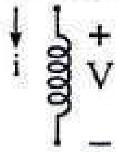
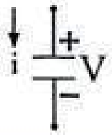
$$= \left(\frac{\int_o^T i^2.dt}{T}\right)^{\frac{1}{2}} = \left[\frac{\int_o^T (10+\frac{10}{T}t)^2.dt}{T}\right]^{\frac{1}{2}}$$

$$= 15.27\,A$$

3.5 Solution of circuits containing energy storage elements

In this section we shall discuss how the energy storage elements respond to a time varying source. The passive circuit elements resistance R, inductance L and capacitance C are defined by the manner in which the voltage and current are related for the individual element. For example, if the voltage V and current i for a single element are related by a constant then the element is a resistance, R is the constant of proportionality and V = Ri. Similarly, if the voltage is the time derivative of the current then the element is an inductance, L is the constant of proportionality and $V = L\frac{di}{dt}$. Finally, if the current in the element is the time derivative of the voltage then the element is a capacitance, C is the constant of proportionality and $i = C\,\frac{dv}{dt}$. Table below summarizes these relationships for the three passive circuit elements. Note the current directions and the corresponding polarity of the voltages.

Table

Circuit element	Units	Voltage	Current	Power
 Resistance	$ohm\,(\Omega)$	$V = Ri$	$i = \frac{V}{R}$	$P = Vi = i^2R$
 Inductance	henries (H)	$V = L\cdot\frac{di}{dt}$	$i = \frac{1}{L}\int V\cdot dt + K_1$	$P = Vi = Li\frac{di}{dt}$
 capacitance	farads (F)	$V = \frac{1}{c}\int i\cdot dt + K_2$	$i = C\cdot\frac{dv}{dt}$	$P = Vi = CV\frac{dv}{dt}$

3.5.1 Sinusoidal AC through Pure resistor

Consider an a.c circuit containing a pure resistance (R) connected in series with an a.c source of voltage $V = V_m \sin \omega t$ as shown in fig. 3.14 (a).

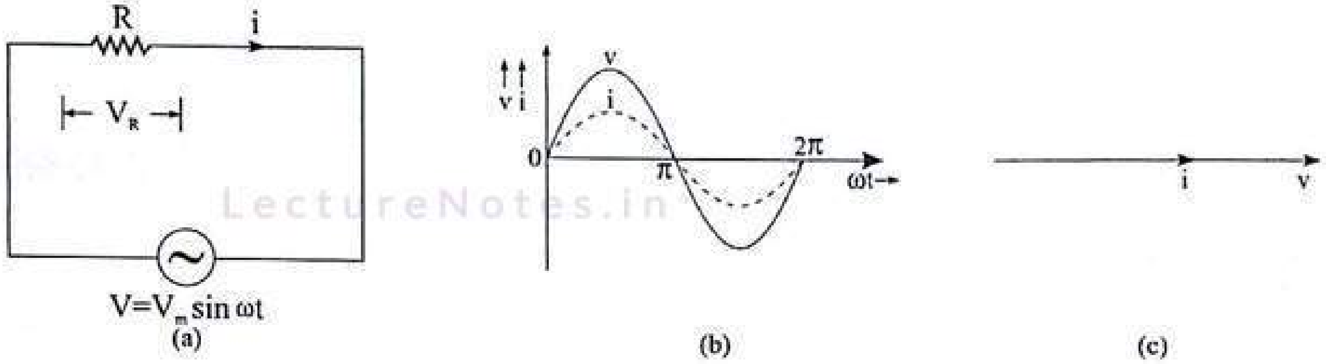


Fig 3.14

Let V_R = voltage drop across resistance R.

Applying KVL, $V = V_R$

$$\Rightarrow V_m \sin \omega t = iR \qquad (\because V_R = IR)$$

$$\Rightarrow i = \frac{V_m \sin \omega t}{R}$$

$$\Rightarrow i = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R}\right)$$

Thus the current will be in phase with applied voltage as shown in fig 3.14(b).

Also $I = \frac{V}{R}$

Where I = r.m.s value of current

V = r.m.s value of the applied voltage

R = resistance in ohm.

Polar form : Let V = Voltage phasor taken as reference phasor = $V\angle 0$

In purely resistive a.c circuit, current and voltage are in same phase.

so current, $I = I\angle 0$

The complex value of circuit impedance,

$$Z = \frac{V}{I} = \frac{V\angle 0}{I\angle 0} = R\angle 0 = R + jo$$

Thus the impedance of a resistor is Z = R.

3.5.2 Sinusoidal AC through pure inductance

Consider an a.c circuit containing a Pure, inductance (L) connected in series with an a.c source of voltage $V = V_m \sin \omega t$ as shown in fig. 3.15 (a)

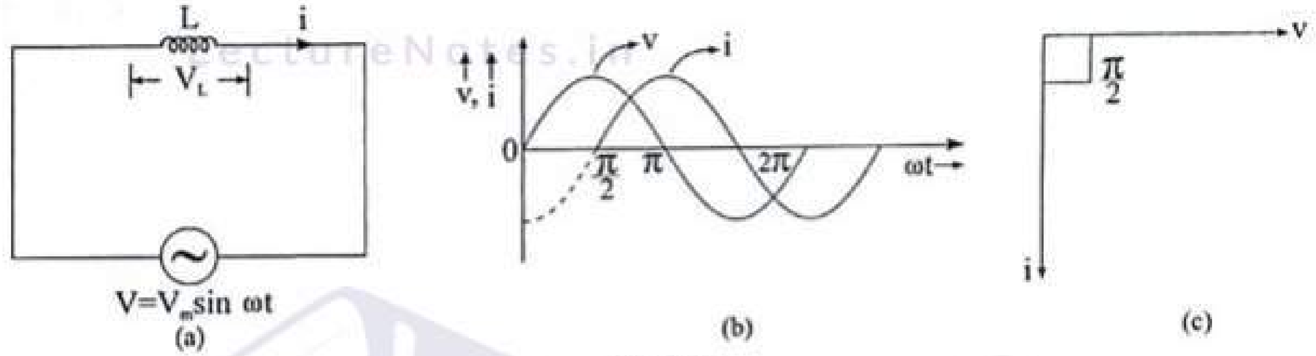


Fig 3.15

Whenever an alternating sinusoidal voltage is applied to a purely inductive coil, a back emf is produced due to the self inductance of the coil. The applied voltage has to over come this self induced emf.

$V = V_m \sin \omega t$ = alternating sinusoidal voltage applied to the coil.

V_L = back emf induced in the coil

According to KVL, $V = V_L$

$$\Rightarrow V_m \sin \omega t = L \cdot \frac{di}{dt} \qquad \left(\because V_L = L \cdot \frac{di}{dt}\right)$$

$$\Rightarrow \frac{di}{dt} = \frac{V_m \sin \omega t}{L}$$

$$\Rightarrow i = \frac{V_m}{L}\int \sin \omega t \cdot dt = -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Where $X_L = \omega L$ = inductive reactance.

Current flows through the inductor $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

where $I_m = \frac{V_m}{X_L}$

So the current lags behind the applied voltage by $\frac{\pi}{2}$ and phasor diagram is shown in fig. 3.15 (b)

Also $I = \dfrac{V}{X_L}$

Where I = r.m.s value of current

V = r.m.s value of voltage

$X_L = \omega L$ = inductive reactance in ohm.

Polar form : Let V = voltage phasor taken as reference phasor = $V\angle 0$

In purely inductive circuit, current lags behind the applied voltage by $\dfrac{\pi}{2}$

So current $I = I\angle -90$

The complex value of circuit impedance.

$$Z = \frac{V\angle 0}{I\angle -90} = X_L \angle 90 = 0 + jX_L = jX_L$$

$\Rightarrow Z = j\omega L$

Thus impedance of an inductor $Z = jX_L = j\omega L$

3.5.3 Sinusoidal A.C through Capacitance

Consider an a.c circuit containing a pure capacitance (C) connected in series with an a.c source of voltage $V = V_m \sin \omega t$ as shown in fig. 3.16 (a)

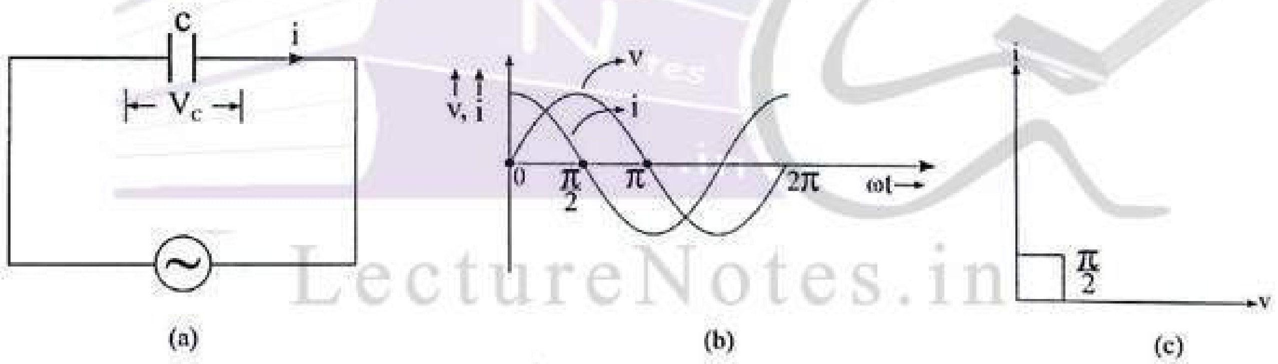


Fig 3.16

Whenever an alternating sinusoidal voltage is applied to the plates of a capacitor of capacitance 'C' then the instantaneous charge in the capaciter is $q = cV_c$ Where V_c is the voltage across capacitance.

Applying KVL, $V = V_c$

$$\Rightarrow V_m \sin \omega t = \frac{q}{c} \qquad \left\{\because V_c = \frac{q}{c}\right\}$$

$$q = CV_m \sin \omega t$$

$$\Rightarrow \frac{dq}{dt} = \omega C \quad V_m \cos \omega t$$

$$\Rightarrow \frac{dq}{dt} = \frac{V_m}{1/\omega C}\cos\omega t \qquad \left(\because X_c = \frac{1}{\omega c}\right)$$

$$\Rightarrow i = \frac{V_m}{X_c}\cos\omega t = \frac{V_m}{X_c}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{Where } I_m = \frac{V_m}{X_c}$$

So the current leads the applied voltage by $\frac{\pi}{2}$ and the phasor diagram is shown in fig.3.16(b)

Also $I = \frac{V}{X_c}$

Where I = r.m.s value of current

V = r.m.s value of voltage

X_c = Capacitive reactance in ohm.

Polar form :

Let V = voltage phasor taken as reference phasor = $V\angle 0$

In a purely capacitive circuit, the current leads the voltage by 90^0.

So current $I = I\angle 90^\circ$

The complex value of circuit impedance,

$$Z = \frac{V\angle 0}{I\angle 90^\circ} = X_c\angle -90 = 0 - jX_c = -jX_c$$

$$\Rightarrow Z = \frac{-j}{\omega c}$$

Thus impedance of capacitor $Z = -jX_c = \frac{-j}{\omega c}$

3.6 Phasor Solution of circuits with sinusoidal excitation

We have seen that sinusoids can be represented by phasors. Since phasors can be expressed in terms of complex numbers, so sinusoids can also be expressed interms of complex numbers. In this section, we introduce an efficient notation to make it possible to represent sinusoidal signals as complex numbers and to eliminate the need for solving differential equations.

3.6.1 Euler's Formula

Euler's formula is a mathmatical formula in complex analysis that establishes the deep relationship between the trigonmetric functions and complex exponential functions.

It states that, $e^{j\theta} = \cos\theta + j\sin\theta$

where j is used to express the imaginary number $\sqrt{-1}$.

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

The magnitude of $e^{j\theta}$ or $e^{-j\theta}$ is equal to one. This formula provides a means of conversion between cartesian co-ordinates and polar co-ordinates. Consider any complex number Z = x + jy as shown in fig. 3.17

Magnitude of Z is,

$$|Z| = \sqrt{x^2 + y^2}$$

Where

$$x = |Z|\cos\theta = \textit{real part.}$$

$$y = |Z|\sin\theta = \textit{Imaginary Part}.$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \textit{ in radians.}$$

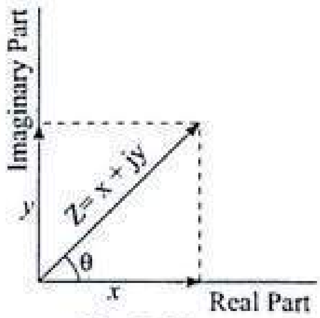


Fig 3.17

$$Z = x + jy = |Z|\cos\theta + j|Z|\sin\theta = |Z|(\cos\theta + j\sin\theta)$$
$$= |Z|e^{j\theta} = |Z|\angle\theta$$

[Note :- We know $A\angle\theta = A(\cos\theta + j\sin\theta)$]

3.6.2 Phasors

A phasor is a complex number that contains the amplitude and phase angle information of a sinusoidal function. Its concept can be developed by Euler's formula.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

If $\theta = \omega t$, $e^{\pm j\omega t} = \cos\omega t \pm j\sin\omega t$ .... (1)

From equation (1),

Cos ωt = $\mathrm{Re}\left[e^{j\omega t}\right]$ = *real part of* $e^{j\omega t}$ = phase (active) component

Sin ωt = $I_m\left[e^{j\omega t}\right]$ = Imaginary part of $e^{j\omega t}$ = quadrature (reactive) component.

Similarly we can write,

$V_m \cos\omega t = \mathrm{Re}\left[V_m e^{j\omega t}\right]$ .... (2)

and $V_m \sin\omega t = I_m\left[V_m e^{j\omega t}\right]$ .... (3)

Also $V_m \sin(\omega t + \phi) = I_m\left[V_m e^{j(\omega t + \phi)}\right] = I_m\left[V_m e^{j\omega t} . e^{j\phi}\right]$ ....(4)

If we compare equations (3) and (4) i.e. $V_m \sin \omega t$ and $V_m \sin(\omega t + \phi)$, we get the factor $e^{j\omega t}$ is common to both and it contains the information about angular frequency (ω). Once the sinusoids of the same frequency are involved, this information is redundant. Then the representations of $V_m \sin \omega t$ and $V_m \sin(\omega t + \phi)$ are simplified, by dropping the term $e^{j\omega t}$ to the following form :

$$V_m \sin \omega t = V_m e^{jo} = V_m \angle o$$

$$V_m \sin(\omega t + \phi) = V_m e^{j\phi} = V_m \angle\phi$$

Example 3.4

Add the sinusoidal voltages $V_1(t) = A\cos(\omega t + \phi)$ and $V_2(t) = B\cos(\omega t + \theta)$ using phasor notation and then convert back to time - domain form. $A = 1.5V$, $\phi = 10^\circ$, $B = 3.2V \; and \; \theta = 25^\circ$.

Solution :

Given $V_1(t) = A\cos(\omega t + \phi) = 1.5\cos(\omega t + 10^\circ) = -1.5\sin(\omega t - 80)$

and $V_2(t) = B\cos(\omega t + \theta) = 3.2\cos(\omega t + 25^\circ) = -3.2\sin(\omega t - 65)$

These two voltages are in phasor form:

$V_1(j\omega) = -1.5\angle -80 = -0.2604 + j1.477$

and $V_2(j\omega) = -3.2\angle -65 = -1.352 + j2.9$

$\therefore \; V_1(j\omega) + V_2(j\omega) = -0.2604 + j1.477 - 1{\cdot}352 + j29$

$\Rightarrow V(j\omega) = -1.6124 + j4.377 = 4.66\angle 110.22^\circ$

In time domain form, $V(t) = 4.66\sin(\omega t + 110.22^\circ)$

$$\Rightarrow V(t) = 4.66\cos(\omega t + 20.22^\circ)$$
$$\Rightarrow V(t) = 4.66\cos(\omega t + 0.3527 \; rad)$$

Note : *Any sinusoidal mathematically represented in two ways.*

(i) in time domain form, $V(t) = A\sin(\omega t + \theta)$

(ii) in phasor form, $V(j\omega) = A e^{j\theta} = A\angle\theta$

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Basic Electrical Engineering

Topic:

Super Position Of AC Signals

Contributed By:

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3.6.3 Super position of AC Signals

Two sinusoidal sources of different phase and amplitude can be added if they have same frequency. If the frequency of two or more sinusoidal sources are different then they can not be added. To complete the analysis of any circuit with two or more sinusoidal sources at different frequencies using phasors, it is necessary to solve the circuit separately for each signal and then add the individual answers obtained for the different sources. It can be explained by the following examples.

Example 3.5 In fig. 3.18 calculate two voltage drops $V_1(t)$ and $V_2(t)$ across two resistors R_1 and R_2. Given $i(t)=0.5\cos 628t\,A$, $V(t)=20\cos 6280t\,V$, $R_1=150\Omega$ and $R_2=50\Omega$.

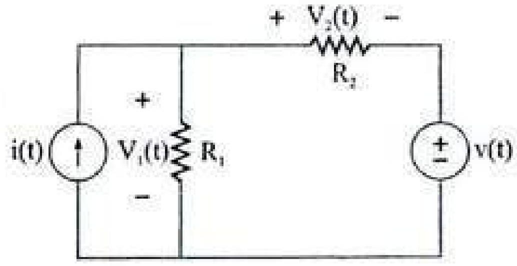


Fig 3.18

Solution :

Here two sources i (t) and V (t) have different frequencies. So we must compute a separate solution for each.

(i) First consider the current source, with the voltage source set to zero (i.e. short circuit) as shown in fig. 3.19

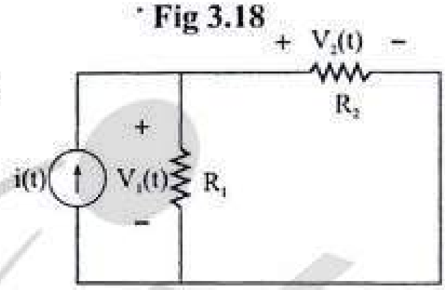


Fig 3.19

$i(t)=0.5\cos 628t=-0.5\sin(628t-90)$

In phasor form,

$I(j\omega)=-0.5\angle-90A, \omega=628\text{ rad.sec}^{-1}$

According to current division rule, Current through

$$R_1=-0.5\angle-90\times\frac{R_2}{R_1+R_2}=-0.5\angle-90\times\frac{50}{200}=-0\cdot125\angle-90A$$

Current through $R_2=-0.5\angle-90\times\dfrac{R_1}{R_1+R_2}=-0.5\angle-90\times\dfrac{150}{200}=-0.375\angle-90A$

Voltage drop across $R_1=-0.125\angle-90\times150=-18.75\angle-90V$

Voltage drop across $R_2=-0.375\angle-90\times50=-18.75\angle-90V$.

(ii) Next we consider the voltage source, with the current source set to zero (i.e. open circuit) as shown in fig. 3.20

$V(t)=20\cos 6280t \qquad V=-20\sin(6280t-90)V$

In phasor form,

$V(j\omega)=-20\angle-90V, \omega=6280\text{ rad.sec}^{-1}$

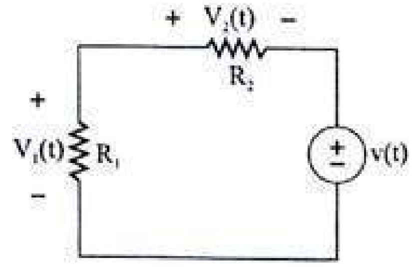


Fig 3.20

According to voltage division rule,

Voltage drop across $R_1 = -20\angle -90° \times \frac{R_1}{R_1 + R_2}$

$$= -20\angle -90 \times \frac{150}{200} = -15\angle -90° V.$$

Voltage drop across $R_2 = -\left[-20\angle -90 \times \frac{R_2}{R_1 + R_2}\right] = 20\angle -90 \times \frac{50}{200} = 5\angle -90° V$

Now we can determine the voltage across each resistor by adding the contributions from each source.

$\therefore$ *Voltage drop across* $R_1 = V_1(j\omega) = -18.75\angle -90° - 15\angle -90°$

In time domain form,

$$V_1(t) = -18.75\sin(628t - 90) - 15\sin(6280t - 90)$$
$$= 18.75\cos 628t + 15\cos 6280t\, V$$

Voltage drop across $R_2 = V_2(j\omega) = -18.75\angle -90° + 5\angle -90°$

In time domain form,

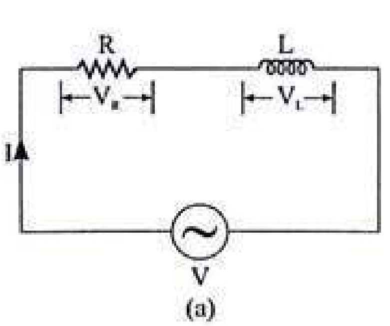
$$V_2(t) = -18.75\sin(628t - 90) + 5\sin(6280t - 90)$$
$$= 18.75\cos 628t - 5\cos 6280t$$
$$= 18.75\cos 628t + 5\cos(6280t + \pi)$$

3.7 A.C Series circuit

A circuit in which the same a.c flows through all the circuit elements (i.e. R, L, C) is called series a.c circuit. The general rules of series a.c circuits are same as d.c circuits. However, we work with phasors.

3.7.1 Series RL Circuit

Consider an ac circuit containing a resistor of resistance R ohms and an inductor of inductance L henries across an a.c source of r.m.s voltage V volts as shown in fig 3.21 (a)



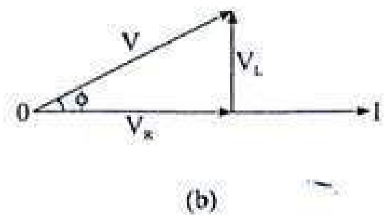


Fig 3.21

Let V = r.m.s Value of the applied voltage

I = r.m.s value of the circuit current

V_R = voltage drop across R

V_L = voltage drop across L

The phasor diagram of RL series circuit is shown in fig. 3.21 (b). The voltage drop V_R is in phase with current and voltage drop V_L leads the current by 90^o in phase. Since current is taken as the reference phasor, we have

$$\mathbf{I} = I + j\,o$$

Now

$$\mathbf{V} = \mathbf{V_R} + \mathbf{V_L}$$

$$\Rightarrow \mathbf{V} = (IR + jo) + (o + j\mathbf{I}\,X_L)$$

$$\Rightarrow \mathbf{V} = \mathbf{I}(R + jX_L)$$

$$\Rightarrow \mathbf{V} = \mathbf{IZ}.$$

Where $\mathbf{Z} = R + jX_L$ = impedance of RL circuit which offers opposition to current flow.

Magnitude of impedance, $Z = \sqrt{R^2 + X_L^2}$

Phase angle, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

It is clear that current lags the voltage by ϕ^o.

Polar form : In Polar form, we have

$$\mathbf{I} = I + j\,o = I\angle o$$

$$\mathbf{Z} = R + jX_L = Z\angle\phi$$

$$\mathbf{V} = \mathbf{IZ} = I\angle o \times Z\angle\phi = IZ\angle\phi$$

This shows that applied voltage leads the current by ϕ (or current lags the voltage by ϕ)

Voltage phasor as reference :

If we take the applied voltage V as the reference phasor (i.e. along X - axis) then

$$\mathbf{V} = V + jo = V\angle 0$$

$$\mathbf{Z} = R + jX_L = Z\angle\phi$$

$$\therefore \quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle o}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$$

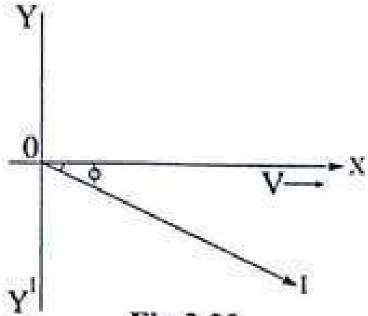
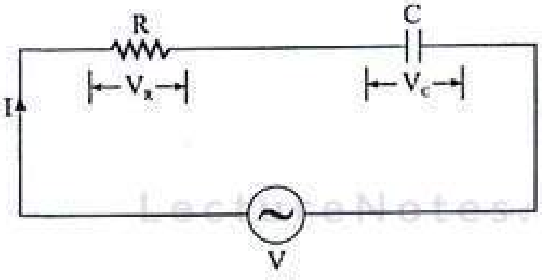


Fig 3.22

Fig. 3.22 shows the phasor diagram of the circuit with voltage as the reference phasor.

3.7.2 Series RC Circuit

Consider an ac circuit containing a resistor of resistance R ohms and a capacitor of capacitance C farad across an a.c source of rms voltage V volts as shown in fig. 3.23 (a)



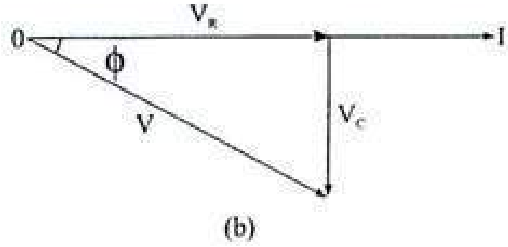


Fig 3.23

Let V = r.m.s value of the applied voltage.

I = r.m.s value of the circuit current

V_R = voltage drop across R

V_C = voltage drop across C

The phasor diagram of RC series circuit is shown in fig. 3.23 (b). The voltage drop V_R is in phase with current and voltage drop V_C lags the current by 90^0 in phase. Since current is taken as the reference phasor, we have

$$\mathbf{I} = I + jo$$

Now $$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C$$

$$\Rightarrow \mathbf{V} = (IR + jo) + (0 - jIX_C)$$

$$\Rightarrow \mathbf{V} = \mathbf{I}\,(R - jX_C)$$

$$\Rightarrow \mathbf{V} = \mathbf{IZ}$$

Where $\mathbf{Z} = R - jX_C$ = impedance of RC circuit which offers opposition to current flow

Magnitude of impedance, $Z = \sqrt{R^2 + X_C^2}$

Phase angle, $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

It is clear that current leads the voltage by $\phi°$.

Polar from : In polar form, we have

$$\mathbf{I} = I + jo = I\angle o$$
$$\mathbf{Z} = R - jX_C = Z\angle -\phi$$
$$\mathbf{V} = \mathbf{I}\,\mathbf{Z} = I\angle o \times Z\angle -\phi = I\,Z\angle -\phi$$

This shows that applied voltage lags the current by ϕ (or current leads the voltage by ϕ)

Voltage phasor as reference :

If we take the applied voltage V as the reference phasor (i.e. along x-axis) then

$$\mathbf{V} = V + jo = V\angle o$$

$$\mathbf{Z} = R - jX_C = Z\angle -\phi$$

$$\therefore \quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle o}{Z\angle -\phi} = \frac{V}{Z}\angle\phi$$

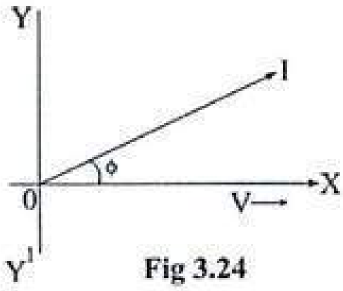


Fig 3.24

Fig.3.24 shows the phasor diagram of the circuit with voltage as the reference phasor.

3.7.3 Series R L C Circuit

Consider an ac circuit containing a resistor of resistance R ohms, an inductor of inductance L henry and a capacitor of capacitance C farad across an ac source of r m s voltage V volts as shown in fig.3.25(a)

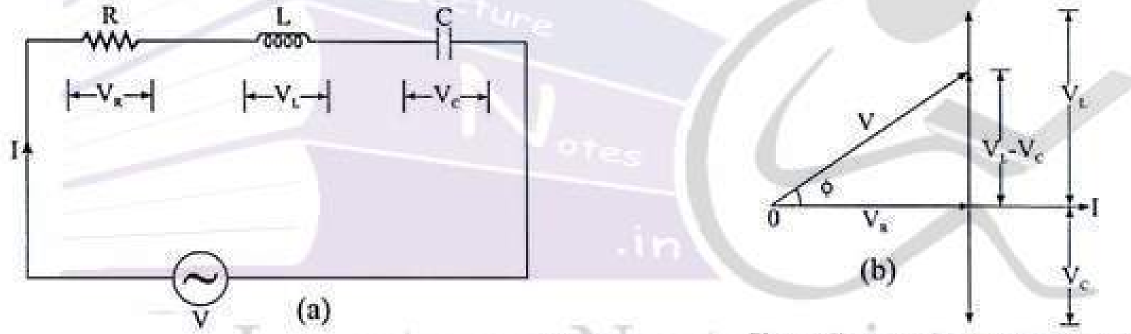


Fig 3.25

Phasor diagram of series RLC circuit when $X_L > X_C$ i.e. $V_L > V_C$

Let $\quad V$ = r.m.s value of the applied voltage.

I = r.m.s value of the circuit current.

V_R = voltage drop across R

V_L = voltage drop across L

V_C = voltage drop across C

The phasor diagram of RLC Series circuit is shown in fig 3.25 (b). The voltage drop V_R is in phase with current, voltage drop V_L leads the current by 90^0 and voltage drop V_C lags the current by 90^0 in phase. Since current is taken as the reference phasor, we have

$$\mathbf{I} = I + jo$$

Now $\quad \mathbf{V} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \qquad \begin{bmatrix} where\ \mathbf{V}_R = \mathbf{I}R, \\ \mathbf{V}_L = j\mathbf{I}X_L,\ \mathbf{V}_c = -j\mathbf{I}X_c \end{bmatrix}$

$\Rightarrow \quad \mathbf{V} = (\text{IR} + j\text{o}) + (\text{o} + j\text{IX}_L) + (\text{o} - j\text{IX}_C)$

$\Rightarrow \quad \mathbf{V} = \mathbf{I}\,(\text{R} + jX_L - jX_C)$

$\Rightarrow \quad \mathbf{V} = \mathbf{I}\,[\text{R} + j\,(X_L - X_C)]$

$\Rightarrow \quad \mathbf{V} = \mathbf{I\,Z}$

Where $Z = R + j\,(X_L - X_C)$ = impedance of RLC circuit which offers opposition to current flow.

Magnitude of impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase angle, $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Polar form : (i) Let $X_L > X_C$ *i.e* $X_L - X_C$ = positive.

In polar form, we have $\mathbf{I} = \text{I} + j\text{o} = \text{I}\angle \text{o}$

$$\mathbf{Z} = \text{R} + j(\text{X}_L - \text{X}_C) = Z\angle\phi$$

$$\therefore \quad \mathbf{V} = \mathbf{IZ} = \text{I}\angle\text{o} \times Z\angle\phi = \text{IZ}\angle\phi$$

$$\text{Where} \quad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

This shows that applied voltage leads the current by ϕ (or current lags the voltage by ϕ) as shown in fig 3.25 (b)

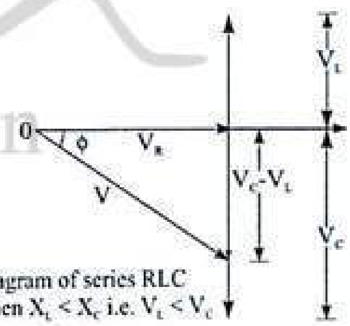
(ii) Let $X_C > X_L$ *i.e* $X_L - X_C$ = negative

In polar form, we have $\mathbf{I} = \mathbf{I} + j\text{o} = \text{I}\angle\text{o}$

$$\mathbf{Z} = \text{R} - j(\text{X}_L - \text{X}_C) = Z\angle -\phi$$

$$\mathbf{V} = \mathbf{IZ} = \text{I}\angle\text{o} \times Z\angle-\phi = \text{IZ}\angle-\phi$$

$$\text{Where} \quad \phi = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$



Phasor diagram of series RLC circuit when $X_L < X_C$ i.e. $V_L < V_C$

Fig 3.26

This shows that applied voltage Lags the current by ϕ (or current leads the voltage by ϕ) as shown in fig 3.26

3.7.4 Series Resonance

In R-L-C series circuit, both X_L and X_C are frequency dependent. If we vary the supply frequency then values of X_L and X_C are vary. At a certain frequency, called resonant frequency (f_r), X_L becomes equal to X_C and series resonance occurs.

An ac circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

At series resonance, $X_L = X_C$

$$\Rightarrow 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

If L is henry and C is in farad then f_r will be in Hz.

Impedance of RLC series circuit is given by,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow Z = \sqrt{R^2 + o} \qquad [\because \textit{In series resonance } X_L = X_C]$$

$$\Rightarrow Z = R$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

Properties of Series resonance :

In series resonance,

(i) the circuit impedance Z is minimum and equal to the circuit resistance R.

(ii) the circuit current $I = \frac{V}{Z} = \frac{V}{R}$, and the current is maximum.

(iii) the power dissipated is maximum *i.e* $P = \frac{V^2}{R}$

(iv) the resonant frequency is $f_r = \frac{1}{2\pi\sqrt{LC}}$

(v) the voltage across inductor is equal and opposite to the voltage across capacitor.

Resonance Curve

The curve between current and frequency is called as resonance curve. Fig 3.27 shows the resonance curve of R-L-C series circuit. If the frequency is below resonant frequency (f_r) then $X_C > X_L$ and net reactance is not zero. (i.e. current falls). If the frequency is above resonant frequency (f_r) then $X_L > X_C$ and the net reactance is again not zero (i.e. current falls). The current reaches its maximum value at resonant frequency (f_r), falling off rapidly on either side at that point.

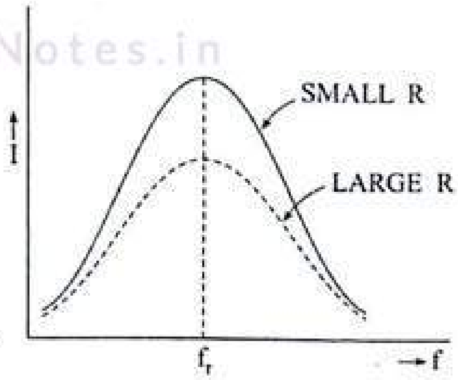


Fig 3.27

Note the effect of resistance in the circuit under resonance condition. The smaller the resistance, the greater the current at resonance and sharper is the curve. On the other hand, the greater the resistance, the smaller the current at resonance and flatter is the curve.

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Basic Electrical Engineering

Topic:

AC Parallel Circuit

Contributed By:

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3.8 A.C Parallel Circuit

Parallel circuits are used more frequently in electrical systems than series circuits. In parallel a.c circuits the voltage across all branches is same, but current in any branch depends upon the impedance of that branch. The total circuit current is the phasor sum of all branch currents.

By three methods we can solve the parallel a.c circuits. These are (i) By phasor diagram method. (ii) By phasor algebra method (iii) By admittance method. Here we use phasor algebra method to solve parallel a.c circuits

By Phasor Algebra Method :

In this method currents, voltages and impedances are expressed in complex form (i.e either in rectangular form or polar form). Consider a parallel circuit consisting of two branches and connected to an alternating voltage of V volts (r.m.s) as shown in fig 3.28.

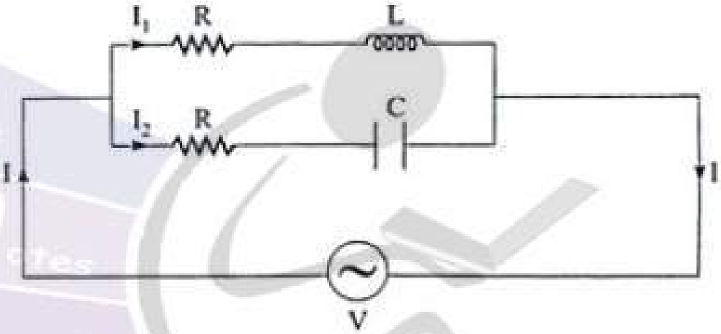

Fig 3.28

(i) Rectangular form :

Take voltage as the reference phasor,

$\mathbf{V} = V + j0 = V$

Impedance of branch 1, $\mathbf{Z_1} = R_1 + jX_L$

Impedance of branch 2, $\mathbf{Z_2} = R_2 - jX_C$

Current in branch 1, $\mathbf{I_1} = \dfrac{\mathbf{V}}{\mathbf{Z_1}} = \dfrac{V}{R_1 + jX_L}$

Current in branch 2, $\mathbf{I_2} = \dfrac{\mathbf{V}}{\mathbf{Z_2}} = \dfrac{V}{R_2 - jX_C}$

$\therefore$ *Total circuit current* $\mathbf{I} = \mathbf{I_1} + \mathbf{I_2} = \dfrac{V}{R_1 + jX_L} + \dfrac{V}{R_2 - jX_C}$

By using the rules of phasor algebra, we can calculate magnitude and phase angle of **I**.

(ii) Polar form :

Take voltage as the reference phasor,

$$\mathbf{V} = V\angle o$$

$$\mathbf{Z}_1 = Z_1\angle\phi_1 \quad \text{where} \quad Z_1 = \sqrt{R_1^2 + X_L^2}, \phi_1 = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\mathbf{Z}_2 = Z_2\angle-\phi_2, \quad \text{where} \quad Z_2 = \sqrt{R_2^2 + X_C^2}, \phi_2 = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\therefore \quad \mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{V\angle o}{Z_1\angle\phi_1} = \frac{V}{Z_1}\angle-\phi_1{}^{\circ}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{V\angle o}{Z_2\angle-\phi_2} = \frac{V}{Z_2}\angle\phi_2$$

$$\therefore \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \frac{V}{Z_1}\angle-\phi_1 + \frac{V}{Z_2}\angle\phi_2$$

3.8.1 Resonance in Parallel circuits

A parallel circuit containing reactive elements (L and C) is said to be in resonance when reactive component of line current is zero. The frequency at which reactive component of line current is zero is called as resonant frequency. It is represented by f_r.

Let us consider a circuit where a capacitance C is connected in parallel with an inductive coil of resistance R and inductance L as shown in fig 3.29 (a). This circuit is called ***tank circuit***

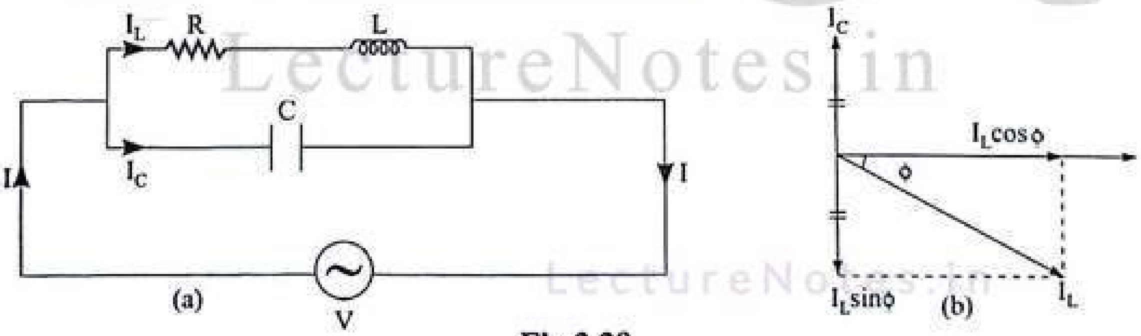

Fig 3.29

The phasor diagram of this circuit is shown in fig 3.29 (b) by considering the voltage V as the reference phasor.

Let I_L = Current through the coil which lags the voltage V by an angle φ

I_C = Current through the capacitor which leads the voltage V by an angle 90^0.

I = total circuit current

Current I_L resolves into two components, $I_L \cos\phi$ and $I_L \sin\phi$. The component $I_L \sin\phi$ is opposite to current I_C. So reactive component of line current is $I_c - I_L \sin\phi$. The circuit will be in resonance when the reactive component of line current is zero. This can be achieved by changing the supply frequency because both I_C and $I_L \sin\phi$ are frequency dependent. At some frequency called resonant frequency (f_r) the reactive component of line current will be zero and resonance takes place.

At parallel resonance, $I_C - I_L \sin\phi = 0$

$$\Rightarrow I_c = I_L \sin\phi$$

$$\Rightarrow \frac{V}{X_c} = \frac{V}{Z_L} \times \frac{X_L}{Z_L} \qquad \left(where\, Z_L = \sqrt{R^2 + {X_L}^2} \text{ and } X_L = Z_L \sin\phi \right)$$

$$\Rightarrow X_L X_C = {Z_L}^2$$

$$\Rightarrow \frac{\omega L}{\omega C} = {Z_L}^2$$

$$\Rightarrow \frac{L}{C} = {Z_L}^2 \text{ .... (1)}$$

$$\Rightarrow \frac{L}{C} = R^2 + {X_L}^2$$

$$\Rightarrow \frac{L}{C} - R^2 = {X_L}^2$$

$$\Rightarrow \frac{L}{C} - R^2 = (2\pi f_r L)^2$$

$$\Rightarrow f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$\Rightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This is the expression for resonant frequency in parallel circuits. If the coil resistance is small then,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Impedance at Resonance :

Under resonance condition $I_L \sin\phi$ is equal and opposite to I_c, hence cancell.

So line current (I) is equal to $I_L \cos\phi$.

$$\therefore I = I_L \cos\phi$$

$$\Rightarrow \frac{V}{Z} = \frac{V}{Z_L} \times \frac{R}{Z_L} \qquad \left[\begin{array}{l} where\ z = impedance\ at\ resonance \\ \quad R = Z_L \cos\phi \end{array} \right]$$

$$\Rightarrow \frac{1}{Z} = \frac{R}{{Z_L}^2}$$

$$\Rightarrow \frac{1}{Z} = \frac{R}{\frac{L}{C}}$$

$$\Rightarrow Z = \frac{L}{CR} \qquad \left[where\, Z_L{}^2 = \frac{L}{C}\, from\, eqn(1) \right]$$

The value of $\frac{L}{C}$ is very large at parallel resonance, so value of impedance Z is very high.

Line current at parallel resonance is minimum and is given by $I = \frac{V}{Z}$

As Z is very high, so line current I be very small.

Impedance - frequency curve

If we plot impedance - frequency graph for a parallel circuit the shape of the curve will be as shown in fig 3.30. Impedance of the circuit is maximum at resonance. As frequency changes from resonance, the circuit impedance decreases very rapidly. For frequencies below resonance, the capacitive reactance will be higher (i.e. $X_C = \frac{1}{2\pi fc}$).

This high value of X_c opposes capacitive current i.e. magnitude of I_c decreases. As a result more current will flow through the coil. This causes the line current to lag behind the applied voltage and circuit behaves inductive. For frequencies above resonance, inductive reactance is higher (i.e $X_L = 2\pi fl$) and more current will flow through the capacitor. This causes the line current leads the applied voltage and circuit behaves capacitive.

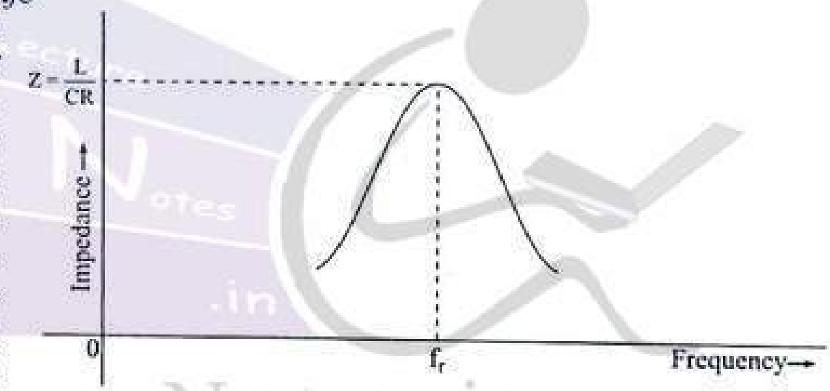

Fig 3.30

3.9 Admittance, conductance and susceptance of AC circuit

The reciprocal of impedance is called ***admittance*** of a.c circuit. It is represented by Y.

$$\therefore Y = \frac{1}{Z} = \frac{I}{V}$$

We know impedance (Z) has two components, resistance (R) and reactance (X). Similarly admittance (Y) has two components, *conductance* (G) along horizontal axis and *susceptance* (B) along vertical axis.

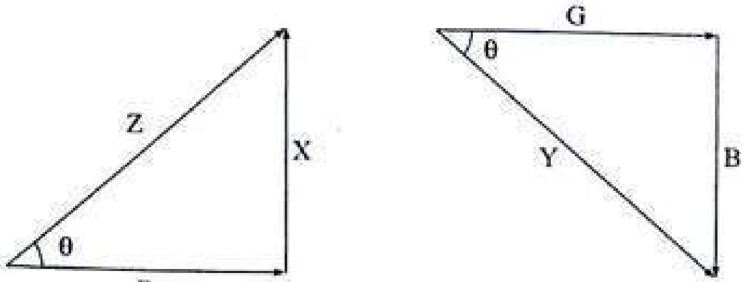

Fig 3.31

From fig 3.31 $G = Y\cos\theta = \frac{1}{Z}\cdot\frac{R}{Z} = \frac{R}{Z^2}$ $\left(\because Y = \frac{1}{Z}\, and \cos\theta = \frac{R}{Z}\right)$

$$B = Y\sin\theta = \frac{1}{Z}\cdot\frac{X}{Z} = \frac{X}{Z^2} \qquad \left(\because \sin\theta = \frac{X}{Z}\right)$$

$$\text{Admittance}\,(Y) = \sqrt{G^2 + B^2}$$

The units of Y, G and B are *mho* or *ohm*$^{-1}$ or *siemens (s)*. The inductive susceptance is considered as negative and capacitive susceptance is considered as positive.

Table

SN	A.C series circuit	Impedance in complex form	Impedence in scalar form	Voltage in complex form	Voltage in scalar form
1.	R-only	Z=R+j0	Z=R	$V_R = IR + j0$	$V_R = IR$
2.	L-only	$Z = 0 + jX_L = 0 + j\omega L$	$Z = X_L = \omega L = 2\pi fL$	$V_L = 0 + jIX_L$	$V_L = IX_L$
3.	C-only	$Z = 0 - jX_c$	$Z = X_C = \frac{1}{\omega C} = \frac{1}{2\pi fc}$	$V_c = 0 - jIX_C$	$V_c = IX_C$
4.	R-L ckt	$Z = R + jX_L$	$Z = \sqrt{R^2 + X_L^2}$	$V = V_R + V_L$ $= IR + jIX_L$ $= I(R + jX_L)$ $= IZ$	$V = \sqrt{V_R^2 + V_L^2}$ $= \sqrt{(IR)^2 + (IX_L^2)}$ $= I\sqrt{R^2 + X_L^2}$ $= IZ$
5.	R-C ckt	$Z = R - jX_c$	$Z = \sqrt{R^2 + X_C^2}$	$V = V_R + V_c$ $= IR - jIX_c$ $= I(R - jX_c)$ $= IZ$	$V = \sqrt{V_R^2 + V_c^2}$ $= \sqrt{(IR)^2 + (IX_c)^2}$ $= I\sqrt{R^2 + X_c^2}$ $= IZ$
6.	R-L-C ckt (i) $V_L > V_c$	(i) if $X_L > X_c$ then $Z = R + j(X_L - X_C)$	(i) $Z = \sqrt{R^2 + (X_L - X_C)^2}$	(i) $V = V_R + V_L + V_C$ $= IR + jIX_L - jIX_C$ $= I[R + j(X_L - X_C)]$ $= IZ$	(i) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ $= \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$ $= I\sqrt{R^2 + (X_L - X_C)^2}$ $= IZ$
	(ii) $V_L < V_c$	(ii) if $X_L < X_C$ then $Z = R - j(X_L - X_c)$	(ii) $Z = \sqrt{R^2 + (X_L - X_C)^2}$	(ii) $V = V_R + V_L + V_C$ $= I[R - j(X_L - X_C)]$ $= IZ$	(ii) $V = \sqrt{V_R^2 + (V_L - V_c^2)}$

Note -*Bold capital letters represent vectors i.e. complex forms*

Example 3.6 Calculate the average value of the sawtooth wave form shown in fig 3.32

Solution : The given wave is symmetrical. So half wave is to be considered.

$$\text{Average value } V_{av} = \frac{area\ of\ half\ cycle}{base\ length\ of\ half\ cycle}$$

$$= \frac{\frac{1}{2}(10)(5)}{10} = 2.5\, volts$$

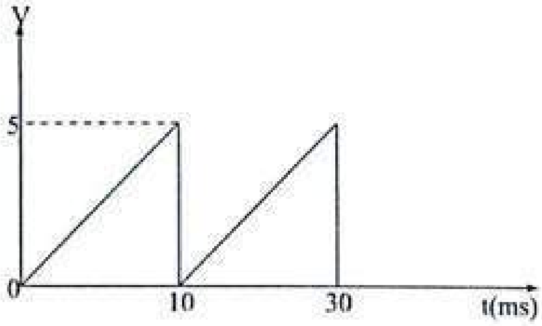

Fig 3.32

Example 3.7 : Calculate the average value of the shifted triangle wave shown in fig 3.33

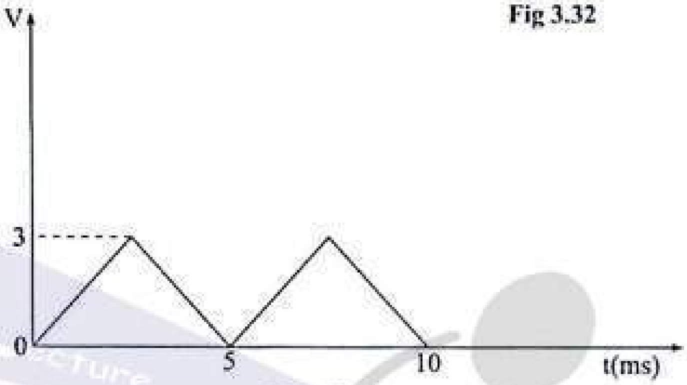

Fig 3.33

Solution: The given wave is symmetrical. So half wave is considered.

$$\text{Average value } V_{av} = \frac{area\ of\ half\ cycle}{base\ length\ of\ half\ cycle}$$

$$= \frac{\frac{1}{2}(5)(3)}{5} = 1 \cdot 5\, volt$$

Example 3.8 Find the impedance of the circuit shown in fig 3.34, $\omega = 10^4 \frac{rad}{sec}$, $R_1 = 100\Omega$, $L = 10mH, R_2 = 50\Omega, C = 10\mu F$

Solution : Let Z = impedance of $R_2 - C$ circuit.

$$Z = \frac{(R_2)\left(\frac{1}{j\omega C}\right)}{R_2 + \frac{1}{J\omega C}} = \frac{R_2}{1 + j\omega c R_2} = \frac{50}{1 + j10^4 \times 10 \times 10^{-6} \times 50}$$

$$= 1 \cdot 92 - j9 \cdot 62\,\Omega$$

Total impedance of the circuit, $Z_T = R_1 + J\omega L + Z$

$$= 100 + j10^4 \times 10^{-2} + 1 \cdot 92 - j9 \cdot 62$$

$$= 101 \cdot 92 + j90 \cdot 38$$

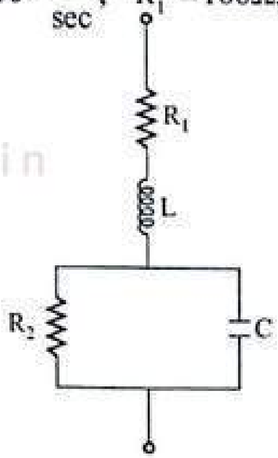

Fig 3.34

Example 3.9 Find the equivalent admittance of the two circuits shown in fig3.35.

$\omega = 2\pi \times 10^3$ rad.sec^{-1}

$R_1 = 50\Omega$, $L = 16mH$, $R_2 = 100\Omega$

$C = 3\mu F$

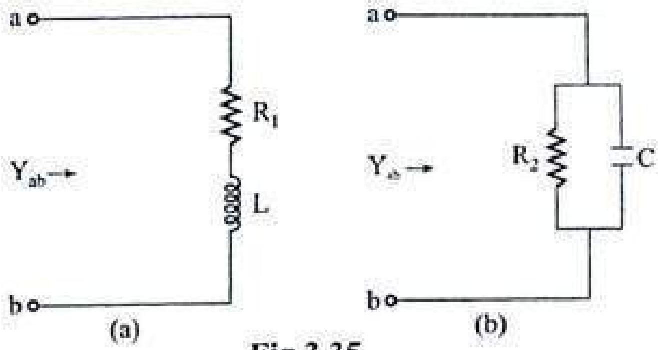

Fig 3.35

Solution : For circuit (a) :

Impedance of the circuit, $Z_{ab} = R_1 + j\omega L$

Admittance of the circuit, $Y_{ab} = \dfrac{1}{Z_{ab}} = \dfrac{1}{R_1 + j\omega L}$

$$= \frac{1}{50 + j2\pi \times 10^3 \times 16 \times 10^{-3}}$$

$$= (3 \cdot 96 - j7 \cdot 97)10^{-3}\ mho$$

For circuit (b) :

Impedance of the circuit, $Z_{ab} = \dfrac{R_2}{1 + j\omega R_2 C}$

Admittance of the circuit, $Y_{ab} = \dfrac{1}{Z_{ab}} = \dfrac{1 + j\omega R_2 C}{R_2} = 0 \cdot 01 + j0 \cdot 019\ mho$

Example 3.10 : Apply phasor analysis method to the circuit of fig 3.36 to determine the source current.

$V_s(t) = 10\cos\omega t$, $\omega = 377$ rad.sec^{-1} $R_1 = 50\Omega$, $R_2 = 200\Omega$,

$C = 100\mu F$

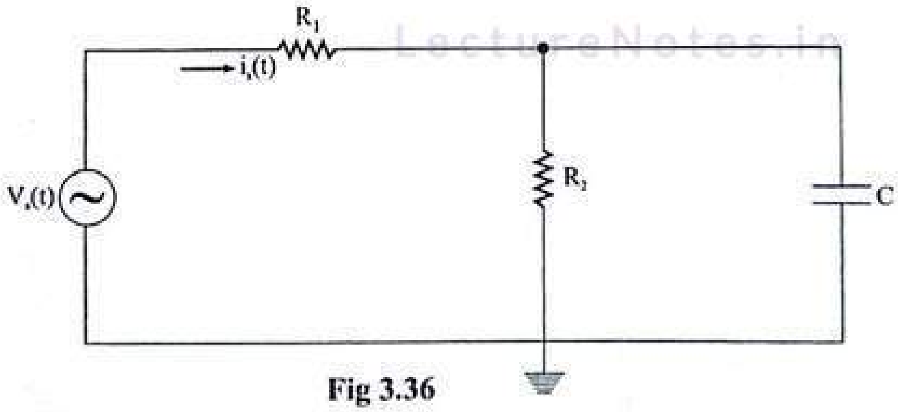

Fig 3.36

Solution : Given $V_s(t) = 10\cos\omega t$

In phasor form, $V_s(j\omega) = 10\angle 0$ volt

Impedance of $R_2 - C$ circuit, $Z = \dfrac{(R_2)\left(\dfrac{1}{j\omega c}\right)}{R_2 + \dfrac{1}{j\omega c}} = \dfrac{R_2}{1 + j\omega R_2 C}$

$= 3\cdot4569 - j26\cdot065\Omega$

$= 26\cdot3\angle -82\cdot4\Omega$

Using nodal analysis, $\dfrac{V_s - V}{R_1} = \dfrac{V}{Z}$

$\Rightarrow (V_s - V)Z = VR_1$

$\Rightarrow V = \dfrac{V_s Z}{R_1 + Z} = \dfrac{10\angle 0 \times 26\cdot3\angle -82\cdot44}{50 + 3\cdot4569 - j26\cdot065} = 4\cdot422\angle -56\cdot44$

From figure $I_s = \dfrac{V_s - V}{R_1} = \dfrac{10\angle 10 - 4\cdot422\angle -56\cdot44}{50} = 0\cdot168\angle 26^0$

$\Rightarrow I_s = 0\cdot168\angle 0\cdot4537^c$

(where $26^0 = 26 \times \dfrac{\pi}{180} = 0\cdot4537$ radian)

In time domain form source current,

$i_s(t) = 0\cdot168\cos(377t + 0\cdot4537)A.$

Example 3.11 : Calculate currents $i_1(t)$ and $i_2(t)$ of the circuit shown in fig 3.37

$V_s(t) = 155\cos(377t)$ volt, $R_s = 0\cdot5\Omega$, $R_1 = 2\Omega$,

$R_2 = 0\cdot2\Omega$, $L_1 = 0\cdot1H$ and $L_2 = 20mH$

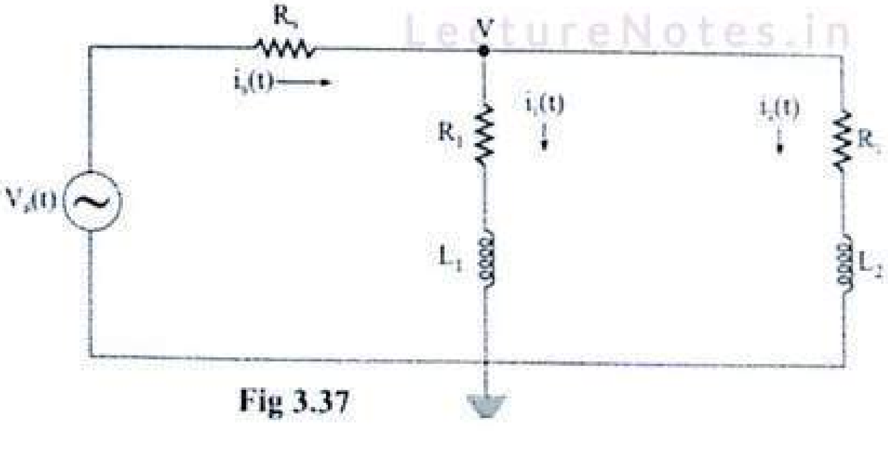

Fig 3.37

Solution : Impedance of $R_1 - L_1$ circuit $Z_1 = R_1 + j\omega L$

Impedance of $R_2 - L_2$ circuit $Z_2 = R_2 + j\omega L_2$

Total impedance of the circuit, $Z = R_s + \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

$$= R_s + \frac{(R_1 + j\omega L_1)(R_2 + j\omega L_2)}{R_1 + j\omega L_1 + R_2 + j\omega L_2}$$

$$= 0\cdot 5 + \frac{(2 + j37\cdot 7)(0\cdot 2 + j7\cdot 54)}{2 + j37\cdot 7 + 0\cdot 2 + j7\cdot 54}$$

$$= 0\cdot 5 + \frac{37\cdot 75\angle 86\cdot 96 \times 7\cdot 54\angle 88\cdot 48}{45\cdot 3\angle 87\cdot 2}$$

$$= 0\cdot 6929 + j6\cdot 28 = 6\cdot 3181\angle 83\cdot 7\Omega$$

Source Current $\quad I_s = \dfrac{V_s}{Z} = \dfrac{155\angle 0}{6\cdot 3181\angle 83\cdot 7} = 24\cdot 53\angle -83\cdot 7\text{ A}$

According to current division rule,

$$I_1 = I_s \frac{Z_2}{Z_1 + Z_2} = \frac{24\cdot 53\angle -83\cdot 7 \times 7\cdot 54\angle 88\cdot 48}{45\cdot 3\angle 87\cdot 2}$$

$$= 4\cdot 08\angle -82\cdot 42^0$$

$$= 4\cdot 08\angle -1\cdot 43^0 A$$

Similarly $I_2 = I_s \dfrac{Z_1}{Z_1 + Z_2} = 20\cdot 44\angle -1\cdot 465 A$

In time domain form the currents are,

$$i_1(t) = 4\cdot 08\cos(377t - 1\cdot 43) A$$

and $\quad i_2(t) = 20\cdot 44\cos(377t - 1\cdot 465) A$

Example 3.12 : The current through a 0.5 H inductor is given by $i_L = 2\cos\left(377t + \dfrac{\pi}{6}\right)$

Write the expression for the voltage across the inductor.

Solution : Voltage across the inductor is,

$$V_L = L\cdot\frac{di_L}{dt} = 0\cdot 5\frac{d}{dt}2\cos\left(377t + \frac{\pi}{6}\right)$$

$$= -377\sin\left(377t + \frac{\pi}{6}\right)$$

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$$= 377\sin\left(377t + \frac{\pi}{6} - \pi\right) \qquad \{\because \sin(\theta - \pi) = -\sin\theta\}$$

$$= 377\sin\left(377t - \frac{5\pi}{6}\right) \text{ volt.}$$

Example 3.13 : The voltage across a 100μF capacitor takes the following values. Calculate the expression for the current through the capacitor in each case.

a) $V_c(t) = 40\cos\left(20t - \frac{\pi}{2}\right)$ volt.

b) $V_c(t) = 20\sin 100t$ volt.

c) $V_c(t) = -60\sin\left(80t + \frac{\pi}{6}\right)$ volt.

d) $V_c(t) = 30\cos\left(100t + \frac{\pi}{4}\right)$ volt.

Solution : a) Current through the capacitor is,

$$i_c(t) = C \cdot \frac{dv_c(t)}{dt} = 100\times10^{-6}\frac{d}{dt}40\cos\left(20t - \frac{\pi}{2}\right)$$

$$= -0\cdot08\sin\left(20t - \frac{\pi}{2}\right)$$

$$= 0\cdot08\sin\left(20t - \frac{\pi}{2} + \pi\right)$$

$$= 0\cdot08\sin\left(20t + \frac{\pi}{2}\right)\text{A}$$

b) Current through the capacitor,

$$i_c(t) = C \cdot \frac{dv_c(t)}{dt} = 100\times10^{-6}\frac{d}{dt}20\sin 100t$$

$$= 0\cdot2\cos 100t\, A$$

c) $$i_c(t) = C \cdot \frac{dv_c(t)}{dt} = 100\times10^{-6}\frac{d}{dt}\left[-60\sin\left(80t + \frac{\pi}{6}\right)\right]$$

$$= -0\cdot48\cos\left(80t + \frac{\pi}{6}\right)$$

$$= 0\cdot48\cos\left(80t + \frac{\pi}{6} - \pi\right)$$

$$= 0\cdot48\cos\left(80t - \frac{5\pi}{6}\right)A$$

d) $$i_c(t) = C \cdot \frac{dv_c(t)}{dt} = 100\times10^{-6}\frac{d}{dt}30\cos\left(100t+\frac{\pi}{4}\right)$$

$$= -0\cdot3\sin\left(100t+\frac{\pi}{4}\right)$$

$$= 0\cdot3\sin\left(100t+\frac{\pi}{4}-\pi\right)$$

$$= 0\cdot3\sin\left(100t-\frac{3\pi}{4}\right)A$$

Example 3.14 : The current through a 250 mH inductor takes the following values. Calculate the expression for the voltage across the inductor in each case.

a) $i_L(t) = 5\sin 25t\,A$

b) $i_L(t) = -10\cos 50t\,A$

c) $i_L(t) = 25\cos\left(100t+\frac{\pi}{3}\right)A$

d) $i_L(t) = 20\sin\left(10t-\frac{\pi}{12}\right)A$

Solution :

a) Voltage across inductor is,

$$V_L = L\cdot\frac{di_L(t)}{dt} = 250\times10^{-3}\frac{d}{dt}(5\sin 25t) = 31\cdot25\cos 25t$$

b) $$V_L = 250\times10^{-3}\frac{d}{dt}(-10\cos 50t) = 125\sin 50t \text{ volt}$$

c) $$V_L = 250\times10^{-3}\frac{d}{dt}\left[25\cos\left(100t+\frac{\pi}{3}\right)\right]$$

$$= -625\sin\left(100t+\frac{\pi}{3}\right)$$

$$= 625\sin\left(100t+\frac{\pi}{3}-\pi\right)$$

$$= 625\sin\left(100t-\frac{2\pi}{3}\right) \text{ volt.}$$

d) $$V_L = 250\times10^{-3}\frac{d}{dt}\left[20\sin\left(10t-\frac{\pi}{12}\right)\right]$$

$$= 50\cos\left(10t-\frac{\pi}{12}\right) \text{ volt}$$

Example 3.15 : In the circuit shown in figure 3.38.

$$\text{Let i (t)} = \begin{cases} 0 & for\ -\infty < t < 0 \\ t & for\ 0 \le t < 10 \\ 10 & for\ 10 \le t < \infty \end{cases}$$

Find the energy stored in the inductor for all time.

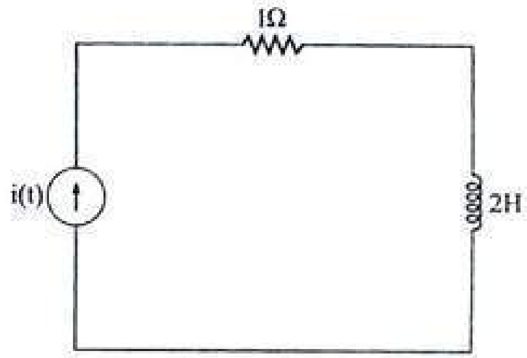

Fig 3.38

Solution : The energy stored in the inductor is magnetic energy

$$\therefore \text{Magnetic energy}, W = \frac{1}{2}Li^2 = \frac{1}{2}(2)(i)^2 = (i)^2$$

When $i(t) = 0\ then\ W = 0$

When $i(t) = t\ then\ W = t^2\ Joule$

When $i(t) = 10\ then\ W = 100\ Joule$

Example 3.16 : In the circuit shown in fig 3.39

$$\text{Let } i\ (t) = \begin{cases} 0 & for\ -\infty < t < 0 \\ t & for\ 0 \le + < 10 \\ 10 & for\ 10 \le t < \infty \end{cases}$$

Find the energy delivered by the source for all time.

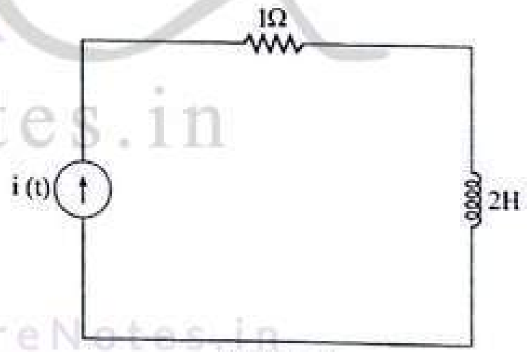

Fig 3.39

Solution : The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the inductor. It is given by,

$$W(t) = i^2R + \frac{1}{2}Li^2 = i^2(1) + \frac{1}{2}(2)i^2 = 2i^2$$

When $i(t) = 0\ \ then\ \ W(t) = 0$

When $i(t) = t\ \ then\ \ W(t) = 2t^2\ Joule$

When $i(t) = 10\ then\ \ W(t) = 2(10)^2 = 200\ Joule$

Example 3.17 : In the circuit shown in fig 3.40

$$\text{Let } V(t)=\begin{cases}0 & for\ -\infty<t<0\\ t & for\ 0\le t<10\\ 10 & for\ 10\le t<\infty\end{cases}$$

Find the energy stored in the capacitor for all time.

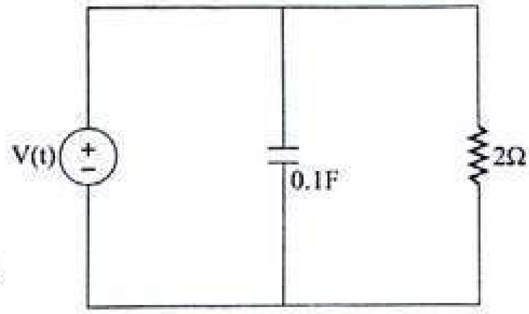

Fig 3.40

Solution : The energy stored in the capacitor is

$$W=\frac{1}{2}CV^2=\frac{1}{2}(0\cdot1)(V)^2=0\cdot05V^2$$

When $V(t)=0\, then\ W=0\cdot05(0)^2=0$

When $V(t)=t\ then\ W=0\cdot05(t)^2=0\cdot05t^2\ Joule$

$V(t)=10\, then\ \ W=0\cdot05(10)^2=5\, Joule$

Example 3.18 : In the circuit shown in fig 3.41

$$\text{Let } V(t)=\begin{cases}0 & for\ -\infty<t<0\\ t & for\ 0\le t<10\\ 10 & for\ 10\le t<\infty\end{cases}$$

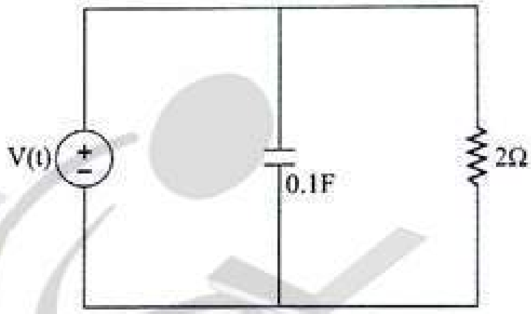

Fig 3.41

Find the energy delivered by the source for all time.

Solution : The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the capacitor. It is given by,

$$W=\frac{V^2}{R}+\frac{1}{2}CV^2=\frac{1}{2}V^2+\frac{1}{2}(0\cdot1)V^2=0\cdot55V^2$$

When $V(t)=0\, then\ \ W=0$

When $V(t)=t\ then\ \ W=0\cdot55t^2\ Joule$

When
$V(t)=10\, then\, W=0\cdot55(10)^2=55\, Joule$

Example 3.19 : Find the energy stored in each Capacitor and inductor, under steady - state conditions, in the circuit shown in fig. 3.42.

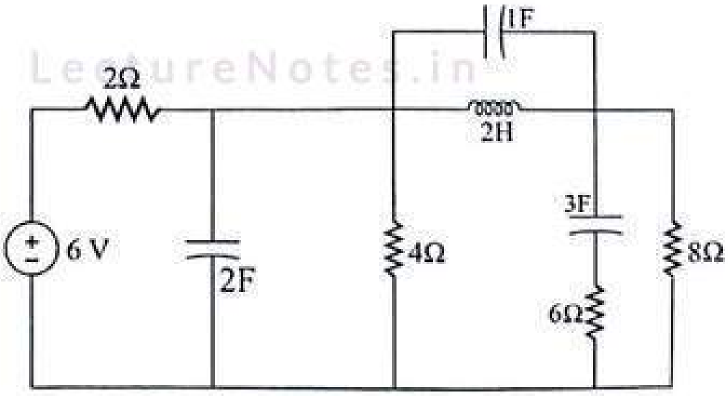

Fig 3.42

Solution : Under steady state conditions, all the currents are constant, no current can flow through the capacitors and the voltage across any inductor is equal to zero.

$$V_{2F} = V_{4\Omega}$$

In steady state currents through IF and 3F are zero. So voltage drop across 6Ω is zero. The voltage across 3F is $V_{4\Omega}$.

$$\frac{6 - V_{4\Omega}}{2} = \frac{V_{4\Omega}}{4} + \frac{V_{4\Omega}}{8}$$

$$\Rightarrow V_{4\Omega} = 3 \cdot 43 \text{ volt.}$$

Energy stored across 2F is,

$$W_{2F} = \frac{1}{2}(2)(3 \cdot 43)^2 = 11 \cdot 76\, Joule$$

$$V_{IF} = V_{2H} = 0$$

Energy stored across 1F is, $W_{IF} = \frac{1}{2}(1)(0)^2 = 0$

Current through 2H is, $i_{2H} = \frac{V_{4\Omega}}{8} = 0 \cdot 43A$

Energy Stored across 2H is, $W_{2H} = \frac{1}{2}(2)(0 \cdot 43)^2 = 0 \cdot 18\, Joule$

Voltage across 3F is $V_{4\Omega}$

Energy stored across $3F = \frac{1}{2}(3)(3 \cdot 43)^2 = 17 \cdot 65\, Joule$

Example 3.20 : Find the average and rms value of x (t).

$$x(t) = 2\cos(\omega t) + 2 \cdot 5$$

Solution : The average value is,

$$X_{avg} = \frac{1}{T}\int_0^T (2\cos\omega t + 2 \cdot 5) \cdot dt$$

$$= \frac{1}{T}\int_0^T 2\cos\omega t \cdot dt + \frac{1}{T}\int_0^T 2 \cdot 5\, dt$$

$$= \frac{1}{T}\left[\frac{2\sin\omega t}{\omega}\right]_0^T + \frac{1}{T}[2 \cdot 5t]_0^T$$

$$= \frac{1}{T}\left[\frac{2\sin\omega T}{\omega} - 0\right] + \frac{1}{T}[2 \cdot 5T - 0]$$

$$= \frac{1}{T}\left[\frac{2\sin\frac{2\pi}{T}\cdot T}{\omega}\right] + 2\cdot 5 \qquad \left(\text{where as } \omega = \frac{2\pi}{T}\right)$$

$$= 0 + 2\cdot 5 = 2.5$$

The rms value is,

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T (2\cos\omega t + 2\cdot 5)^2 \cdot dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \left(4\cos^2\omega t + 10\cos\omega t + 6\cdot 25\right) dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T 4\cos^2\omega t \cdot dt + \frac{1}{T}\int_0^T 10\cos\omega t \cdot dt + \frac{1}{T}\int_0^T 6\cdot 25 \cdot dt}$$

$$= 2.87$$

Example 3.21 : Find the ratio between average and rms value of the wave form of fig 3.43.

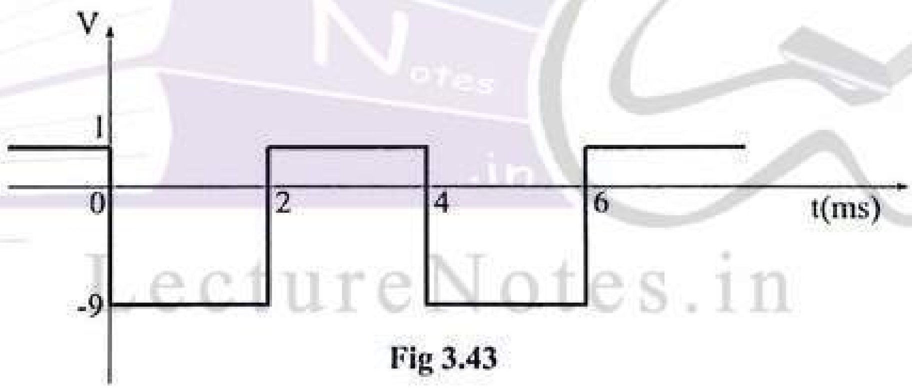

Fig 3.43

Solution : The average value is,

$$V_{av} = \frac{1}{T}\int_0^T V \cdot dt = \frac{1}{4\times 10^{-3}}\left[\int_0^{2\times 10^{-3}} -9\cdot dt + \int_{2\times 10^{-3}}^{4\times 10^{-3}} 1\cdot dt\right] = -4 \text{ volt.}$$

The r.m.s value is,

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 . dt}$$

$$= \sqrt{\frac{1}{4\times10^{-3}}\left[\int_0^{2\times10^{-3}} (-9)^2 \cdot dt + \int_{2\times10^{-3}}^{4\times10^{-3}} (1)^2 \cdot dt\right]}$$

= 6.40 volt.

$$\therefore \frac{V_{av}}{V_{rms}} = \frac{-4}{6\cdot40} = -0\cdot625$$

Example 3.22 : Given the current wave form shown in fig. 3.44, find the power dissipated by a 1Ω resistor.

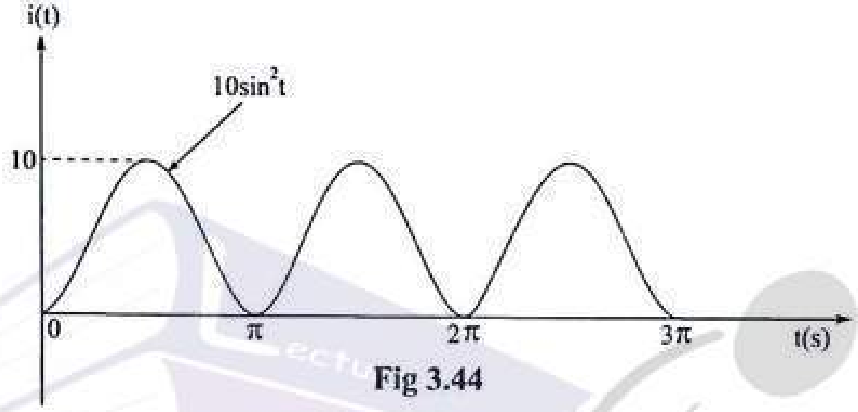

Fig 3.44

Solution :

The rms value of current is,

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \cdot dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} \left(10\sin^2 t\right)^2 \cdot dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} 100\sin^4 t \cdot dt}$$

$$= \sqrt{\frac{1}{\pi}\times100\int_0^{\pi}\left(\frac{1-\cos2t}{2}\right)^2 \cdot dt}$$

$$= \sqrt{\frac{1}{\pi}\times100\times\frac{1}{4}\int_0^{\pi}(1-\cos2t)^2 \cdot dt}$$

$$= \sqrt{\frac{25}{\pi}\int_0^{\pi}\left(1+\cos^2 2t - 2\cos2t\right)\cdot dt}$$

$$= \sqrt{\frac{25}{\pi} \times \frac{3\pi}{2}} = 6.123 \text{ A}$$

Therefore, the power dissipated by 1Ω resistor is,

$$P = i_{rms}{}^2 R = (6 \cdot 123)^2 (1) = 37 \cdot 5 \text{ watt}$$

Example 3.23 : Find the ratio between average and rms value of the wave form of fig 3.45.

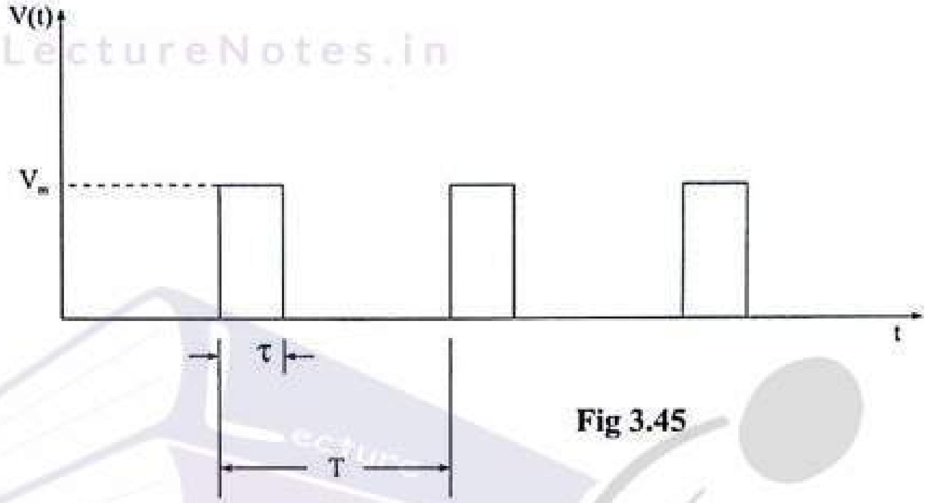

Fig 3.45

Solution : The average value is,

$$V_{av} = \frac{1}{T}\int_0^{0+\tau} V_m \cdot dt = \frac{t}{T} V_m$$

The rms value is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{0+\tau} V_m{}^2 \cdot dt} = \sqrt{\frac{t}{T}} \cdot V_m$$

$$\text{Therefore } \frac{V_{av}}{V_{rms}} = \sqrt{\frac{t}{T}}$$

Example 3.24 : Determine the rms value of

$$V(t) = 50 + 70 \cdot 7\cos(377t) \text{ volt.}$$

Solution : The rms value is,

$$V_{rms} = \sqrt{(50)^2 + \left(\frac{70 \cdot 7}{\sqrt{2}}\right)^2}$$

$$= 70 \cdot 7 \text{ volt}$$

Example 3.25 : If the current through and the voltage across a component in an electric circuit are.

$$i(t) = 17\cos\left(\omega t - \frac{\pi}{12}\right) mA$$

$$v(t) = 3\cdot5\cos(\omega t + 1\cdot309) \text{ volt}$$

Where $\omega = 628\cdot3 \frac{rad}{sec}$, determine

(a) Whether the component is a resistor, capacitor or inductor.

(b) The value of the component in ohms, farads or henrys.

Solution : (a) The current and voltage can be expressed in phasor form,

$$I = 17\angle -15^0 \, mA$$

$$V = 3\cdot5\angle 75^0 \text{ volt}$$

$$Z = \frac{V}{I} = \frac{3\cdot5\angle 75^0}{17\angle -15^0} = 205\cdot9\angle 90^0 = 0 + j205\cdot9\Omega$$

The impedance has a positive imaginary or reactive component and a positive angle of 90^0 indicating that this is an inductor.

(b) $Z_L = j\omega L = j205\cdot9$ $\{\because \omega L = 205\cdot9\Omega\}$

$$\Rightarrow L = \frac{205\cdot9}{\omega} = \frac{205\cdot9}{628\cdot3} = 327\cdot7 \, mH$$

Example 3.26 : Determine the equivalent impedance in the circuit shown in fig 3.46

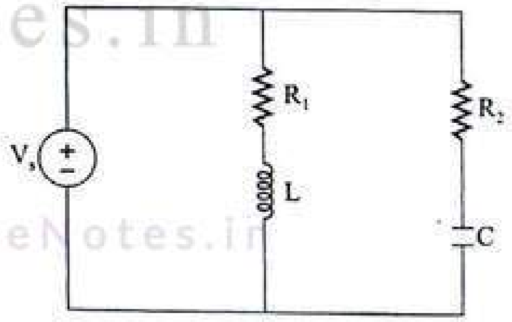

Fig 3.46

$$V_s(t) = 7\cos\left(3000t + \frac{\pi}{6}\right) \text{ volt}$$

$R_1 = 2\cdot3K\Omega$, $R_2 = 1\cdot1K\Omega$

L=190 mH, C=55nF

Solution : Given $\omega = 3000 \frac{rad}{sec}$

$$X_L = \omega L = 3000 \times 190 \times 10^{-3} = 0\cdot57 \times 10^3\,\Omega = 0\cdot57K\Omega$$

$$X_L = \frac{1}{\omega C} = \frac{1}{3000 \times 55 \times 10^{-9}} = 6\cdot061 \times 10^3\,\Omega = 6\cdot061K\Omega$$

$$Z_1 = R_1 + jX_L = 2\cdot3 + j0\cdot57 = 2\cdot37\angle 13\cdot92^0\, K\Omega$$

$Z_2 = R_2 - jX_c = 1 \cdot 1 - j6 \cdot 061 = 6 \cdot 16\angle -79 \cdot 71^0\ K\Omega$

$\therefore Z_{eq} = \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{2 \cdot 37\angle 13 \cdot 92 \times 6 \cdot 16\angle -79 \cdot 71}{2 \cdot 3 + j0 \cdot 57 + 1 \cdot 1 - j6 \cdot 061}$

$= 2 \cdot 261\angle -7 \cdot 56^0\ K\Omega$

Example 3.27 : In the circuit of fig 3.47

$i_s(t) = I_0 \cos\left(\omega t + \dfrac{\pi}{6}\right)$

$I_0 = 13\,mA, \qquad \omega = 1000\dfrac{rad}{sec}, \quad C = 0 \cdot 5\mu F$

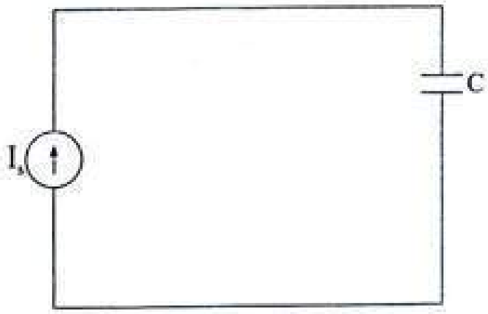

Fig 3.47

a) State, using phasor notation the source current.

b) Determine the impedance of the circuit.

c) Using phasor notation only and showing all work, determine the voltage across the capacitor.

Solution : a) Phasor notation,

$I_s = I_0\angle\phi = 13\angle 30^0\ mA$

b) $Z_c = -jX_c = -j\dfrac{1}{\omega C} = -j\dfrac{1}{1000 \times 0 \cdot 5 \times 10^{-6}}$

$= 0 - j2 \times 10^3$

$= 2\angle -90^0\ K\Omega$

c) $V_c = I_s Z_c = 13\angle 30^0 \times 10^{-3} \times 2\angle -90 \times 10^3 = 26\angle -60^0$ volt.

$V_C(t) = 26\cos\left(1000t - 60^0\right)$ volts.

Example 3.28 : Determine $i_3(t)$ in the circuit shown in fig. 3.48.

$i_1(t) = 141 \cdot 4\cos(\omega t + 2 \cdot 356)\,mA$

$i_2(t) = 50\sin(\omega t - 0 \cdot 927)\,mA$

$\omega = 377\dfrac{rad}{sec}$

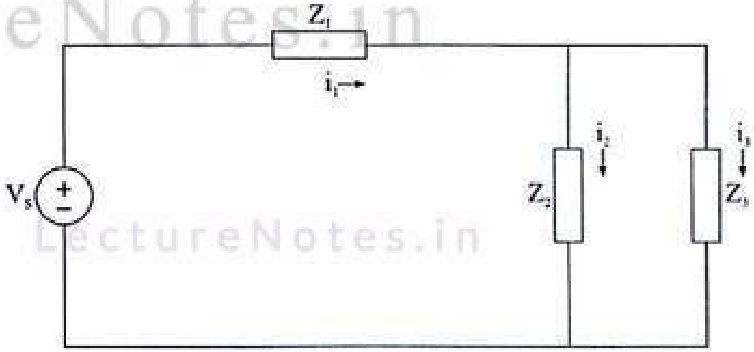

Fig 3.48

Solution : Applying KCL,

$-i_1(t) + i_2(t) + i_3(t) = 0$

$\Rightarrow i_3(t) = i_1(t) - i_2(t)$

$= 141 \cdot 4\cos\left(\omega t + 135^0\right) - 50\cos\left(\omega t - 53 \cdot 13^0 - 90^0\right)$

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In phasor form $I_3 = 141\cdot 4\angle 135^0 - 50\angle -143\cdot 13^0$

$$= -99\cdot 98 + j99\cdot 98 - (-40 - j30)$$

$$= -59\cdot 98 + j129\cdot 98$$

$$= 143\cdot 2\angle 114\cdot 8^0\ mA$$

$$\therefore i_3(t) = 143\cdot 2\cos(\omega t + 114\cdot 8^0)\ mA$$

Example 3.29 : Determine the current through Z_3 in the circuit of fig 3.49

$V_{s1} = V_{s2} = 170\cos(377t)$ volt

$Z_1 = 5\cdot 9\angle 0\cdot 122\Omega$

$Z_2 = 2\cdot 3\angle 0\Omega$

$Z_3 = 17\angle 0\cdot 192\Omega$

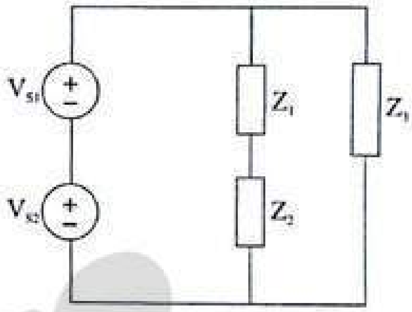

Fig 3.49

Solution : $V_{s1} = V_{s2} = 170\angle 0^0 = (170 + j0)$ volt

Applying KVL,

$$V_{s1} - V_{s2} + I_3 Z_3 = 0$$

$$\Rightarrow I_3 = \frac{V_{s1} + V_{s2}}{Z_3} = \frac{170\angle 0 + 170\angle 0}{17\angle 11^0} = \frac{340\angle 0}{17\angle 11^0} = 20\angle -11^0 A$$

$$i_3(t) = 20\cos(377t - 11^0) A$$

Example 3.30 : Determine the frequency so that the current I and voltage V_0 in the circuit of fig.3.50 are in phase.

$Z_s = 13000 + j\omega 3\Omega$, $R = 120\Omega$, L=19mH, C=220pF

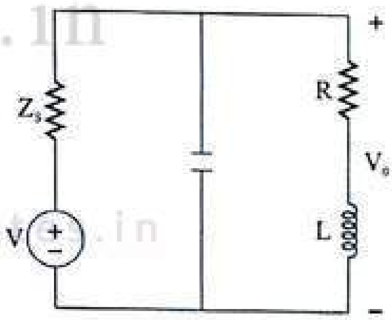

Fig 3.50

Solution : Z_s is not a factor in this solution only R, L and C will determine the frequency. If the voltage and current are in phase then the equivalent impedance must have an imaginary or reactive part which is zero.

$$\therefore Z_{eq} = \frac{(R + jX_L)(-jX_c)}{R + jX_L - jX_C} = \frac{(R + jX_L)(-jX_c)[R - j(X_L - X_C)]}{[R + j(X_L - X_C)][R - j(X_L - X_C)]}$$

$$= \frac{[X_L X_C R - RX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C(X_L - X_C)]}{R^2 + (X_L - X_C)^2}$$

Put imaginary part of Z_{eq} to zero

$$\frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0$$

$$\Rightarrow R^2 + X_L (X_L - X_C) = 0$$

$$\Rightarrow R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{19\times10^{-3}\times220\times10^{-12}} - \frac{(120)^2}{(19\times10^{-3})^2}}$$

$$= 489\cdot1\times10^3 \frac{rad}{sec}$$

Example 3.31 : The coil resistor in series with L models the internal losses of an inductor in the circuit of fig 3.51. Determine the current supplied by the source if

$V_s(t) = V_0 \cos(\omega t + 0)$

$V_0 = 10\, volt$ $\qquad \omega = 6\times10^6\, rad/sec$

$R_s = 50\,\Omega,$ $\qquad R_C = 40\Omega$

$L = 20\mu H,$ $\qquad C = 1\cdot25\times10^{-9}\, F$

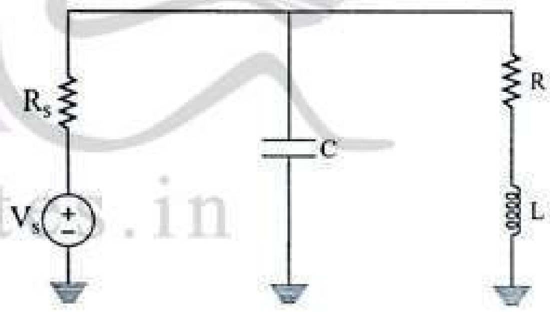

Fig 3.51

Solution : Equivalent impedance of the circuit is,

$$Z = R_s + \frac{(R_C + J\omega L)(-jX_C)}{R_C + J\omega L - jX_C} = 50 + \frac{(40 + j120)(-j133\cdot3)}{40 + j120 - j133\cdot3}$$

$$= 50 + \frac{126\cdot5\angle71\cdot56^0 \times 133\cdot3\angle-90}{42\cdot153\angle-18\cdot44}$$

$$= 450\angle0^0\,\Omega$$

Current supplied by source $I_s = \frac{V_s}{Z} = \frac{10\angle0}{450\angle0} = 22\cdot22\angle0\, mA$

In time domain form i (t) = 22.22 cos (ωt + 0) mA.

Example 3.32 : Using phasor techniques, solve for the current in the circuit shown in fig 3.52

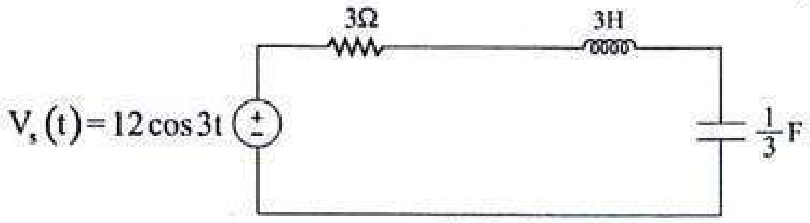

Fig 3.52

Solution : Total impedance of the circuit,

$$Z = R + jX_L - jX_c = R + J\omega L - j\frac{1}{\omega C} \quad = 3 + j3\times 3 - j\frac{1}{3\times\frac{1}{3}} \qquad \left(\because \omega = 3\,{}^{rad}/_{sec}\right)$$

$$= 3 + j9 - j \quad = 3 + j8 \quad = 8\cdot 544\angle 69\cdot 44^0\,\Omega$$

$$I = \frac{V}{Z} = \frac{12\angle 0}{8\cdot 544\angle 69\cdot 44} = 1\cdot 404\angle -69\cdot 44A$$

In time domain form $i(t) = 1\cdot 404\cos\left(3t - 69\cdot 4^0\right)A$

Example 3.33 : Using phasor techniques solve for the voltage V in the circuit shown in fig 3.53

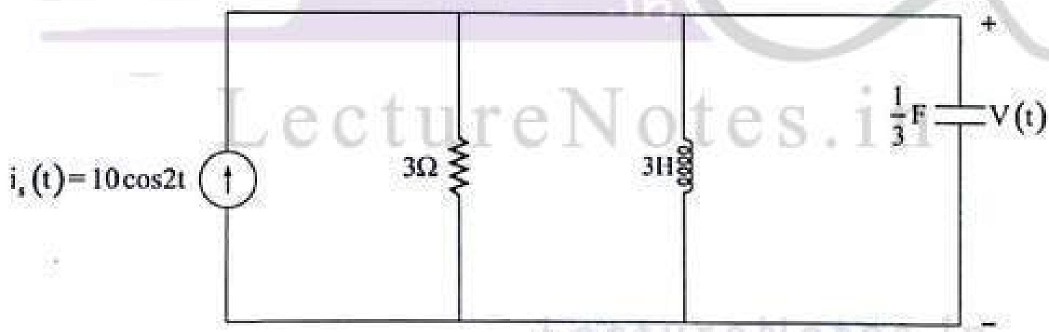

Fig 3.53

Solution : Here $\omega = 2\frac{rad}{sec}$

$$I_s = 10\angle 0^0$$

$$Z_L = J\omega L = j6$$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{3}{2}\Omega$$

Total circuit impedance, $Z=\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{Z_L}+\dfrac{1}{Z_C}} = \dfrac{1}{\dfrac{1}{3}+\dfrac{1}{j6}+\dfrac{1}{-j\,3/2}} = \dfrac{1}{\dfrac{1}{3}-\dfrac{j}{6}+j\dfrac{2}{3}}$

$$=\frac{1}{0\cdot 33+j0\cdot 5}$$

$$=0\cdot 9231-j1\cdot 3846\Omega \quad =1\cdot 664\angle -56\cdot 308\Omega$$

$$V=I_s Z=10\angle 0\times 1\cdot 664\angle -56\cdot 308$$

$$=16\cdot 64\angle -56\cdot 308^0 \text{ volt}$$

Example 3.34 : Solve for I_1 in the circuit shown in fig 3.54

Solution :

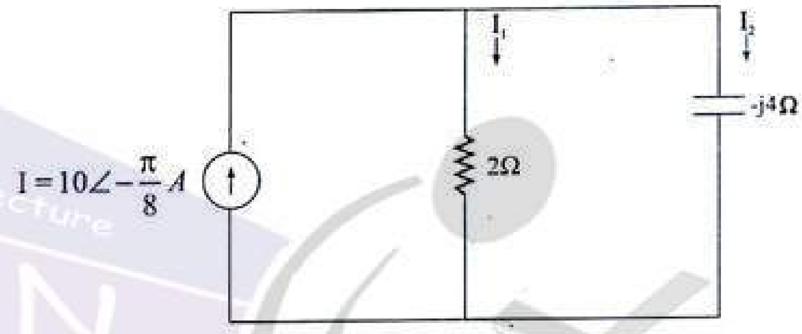

Fig 3.54

Total circuit impedance,

$$Z=\frac{1}{\frac{1}{2}+\frac{1}{-j4}}$$

$$=1\cdot 79\angle 26\cdot 56^0\,\Omega$$

$$V_s=I_s Z=10\angle -22\cdot 5^0\times 1\cdot 79\angle 26\cdot 56=17\cdot 9\angle 4\cdot 06^0 \text{ volt}$$

$$I_1=\frac{V_s}{R}=8\cdot 95\angle 4\cdot 06^0 A$$

Example 3.35 : Solve for V_2 in the circuit shown in fig 3.55 - Assume $\omega=2\dfrac{rad}{sec}$

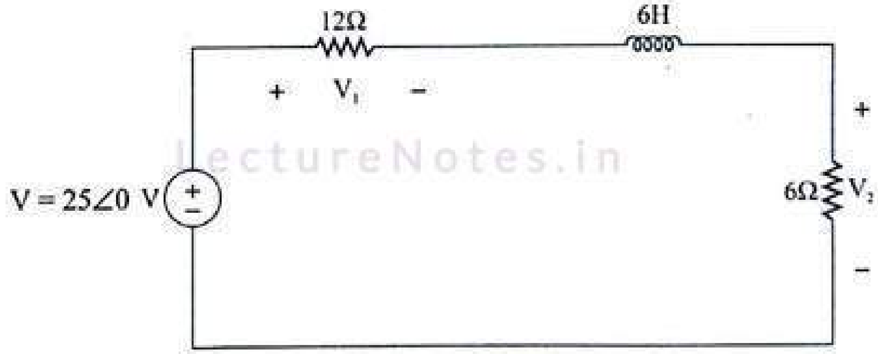

Fig 3.55

Solution : According to voltage division rule,

$$V_2=V\cdot\frac{6}{12+j2\times 6+6}=25\angle 0\times\frac{6}{18+j12}$$

$$= \frac{150\angle 0^0}{21\cdot 63\angle 33\cdot 7}$$

$$= 6\cdot 93\angle -33\cdot 7^0 \text{ volt}$$

Example 3.36 : Find V_{out} for the circuit shown in fig 3.56

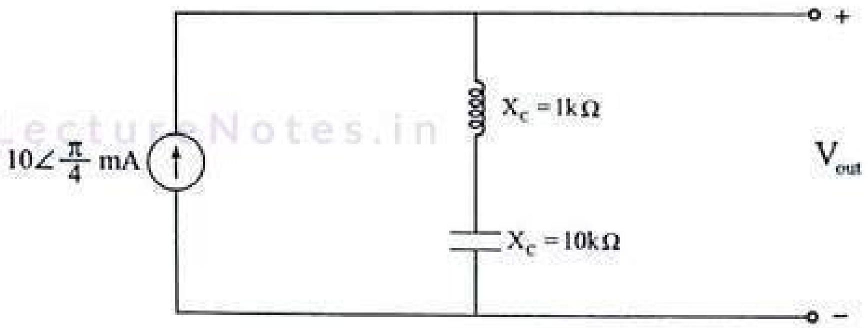

Fig 3.56

Solution : Total impedance of the circuit is,

$$Z = jX_L - jX_C = j1 - j10 = -j9k\,\Omega = 9\angle -90^0\,k\Omega$$

$$V_{out} = IZ = \left(10\angle 45^0 \times 10^{-3}\right)\left(9\angle -90^0 \times 10^3\right)$$

$$= 90\angle -45^0 \text{volt.}$$

In time domain form, $V_{out} = 90\cos\left(\omega t - 45^0\right)$ volt

Example 3.37 : For the circuit shown in fig 3.57 find the impedance Z, given $\omega = 4$ rad.sec^{-1}

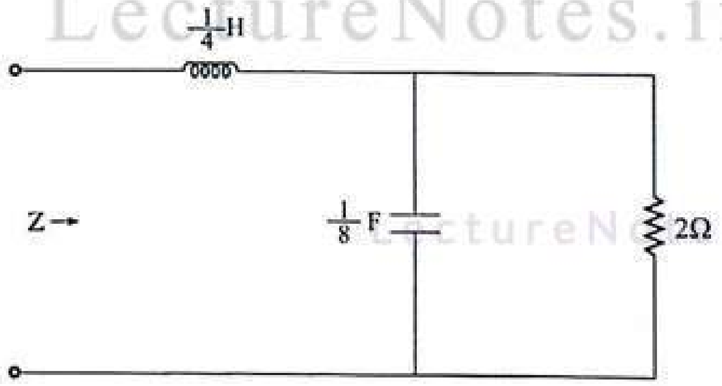

Fig 3.57

Solution : Total impedance of the circuit is,

$$Z = Z_L + Z_C || R$$

$$= Z_L + \frac{1}{\frac{1}{Z_C} + \frac{1}{R}}$$

$$= j\omega L + \frac{1}{\frac{1}{-jX_C} + \frac{1}{R}}$$

$$= j(4)\left(\frac{1}{4}\right) + \frac{1}{\frac{1}{-j2} + \frac{1}{2}}$$

$$= j + \frac{j2}{-1+j}$$

$$= 1\Omega$$

Example 3.38 : Find the admittance of the circuit shown in fig 3.58

when $\omega = 5$ rad.sec^{-1}

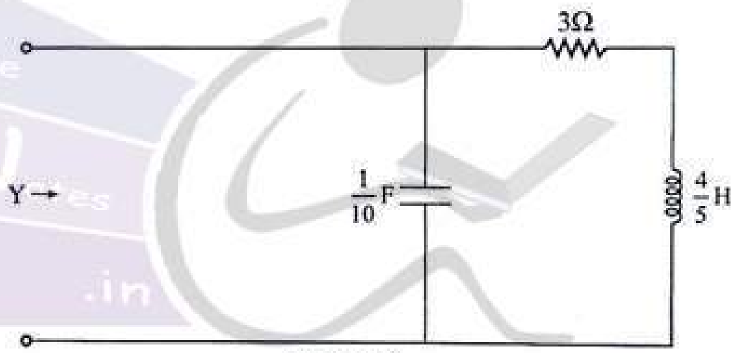

Fig 3.58

Solution :

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(5)\left(\frac{1}{10}\right)} = -j2\Omega$$

$$Z_L = j\omega L = j(5)\left(\frac{4}{5}\right) = j4\Omega$$

$$Y = \frac{1}{Z} = \frac{1}{Z_C \| (R + Z_L)}$$

$$= \frac{1}{\frac{1}{\frac{1}{Z_C} + \frac{1}{R+Z_L}}}$$

$$= \frac{1}{Z_C} + \frac{1}{R + Z_L}$$

$$= \frac{1}{-j2} + \frac{1}{3 + j4}$$

$$= j\frac{1}{2} + \frac{(3 - j4)}{(3 - j4)(3 + j4)}$$

$$= 0 \cdot 12 + j0 \cdot 34 S.$$

Example 3.39 : Using phasor techniques, solve for V in the circuit shown in fig.3.59.

Solution : Here $\omega = 3\text{rad.sec}^{-1}$

$V_s = 36\angle - 60^0$ volt

$Z_{L2} = J\omega L_2 = j3(3) = j9\Omega$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{(3)\left(\frac{1}{18}\right)} = -j6\Omega$$

$Z_{l3} = J\omega L_3 = j(3)(3) = j9\Omega$

$$Z_{eq} = \frac{1}{Z_{L3} || (Z_{l2} + Z_C)}$$

$$= \frac{1}{\frac{1}{Z_{l3}} + \frac{1}{Z_{l2} + Z_C}}$$

$$= \frac{1}{\frac{1}{j9} + \frac{1}{j9 - j6}}$$

$$= \frac{1}{\frac{1}{j9} + \frac{1}{j3}}$$

$$= j\frac{9}{4} = 2 \cdot 25\angle 90^0\,\Omega$$

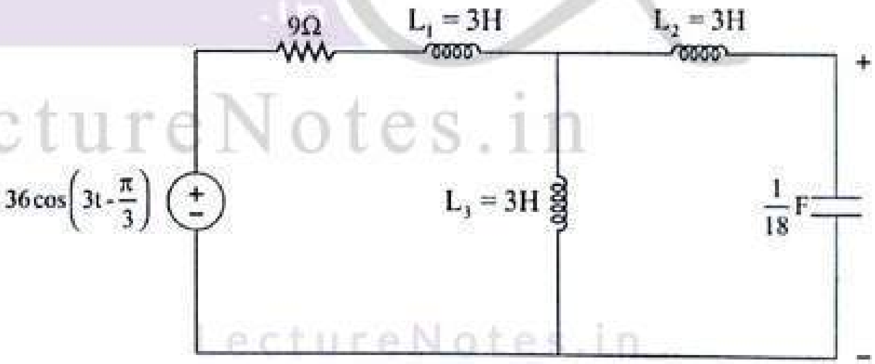

Fig 3.59

Total circuit impedance $Z = 9 + j(3)(3) + Z_{eq}$

$$= 9 + j9 + j\frac{9}{4}$$

$$= 9 + j11 \cdot 25$$

$$= 14 \cdot 407\angle 51 \cdot 34^0\,\Omega$$

Current flows through circuit, $I = \frac{V_s}{Z}$

$$\Rightarrow I = \frac{36\angle -60^0}{14\cdot 407\angle 51\cdot 340} = 2\cdot 499\angle -111\cdot 34^0\, A$$

Voltage across Z_{eq} is,

$V_{eq} = IZ_{eq} = 2\cdot 499\angle -111\cdot 34^0 \times 2\cdot 25\angle 90^0 = 5\cdot 623\angle -21\cdot 34^0$ volt

According to voltage division rule,

$$V = V_{eq} \times \frac{Z_C}{Z_{l2} + Z_C} = 5\cdot 623\angle -21\cdot 34^0 \times \frac{-j6}{j9 - j6}$$

$= 11\cdot 25\angle -158\cdot 66^0$ volt.

In time domain form $V(t) = 11\cdot 25\cos\left(3t - 158\cdot 66^0\right)$ volt.

Example 3.40 : Using phasor techniques solve for i in the circuit shown in fig 3.60

Solution :

Here $\omega = 2$ rad.sec^{-1} and current $I_s = 6\angle 0\, A$

From fig 3.60, 5Ω and L_2 are connected in series.

So $Z_1 = 5 + J\omega L_2 = 5 + j(2)(1) = 5 + j2\Omega$

Also $\frac{1}{2}F$ and L_3 are connected in series,

So, $$Z_2 = -j\frac{1}{\omega C} + j\omega L_3 = -j\frac{1}{(2)\left(\frac{1}{2}\right)} + j(2)(10) = -j + j20 = 0 + j19\Omega$$

According to current division rule,

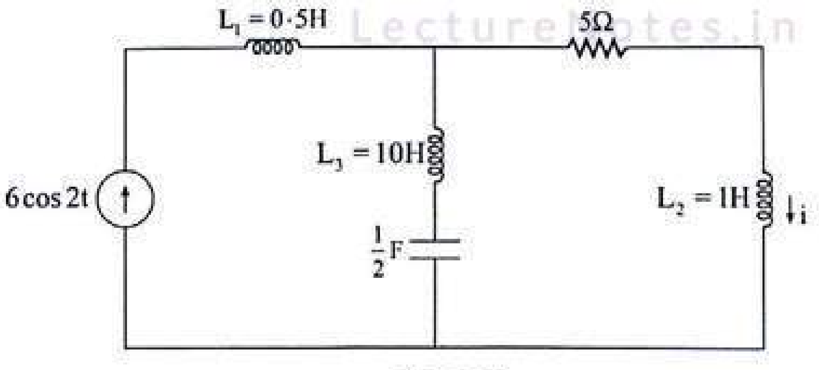

Fig 3.60

$$I = I_s \frac{Z_2}{Z_1 + Z_2} = 6\angle 0 \times \frac{0 + j19}{5 + j2 + 0 + j19} = 6\angle 0 \times \frac{j19}{5 + j21}$$

$$= 5 \cdot 28\angle 13 \cdot 4^0 A$$

In time domain form $i(t) = 5 \cdot 28\cos\left(2t + 13 \cdot 4^0\right)$

Example 3.41 : Solve for i (t) in the circuit of fig 3.61, using phasor techniques, if

$V_s(t) = 2\cos 2t$ $\quad R_1 = 4\Omega \quad R_2 = 4\Omega$

L=2H and $C = \frac{1}{4}F$

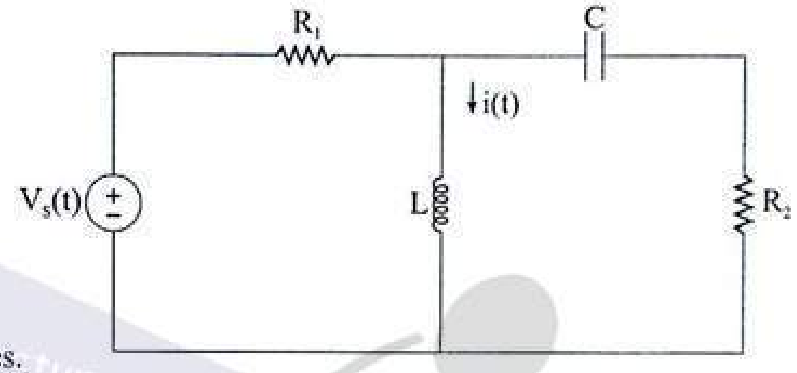

Fig 3.61

Solution : Here $\omega = 2\frac{rad}{sec}$

$V_s = 2\angle 0^0$ volt.

From fig 3.61

R_2 and C are connected in series.

So $Z_1 = R_2 - j\frac{1}{\omega C} = 4 - j\frac{1}{(2)\left(\frac{1}{4}\right)} = 4 - j2\Omega$

Also $Z_L = j\omega L = j(2)(2) = j4\Omega$

$$\therefore Z_{eq} = Z_1 || Z_L = \frac{Z_1 Z_L}{Z_1 + Z_L} = \frac{(4 - j2)(j4)}{4 - j2 + j4}$$

$$= \frac{4 \cdot 472\angle -26 \cdot 56^0 \times 4\angle 90}{4 + j2}$$

$$= \frac{17 \cdot 888\angle 63 \cdot 44^0}{4 \cdot 472\angle 26 \cdot 56^0}$$

$$= 4\angle 36 \cdot 8^0 = 3 \cdot 203 + j2 \cdot 396\Omega$$

Now Z_{eq} and R_1 are connected in series,

According to Voltage division rule,

Voltage across Z_{eq} is,

$$V = V_s \frac{Z_{eq}}{R_1 + Z_{eq}} = \frac{2\angle 0^0 \times 4\angle 36 \cdot 8^0}{4 + 3 \cdot 203 + j2 \cdot 396} = 1 \cdot 05\angle 18 \cdot 4^0 \text{ volt}$$

As Z_L and Z_1 are connected in parallel so,

Voltage across Z_{eq} = voltage across L = voltage across Z_1

$\therefore$ Voltage across $L = V_L = 1\cdot05\angle18\cdot4^0$ volt.

Current through $L = \dfrac{V_L}{Z_L} = \dfrac{1\cdot05\angle18\cdot4^0}{j4} = \dfrac{1\cdot05\angle18\cdot4^0}{4\angle90}$

$= 0\cdot2625\angle-71\cdot6^0\,A$

In domain form $i(t) = 0\cdot2625\cos\left(2t - 71\cdot6^0\right)A$

Example 3.42 : Determine V_o in the circuit of fig 3.62, if $V_i = 4\cos\left(1000t + \dfrac{\pi}{6}\right)$ volt

$L = 60mH,$ $C = 12\cdot5\mu F$ and $R_L = 120\Omega$

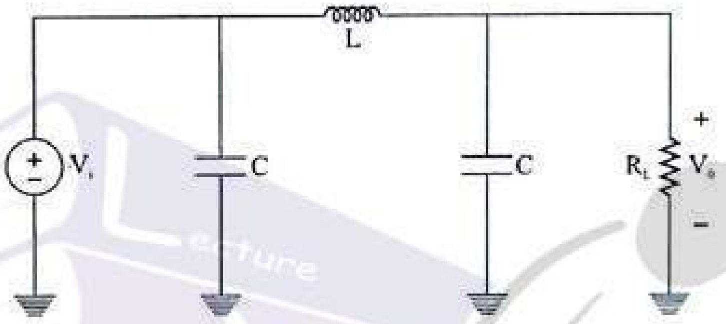

Fig 3.62

Solution : Here $\omega = 1000\dfrac{rad}{\sec}$

$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{(1000)\left(12\cdot5\times10^{-6}\right)} = -j80 = 80\angle-90^0\,\Omega$$

$$Z_L = j\omega L = j(1000)\left(60\times10^{-3}\right) = j60 = 60\angle90^0\text{ ohm}$$

$V_i = 4\angle30^0$ volt.

Apply KCL to the node as shown,

$$\frac{0-V_0}{R_L} + \frac{0-V_o}{Z_C} + \frac{V_i - V_o}{Z_L} = 0$$

$$\Rightarrow \frac{-V_0}{R_L} - \frac{V_0}{Z_C} + \frac{V_i}{Z_L} - \frac{V_0}{Z_L} = 0$$

$$\Rightarrow \frac{V_i}{Z_L} = V_0\left(\frac{1}{R_L} + \frac{1}{Z_C} + \frac{1}{Z_L}\right)$$

BURN IGNORANCE

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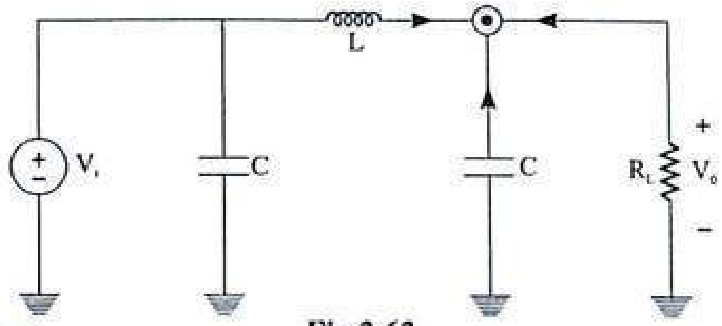


Fig 3.63

$$\Rightarrow V_0 = \frac{\frac{V_i}{Z_L}}{\frac{1}{R_L} + \frac{1}{Z_C} + \frac{1}{Z_L}}$$

$$\Rightarrow V_0 = \frac{V_i}{\frac{Z_L}{R_L} + \frac{Z_L}{Z_C} + 1} = \frac{4\angle 30^0}{\frac{60\angle 90^0}{120\angle 0^0} + \frac{60\angle 90^0}{80\angle -90^0} + 1}$$

$$= \frac{4\angle 30^0}{0\cdot 5\angle 90^0 + 0\cdot 75\angle 180^0 + 1} = \frac{4\angle 30^0}{0 + j0\cdot 5 - 0\cdot 75 + j0 + 1 + j0}$$

$$= \frac{4\angle 30^0}{0\cdot 25 + j0\cdot 5} = \frac{4\angle 30^0}{0\cdot 559\angle 63\cdot 43^0} = 7\cdot 155\angle -33\cdot 43^0 \text{ volt}$$

In time domain form $V_0(t) = 7\cdot 155\cos\left(1000t - 33\cdot 43^0\right)$ volt

1. What is the value of ratio of effective value to average value of a sinusoidal waveform? **(1st semester 2003)**

Solution : $\frac{\textit{Effective value}}{\textit{Average value}} = 1 \cdot 11$

2. State AC transmission voltage values used in India. **(1st semester 2003)**

Solution : The various transmission voltages used in India are 132 KV, 220 KV, 400 KV and 765 KV.

3. A 5 Henry inductor changes its current by 3A in 0.2 sec. What is the voltage produced at the terminals of the inductor ? **(1st semester 2004)**

Solution : L = 5 Henry

Voltage produced at the terminals of the inductor is, $V = L \cdot \frac{dI}{dt} = 5 \times \frac{3}{0 \cdot 2} = 75\ V$

4. In a single phase AC circuit.

V = 100 + j100 volt, Z = 3 + j4 Ω, find the current in polar form.

Solution : $V = 100 + j100 = 141 \cdot 42\angle 45^0$ volts.

$Z = 3 + j4 = 5\angle 53 \cdot 13^0\ \Omega$

∴ Current $I = \frac{V}{Z} = \frac{141 \cdot 42\angle 45^0}{5\angle 53 \cdot 13^0} = 28 \cdot 28\angle -8 \cdot 13^0$

5. What is the rms value and frequency of the emf $e = 100\sqrt{2}\sin 628t$

(supplementary 2004)

Solution : R.M.S Value $E_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100$ volt.

Frequency, $f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = 100\,HZ$

6. What is the resonant frequency of a circuit containing 10 ohm resistance, 1H inductance and $1\mu F$ capacitance in series ? **(supplementary 2004)**

Solution : $R = 10\Omega$, $L = 1H$, $C = 10^{-6}\,F$

Resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{1 \times 10^{-6}}} = 159 \cdot 23\,HZ$

7. What is the supply voltage and frequency at your house?

Solution : Supply voltage = 230 V and frequency = 50 Hz.

8. An alternating voltage is represented by V = 282 (314 t + 60^0). What is the frequency, rms value, maximum value and phase angle of the voltage ?

(1st semester 2005)

Solution : V = 282 sin (314t+60^0).

We know, $V = V_m \sin \omega t$

Comparing these two equations we get,

$V_m = 282$ volt. = maximum value of A.C.

Phase angle $\phi = 60^0$

RMS value of alternating *emf* $= \dfrac{282}{\sqrt{2}} = 199 \cdot 43$ volt.

Frequency of alternating *emf* , $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2\pi} = 50\,Hz$

9. A series circuit comprises an inductor of resistance 10 ohms and inductance 159 μH and a variable capacitor connected to a 50 mv sinusoidal supply of frequency 1 MHz. What value of capacitance will result in resonant conditions and what will then be the current? Calculate the Q-factor of the circuit.

(1st semester 2005)

Solution : $R = 10\Omega$, $L = 159 \times 10^{-6} H$, C = ?

$E = 50 \times 10^{-3}$ volts, $f = 10^6\,Hz$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \left(10^6\right)^2 \cdot 159 \times 10^{-6}} = 159 \cdot 47\,PF$$

$$\text{Current } I = \frac{50 \times 10^{-3}}{10} = 5 \times 10^{-3} = 5\,mA$$

$$\text{Q-factor of the circuit} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{159 \times 10^{-6}}{159 \cdot 47 \times 10^{-12}}} = 99.85$$

10. An AC emf is given by $e = 200 \sin\left(628t + \dfrac{\pi}{4}\right)$ V. What is its rms value, frequency and phase angle ?

(1st semester 2005)

Solution : $e = 200 \sin\left(628t + \dfrac{\pi}{4}\right)$ volt.

RMS value of AC emf is $E_{rms} = \frac{200}{\sqrt{2}} = 141 \cdot 44$ volt.

Frequency of AC emf is $f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = 100Hz$

Phase angle $\phi = \pi/4 = 45^0$.

11. A circuit contains an inductance of 1H in series with a resistance of 100Ω. A voltage of $e = 100\sqrt{2}\sin 314t$ volt is applied across the circuit. Express the rms value of current in polar form, complex form. Give the expression for instantaneous current under steady state.

(2nd semester 2005)

Solution : L = 1H, $R = 100\Omega$

$e = 100\sqrt{2}\sin 314t$ volt.

Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50Hz$

$X_L = \omega L = 2\pi f L = 2\pi(50)(1) = 314\Omega$

$Z = R + jX_L = 100 + j314 = 329 \cdot 53\angle 72 \cdot 33^0\ \Omega$

$E_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100$ volt.

∴ RMS value of current $I_{rms} = \frac{E_{rms}}{Z} = \frac{100\angle 0}{329 \cdot 53\angle 72 \cdot 33^0}$

$= 0 \cdot 3034\angle -72 \cdot 33^0 A$

In complex form the rms value of current is $I_{rms} = 0 \cdot 092 - j0 \cdot 289A$

Instantaneous current is given by

$i = 0 \cdot 3034\sqrt{2}\sin(314t - 72 \cdot 33^0)A$

12. An RLC series circuit is resonant at a frequency of 100 Hz. The value of the capacitor is changed so that the circuit resonates at a frequency of 200 Hz. Find the percentage changes in the capacitor values & the current at resonance the applied voltage remaining same.

(2nd semester 2005)

Solution : Resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

Clearly $f \propto \frac{1}{\sqrt{C}}$

$$\therefore \quad \frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}}$$

But $\quad f_1 = 100\,Hz$ and $f_2 = 200\,Hz$

$$\therefore \quad \frac{100}{200} = \sqrt{\frac{C_2}{C_1}}$$

$$\Rightarrow \frac{C_2}{C_1} = \frac{1}{4}$$

$$\Rightarrow C_1 = 4C_2$$

$$\% \text{ change in capacitor} = \frac{C_1 - C_2}{C_1} \times 100$$

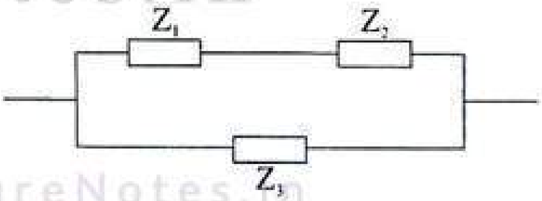
$$= \frac{4C_2 - C_2}{4C_2} \times 100$$

$$= 75\ \%$$

13. Two impedances $15\angle 60^0$ ohm and $10\angle -30^0$ ohm are connected in series and their series combination is connected in parallel with another impedane of $30\angle 45^0$ ohm. Find out the equivalent impedance of this connection.

(1st semester 2006)

Solution : $Z_1 = 15\angle 60^0\ \Omega$, $Z_2 = 10\angle -30^0\ \Omega$, $\quad Z_3 = 30\angle 45^0\ \Omega$



$$Z_{eq} = \frac{(Z_1 + Z_2)(Z_3)}{Z_1 + Z_2 + Z_3}$$

$$= \frac{[15\angle 60 + 10\angle -30][30\angle 45^0]}{15\angle 60^0 + 10\angle -30^0 + 30\angle 45^0}$$

$$= \frac{[7 + j13 + 8\cdot 66 - j5][30\angle 45^0]}{7 + j13 + 8\cdot 66 - j5 + 21\cdot 21 + j21\cdot 21}$$

$$= \frac{17\cdot 58\angle 27\cdot 06 \times 30\angle 45^0}{47\cdot 03\angle 38\cdot 38}$$

$$= 11\cdot 214\angle 33\cdot 68^0 \text{ ohm}$$

$$= 9\cdot 332 + j6\cdot 219\,\Omega$$

14. A circuit having a resistance of 5 ohm, an inductance of 0.5 H and a variable capacitance in series is connected across a 220 V, 50 Hz supply.

Calculate :

(i) The capacitance to give resonance.

(ii) The voltage across the capacitance and the inductance.

(iii) The Q-factor of the circuit. *(1st semester 2006)*

Solution : $R = 5\Omega, L = 0.5$ H, $C = ?$

V = 220 volt, f = 50 Hz

(i) $f = \dfrac{1}{2\pi\sqrt{LC}}$

$$\Rightarrow C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (50)^2 (0\cdot5)} = 20\cdot28\mu F$$

(ii) Current under resonant condition $I = \dfrac{V}{R} = \dfrac{220}{5} = 44\,A$

Angular frequency $\omega = 2\pi f = 2\pi(50) = 314\dfrac{rad}{\sec}$

Voltage across inductance $= V_L = I\omega L = (44)(314)(0\cdot5) = 6908$ volt.

Voltage across capacitance $= V_C = \dfrac{I}{\omega C} = \dfrac{44}{314 \times 20\cdot28 \times 10^{-6}} = 6908$ volt.

(approximately)

(iii) $Q - factor = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{5}\sqrt{\dfrac{0\cdot5}{20\cdot28 \times 10^{-6}}} = 31\cdot4$

15. Two impedances $Z_1 = 2 + j6$ and $Z_2 = 6 - j12$ ohm are connected in series, So that they are additive. Find the resultant impedance in polar form. *(2nd semester 2006)*

Solution : $Z = Z_1 + Z_2 = 2 + j6 + 6 - j12$

$= 8 - j6 = 10\angle -36\cdot87^0$ ohm.

16. A current of 20 A flows in a circuit with a 30^0 logging when the applied voltage is 200 V. Find the resistance and reactance. *(1st semester 2007)*

Solution : $I = 20\angle -30^0\,A$

$V = 200V$

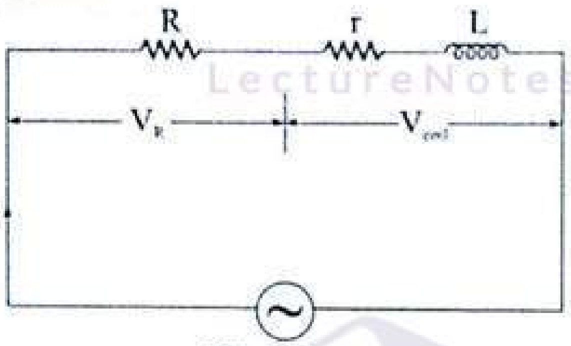
$$Z = \frac{V}{I} = \frac{200}{20\angle -30} = 10\angle 30^0 = 8\cdot66 + j5\text{ ohm}$$

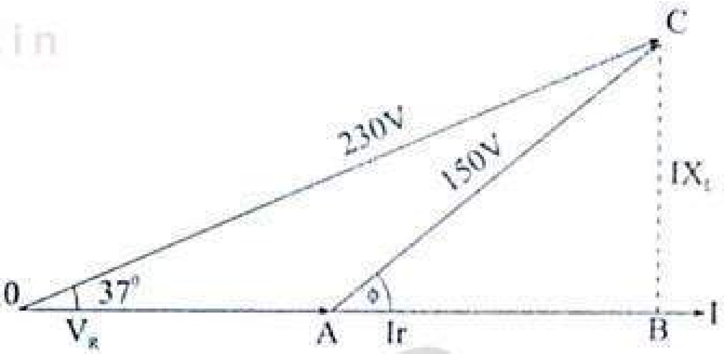
$\therefore$ Resistance $R = 8\cdot66\Omega$

Inductive reactance $= X_L = 5\Omega$

17. A resistor and an inductor are connected in series and the combination is connected across a single phase AC supply voltage of 230V at 50 Hz. The voltage across the inductor is measured and found out to be equal to 150 V. The current drawn by the circuit is 5A lagging 37 degrees behind the supply voltage. Find the resistance of the resistor and the resistance and inductace of the inductor. What is the p.f. of the inductor ? *(1st semester 2007)*

Solution :





Given $V = 230$ volts, $V_{coil} = 150$ volts

$I = 5\angle -37^0\ A$

f = 50 Hz

$\omega = 2\pi f = 314 \dfrac{rad}{sec}$

Total impedance of the circuit, $Z = \dfrac{230\angle 0}{5\angle -37} = 46\angle 37^0\ \Omega = 36\cdot 73 + j27\cdot 68\Omega$

Total resistance of the circuit = R + r = 36.73 Ω

Inductive reactance $= X_L = 27\cdot 68\Omega$

∴ Inductance $L = \dfrac{X_L}{\omega} = \dfrac{27\cdot 68}{314} = 0\cdot 088$ Henry.

In Δ ABC, $AB = \sqrt{AC^2 - BC^2}$

$\Rightarrow Ir = \sqrt{(150)^2 - (IX_L)^2} = \sqrt{(150)^2 - (5\times 27\cdot 68)^2}$

$\Rightarrow Ir = 57\cdot 83$

$\Rightarrow r = \dfrac{57\cdot 83}{I} = \dfrac{57\cdot 83}{5} = 11\cdot 56\Omega$

But $R + r = 36\cdot 73$

$\Rightarrow R = 36\cdot 73 - r = 36\cdot 73 - 11\cdot 56 = 25\cdot 16\Omega$

18. A coil of resistance 60 ohms and inductance 0.8 H is connected in series with a capacitor. The resonant frequency of the circuit is 60 Hz. If the supply given to the above series combination is 230 V, 50 Hz then find

(i) Line Current.

(ii) the Power factor

(iii) the voltage across the coil. *(1st semester 2007)*

Solution : $R = 60\Omega$, L= 0.8 H, C = ?

V = 230 volts, f = 50 Hz

Resonant frequency = 60 Hz

$$\Rightarrow \frac{1}{2\pi\sqrt{LC}} = 60$$

$$\Rightarrow C = \frac{1}{4\pi^2(60)^2(0.8)} = 8.804\times10^{-6}F$$

$$X_L = 2\pi fL = 2\pi(50)(0\cdot8) = 251\cdot2\Omega$$

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)\left(8\cdot804\times10^{-6}\right)} = 361\cdot734\Omega$$

Impedance of the circuit, $Z = R + j\left(X_L - X_C\right)$

$$\Rightarrow Z = 60 - j110\cdot534 = 125\angle-61^0\ \Omega$$

(i) Line Current, $I = \frac{V}{Z} = \frac{230\angle0}{125\angle-61} = 1\cdot84\angle61^0\ A$

(ii) Powert factor = $\cos\phi = \cos 61^0 = 0.484$

Impedance of coil $R + jX_L = 60 + j251\cdot2 = 258\cdot26\angle76\cdot56^0$ ohm.

Voltage across the coil $= 1\cdot84\angle61^0 \times 258\cdot26\angle76\cdot56^0$

$= 475\cdot19\angle137\cdot56^0$ volt.

19. What is the equation of a sinusoidal current of 60 Hz frequency having an rms value of 50 A?

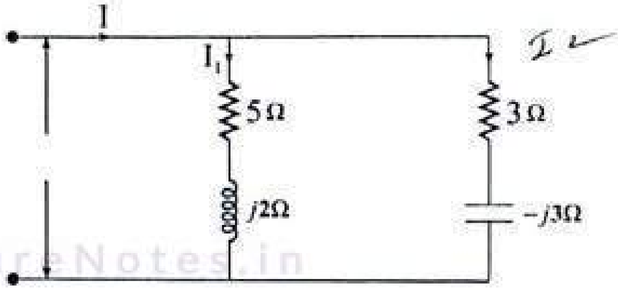
(2nd semester 2007)

Solution : $\omega = 2\pi f = 376\cdot8\frac{rad}{sec}$

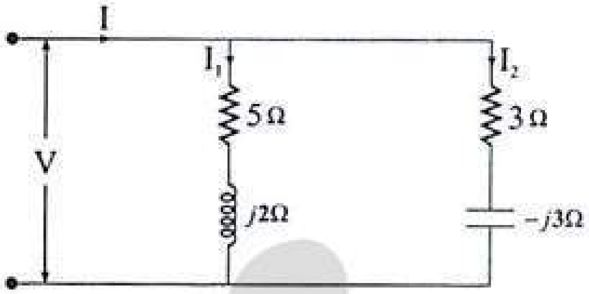
$I_m = 50\sqrt{2} = 70\cdot7\ A$

$\therefore$ $i = 70\cdot7\sin 376\cdot8t\ A$

20. In the parallel circuit shown in figure, the voltage across 3 ohm resistor is 45 volts. Calculate the total current I and draw the complete phasor diagram. ***(2nd semester 2007)***



Solution : Voltage across $3\Omega = 45$ volts.



$\therefore$ Current through $3\Omega = \dfrac{45}{3} = 15A$

$\Rightarrow I_2 = 15A$

$Z_2 = 3 - j3 = 4 \cdot 24\angle - 45^0\ \Omega$

$\therefore \quad \phi_2 = 45^0$ (lead)

$\therefore \quad V_2 = I_2 Z_2$

$= 15 \times 4 \cdot 24$

$= 63 \cdot 6$ volts.

$\therefore \quad V_2 = V = 63 \cdot 6$ volts.

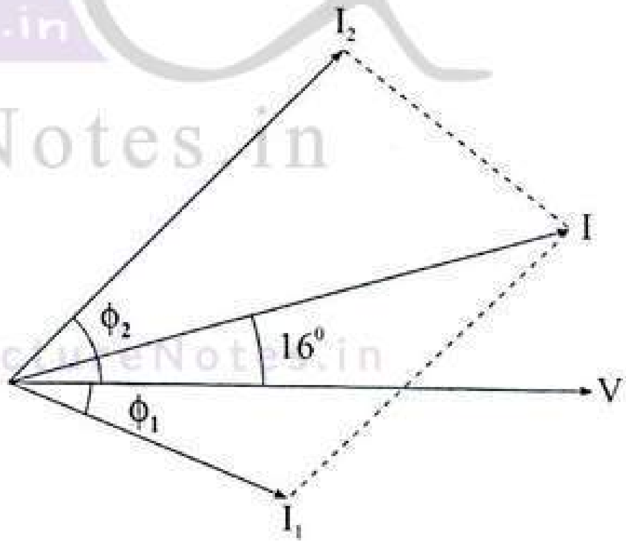
$Z_1 = 5 + j2$

$= 5 \cdot 38\angle 21 \cdot 8^0\ \Omega$

$\phi_1 = 21 \cdot 8^0\ (lag)$

$I_1 = \dfrac{63 \cdot 6}{5 \cdot 38\angle 21 \cdot 8}$

$= 11 \cdot 82\angle - 21 \cdot 8^0\ A$



Total Current $\quad I = I_1 + I_2$

$= 11 \cdot 82\angle - 21 \cdot 8^0 + 15\angle 45$

$= 21 \cdot 574 + j6 \cdot 211$

$= 22\angle 16^0\ A$

21. A generator supplies a variable frequency voltage of constant amplitude 150V (rms) to a series R-L-C circuit having R = 10 ohms, L = 5 milli henry and C = 0.15 micro farad. The frequency is to be varied until maximum current flows in the circuit. Predict the maximum current, the frequency at which it occurs and the resulting voltage across the inductance and capacitance.

(2nd semester 2007)

Solution : $R = 10\,\Omega$, $L = 5 \times 10^{-3}$ H, $C = 0.15 \times 10^{-6}$ F, V = 150 volts.

Maximum current $I = \frac{V}{R} = \frac{150}{10} = 15A$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times10^{-3}\times0\cdot15\times10^{-6}}} = 5814\cdot62\,Hz$$

Voltage across inductance $= V_L = I\omega L = I(2\pi f L)$

$= 15\left(2\pi\times5814\cdot62\times5\times10^{-3}\right)$ volts.

$= 2738\cdot69$ volts.

Voltage across capacitance $= V_C = I\left(\frac{1}{2\pi fc}\right)$

$$=15\left(\frac{1}{2\pi\times5814\cdot62\times0\cdot15\times10^{-6}}\right)$$

$= 2738\cdot69$ volts.

22. A resistor of 20 ohms in series with a 0.5 H inductor is connected across a supply at 250 volt, 60 Hz. Find the current through the inductor.

(1st semester 2008).

Solution : $R = 20\Omega$, $L = 0\cdot5H$, $V = 250$ volts. f = 60 Hz

$X_L = \omega L = 2\pi(60)(0\cdot5) = 188\cdot4\Omega$

$Z = R + jX_L = 20 + j188\cdot4 = 189\cdot45\angle83\cdot94^0$

Current through the inductor, $I = \frac{V}{Z} = \frac{250\angle0}{189\cdot45\angle83\cdot94}$

$= 1\cdot32\angle-83\cdot94^0\,A$

23. A circuit consists of a resistor of 15 ohms in series with a capacitor of 50 micro farads. The frequency is 60 Hz. Calculate the conductance and the susceptance of the circuit.

(1st semester 2008)

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Solution : $R = 15\Omega, \quad C = 50\times10^{-6}F, \quad f = 60Hz$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fc} = 53\cdot07\,\Omega$

$Z = R - jX_C = 15 - j53\cdot07 = 55\cdot15\angle-74^0\,\Omega$

$Y = \frac{1}{Z} = \frac{1}{55\cdot15\angle-74^0} = 0\cdot01813\angle74^0 = 4\cdot997\times10^{-3} + j0\cdot0174$ mho

But $Y = G + jB$.

Conductance $G = 4\cdot997\times10^{-3}$ mho and susceptance $B = 0\cdot0174$ mho.

24. Two impedances $25\angle-50^0$ and $15\angle45^0$ are connected in parallel. Find out the resultant impedance in rectangular form. *(1st semester 2008)*

Solution : $Z_1 = 25\angle-50^0 = 16\cdot07 - j19\cdot15\Omega$

$Z_2 = 15\angle45^0 = 10\cdot606 + j10\cdot606\Omega$

Resultant impedance, $Z = \frac{Z_1Z_2}{Z_1+Z_2} = \frac{25\angle-50\times15\angle45}{16\cdot07 - j19\cdot15 + 10\cdot606 + j10\cdot606}$

$= \frac{375\angle-5^0}{26\cdot676 - j8\cdot544} = 13\cdot05 + j2\cdot95$

25. A coil of resistance 2.5 ohms and inductance 0.02 H is connected in series with a capacitor across 230 V mains. What must be the capacitance in order that maximum current occurs at a frequency of (i) 30 Hz (ii) 60 Hz and (iii) 120 Hz ? Find also the voltage across the capacitor in each case. *(1st semester 2008)*

Solution : $R = 2\cdot5\Omega$, L = 0.02 H, C = ? V = 230 volts.

(i) Under resonant condition, current flows through R-L-C series circuit is maximum.

Resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$\Rightarrow C = \frac{1}{4\pi^2Lf^2} = \frac{1}{4\pi^2(0\cdot02)(30)^2} = 1408.6\mu F$

(ii) When $f = 60Hz$ then $C = \frac{1}{4\pi^2(0\cdot02)(60)^2} = 352\cdot1\mu F$

(iii) When f = 120 Hz then $C = \frac{1}{4\pi^2(0\cdot02)(120)^2} = 87\cdot95\mu F$

26. Two impedances of value (5+j6) ohms and (8-j3) ohms are connected in parallel. What would be the net equivalent impedance of the combination ? What would be the phase current with respect to the supply voltage, when the combination is excited from a 220V, 50 Hz single phase AC supply ? *(1st semester 2009)*

Solution : $Z_1 = 5 + j6 = 7.81\angle 50.194\,\Omega$

$$Z_2 = 8 - j3 = 8\cdot 544\angle -20\cdot 55\,\Omega$$

Equivalent impedance $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

$$= \frac{7\cdot 81\angle 50\cdot 194 \times 8\cdot 544\angle -20\cdot 55}{5 + j6 + 8 - j3}$$

$$= \frac{66\cdot 728\angle 29\cdot 644}{13\cdot 341\angle 13^0}$$

$$= 5\angle 16\cdot 644\,\Omega$$

Phase current $I = \dfrac{V}{Z} = \dfrac{200\angle 0}{5\angle 16\cdot 644} = 40\angle -16\cdot 644^0\ A$

27. Calculate the combined impedance and admittance in polar form when three impedances of value $5\angle 30^0\,\Omega$, (3+j6) ohms and (4-j8) ohms are connected series. *(1st semester 2009)*

Solution : $Z_1 = 5\angle 30^0 = 4\cdot 33 + j2\cdot 5$ ohms

$Z_2 = 3 + j6$ ohms

$Z_3 = 4 - j8$ ohms

$\therefore \quad Z = Z_1 + Z_2 + Z_3 = 4\cdot 33 + j2\cdot 5 + 3 + j6 + 4 - j8$

$= 11\cdot 33 + j0\cdot 5 = 11\cdot 34\angle 2\cdot 5\,\Omega$

$$Y = \frac{1}{Z} = \frac{1}{11\cdot 34\angle 2\cdot 5} = 0\cdot 088\angle -2\cdot 5 \text{ mho.}$$

28. An inductor of inductance 50 milli henries is connected in series with a capacitance of 10 micro-farads. Find the impedance of the circuit when the frequency is (i) 50 Hz and (ii) 5 KHz. *(2nd semester 2009)*

Solution : $L = 50\times 10^{-3}\,H,\ C = 10\times 10^{-6}\,F$

(i) $X_L = \omega L = 2\pi f L = 2\pi(50)(50\times 10^{-3}) = 15\cdot 7\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f c} = \frac{1}{2\pi(50)(10\times 10^{-6})} = 318\cdot 47\,\Omega$$

Impedance of the circuit $= X_C - X_L = 302\cdot 77\,\Omega$ (capacitive)

(ii) $X_L = 2\pi fL = 2\pi\left(5\times10^{-3}\right)\left(50\times10^{-3}\right) = 1570\,\Omega$

$$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi\left(5\times10^{3}\right)\left(10\times10^{-6}\right)} = 3\cdot184\,\Omega$$

Impedance of the circuit $= X_L - X_C = 1566\cdot816\Omega$ (Inductive)

29. A resistor of 25 ohms in series with a 0.45 micro-farad capacitor is connected across a supply at 270 V, 70 Hz. Find the current through the capacitor. ***(2nd semester 2009)***

Solution : $R = 25\,\Omega,\quad C = 0\cdot45\times10^{-6}F$

V = 270 volts. f = 70 Hz.

Capacitive reactance $X_C = \dfrac{1}{2\pi(70)0\cdot45\times10^{-6}}$

$= 5055\cdot10\Omega$

Impedance of the circuit, $Z = R - jX_C$

$= 25 - j5055\cdot10$

$= 5055\cdot16\angle-89\cdot71^0$

Current through capacitor, $I = \dfrac{V}{Z} = \dfrac{270\angle0}{5055\cdot16\angle-89\cdot71}$

$= 0\cdot0534\angle89\cdot71A$

30. A circuit consists of a resistor of 10 ohms in series with an ideal inductor of 4.5 Henries. The frequency is 70 Hz. Calculate the conductance and susceptance of the circuit.

(2nd semester 2009)

Solution : $R = 10\Omega,\quad L = 4\cdot5H,\quad f = 70\,Hz,\quad X_L = 2\pi fL = 1978\cdot2\Omega$

$Z = R + jX_L = 10 + j1978\cdot2 = 1978\cdot22\angle89\cdot7\,\Omega$

Conductance in series circuit, $G = \dfrac{R}{Z^2} = \dfrac{10}{(1978\cdot22)^2} = 2\cdot55\times10^{-6}$ mho

Susceptance in series circuit, $B = \dfrac{X_L}{Z^2} = 5\cdot055$ mho $\times10^{-4}$

31. Two admittances $0\cdot025\angle-50^0$ and $0\cdot015\angle45^0$ are connected in parallel. Find out the resultant impedance in rectangular form. ***(2nd semester 2009)***

Solution : $Y = Y_1 + Y_2 = 0\cdot025\angle-50^0 + 0\cdot015\angle45^0$

$\Rightarrow Y = 0\cdot016 - j0\cdot0191 + 0\cdot010 + j0\cdot010$

$\Rightarrow Y = 0\cdot026 - j9\cdot1\times10^{-3} = 0\cdot0275\angle-19\cdot3^0$ mho

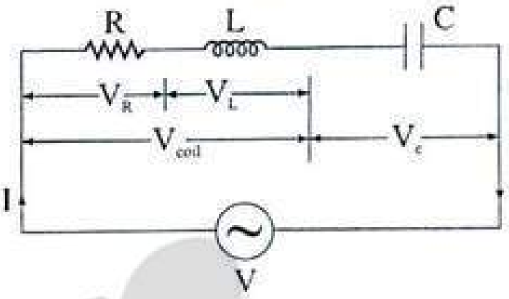
$\therefore \qquad Z = \dfrac{1}{0\cdot0275\angle-19\cdot3^0} = 36\cdot363\angle19\cdot3^0$

$\Rightarrow Z = 34\cdot32 + j12\cdot018$ ohms

32. A single phase AC supply voltage of 230V at 50 Hz is applied to a coil of inductance 4.5 henries and resistance of 2.25 ohms in series with a capacitance 'C'. Calculate the value of capacitance 'C' so as to obtain a p.d. of 255 V across the coil.

(2nd semester 2009)

Solution : V = 230 volts, f = 50 Hz, L = 4.5 H, R = 2.25 Ω C = ?



Inductive reactance

$X_L = 2\pi fL = 2\pi(50)(4\cdot5)$

= 1413 ohms,

Impedance of coil = $Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{(2\cdot5)^2 + (1413)^2} = 1413$ ohms

Current flowing through the coil, $I = \dfrac{V_{coil}}{Z_{coil}} = \dfrac{255}{1413} = 0\cdot1805\,A$

Impedance of the circuit, $Z = \dfrac{V}{I} = \dfrac{230}{0\cdot1805} = 1274\cdot24\Omega$

But $\quad Z^2 = R^2 + X^2$

$\Rightarrow X = \sqrt{Z^2 - R^2} = \sqrt{(1274\cdot24)^2 - (2\cdot5)^2} = 1274\cdot235$ ohms

$\Rightarrow$ Net reactance = 1274.235 ohms.

case (i) $X_L - X_C = 1274\cdot235$

$\Rightarrow X_C = X_L - 1274\cdot235 = 1413 - 1274\cdot235 = 138\cdot764\Omega$

$\Rightarrow C = \dfrac{1}{\omega X_C} = \dfrac{1}{2\pi(50)(138\cdot764)} = 22\cdot95\mu F$ (inductive circuit)

case (ii) Also $\quad X_C - X_L = 1274\cdot235$

$\Rightarrow X_C = 1274\cdot235 + X_L = 1274\cdot235 + 1413 = 2687\cdot23$

$\Rightarrow C = \dfrac{1}{\omega X_C} = \dfrac{1}{2\pi(50)(2687\cdot23)} = 1\cdot185\mu F$ (capacitive circuit)

Do Your Self

T 3.1 The instantaneous voltage and current for an AC circuit are V = 155.6 sin 377t volt and

I = 7.07 sin (377t - 36.870) A

A. Represent these (a) as complex exponentials

(b) In a phasor diagram

(c) The frequency (in hertz),

(d) The period

(e) The phase angle between V and i (in radians).

$$\left[v = 155\cdot 6e^{j377t}(V)\;\; i = 7\cdot 07e^{j(377t-36\cdot 87^0)}(A) 60\,Hz, 0\cdot 0167\,s,\, 0\cdot 64\,rad\right]$$

T 3.2 The voltage wave of Fig. T3.1 is applied to a 20Ω resistor. If electrical energy costs 6 per kWh, how much would it cost to operate the circuit for 24 hours ? **[Rs24]**

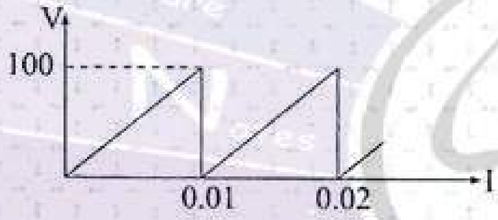


Fig T 3.1

T 3.3 Given v = 200 sin 377t V and i = 8 sin (377t - 300) A for an AC circuit. Determine (a) the power factor, (b) ture power, (c) apparent power and (d) reactive power.

[0.866 lagging, 692.8 W, 800 VA, 400 VAR]

T 3.4 A coil has a resistance of 10Ω and draws a curret of 5 A when connected across a 100 V, 60 Hz source. Determine (a) the inductance of the coil, (b) the power factor of the circuit, and (c) The reactive power **[45.94 mH, 0.5 lag. 433 VAR]**

T 3.5 A series RLC circuit is excited by a 100 V, 79.6 Hz source and has the following data : R = 100 ft, L = 1H, C = 5μF. Calculate (a) the input current, and (b) the voltages across the elements.

$$[0.707\angle -45^0, 70\cdot 7\angle -45^0,\; 282\cdot 8\angle -135^0]$$

T 3.6 For the circuit shown in Fig. T3.2 evaluate the current through, and the voltage across, each element. Then dra a phasor diagram showing all the voltages and currents.

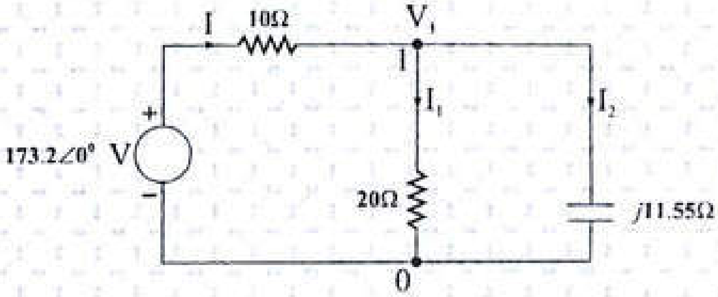


Fig T 3.2

$$\left[100\angle-30^{0}V,\ 100\angle30^{0}V,\ 10\angle30^{0}A, 5\angle-30^{0}A,\ 8\cdot66\angle60^{0}A\right]$$

T 3.7 A 20 Ω resistance is connected in series with a parallel combination of a capacitance C and 1 15 mH pure inductance. At angular frequency w = 1000 rad/s, find C such that the line current is 45^{0} out of phase with the line voltage.

$$\left[16\cdot67\mu F,\ 116\cdot7\mu F\right]$$

T 3.8 A 46-mH inductive coil has a resistance of 10Ω. (a) How much current will it draw if conneced across a 100 V, 60 Hz source ? (b) What is the power factor of the coil ? (c) Determine the value of the capacitance that must be connected across the coil to make the power factor of the overall circuit unity.

$$\left[5.0\angle-60\ A,\ 0\cdot5\ \textit{lagging}, 115uF\right]$$

T 3.9 The instantaneous values of two alternating voltages are given by $V_1 = 5\sin wt$ and $V_2 = 8\sin(wt - \pi/6)$ obtain expressions for (a) $V_1 + V_2$ and (b) $V_1 - V_2$

$$\left[(a)\,12\cdot58\sin(wt-0\cdot324), (b)\,4\cdot44\sin(wt+2\cdot02)\right]$$

T 3.10 A coil of inductance 636.6 mH and negligible resistance is connected in series with a 100Ω resistor to a 250V, 50 Hz supply. Calculate (a) the inductive reactance of the coil, (b) the impedance of the circuit, (c) The current in the circuit, (d) the p.d. across each component, and (e) the circuit phase angle.

$$\left[(a)\,200\Omega, (b)\,223\cdot6\Omega, (c)\,1\cdot118A\ (d)\,223\cdot6V, 111\cdot8V\ (e)\,63\cdot43^{0}\ \textit{lagging}\right]$$

T 3.11 An alternating voltage given by V=100 sin 240t volts is applied across a coil of resistance 32 Ω and inductance 100 mH. Determine (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistance and (d) the p.d. across the inductance.

$$\left[(a)\,40\Omega\ (b)\,1\cdot77A\ (c)\,56\cdot64V\ (d)\,42\cdot48V\right]$$

T 3.12 Al alternating voltage V=250 sin 800t volts is applied across a series containinga 30Ω resistor and 50μF capacitor. Calculate (a) the circuit impedance (b) the current flowing, (c) the p.d. across the resistor, (d) the p.d. across the capacitor, and (e) the phase angle between voltage and current.

$$\left[(a)\,39\cdot05\Omega\ (b)\,4\cdot526A\ (c)\,135\cdot8V\ (d)\,113\cdot2V\ (e)\,39\cdot81^{0}\ \textit{leading}\right]$$

T 3.13. A coil takes a current of 5A from a 20 V d.c. supply. When connected to a 200V, 50 Hz a.c. supply the current is 25A. Calculate the (a) resistance, (b) impedance, (c) inductance of the coil.

[(a) 4Ω (b) 8Ω (c) 22.05 mH]

T. 3.14 A voltage of 35 V is applied across a C-R series circuit. If the voltage across the resistor is 21V, find the voltage across the capacitor. **[28V]**

T3.15 A resistance of 50Ω is connected in series with a capacitance of 20μF. If a supply of 200 V, 100 Hz is connected across the arrangement find (a) the circuit impedance, (b) the current flowing, and (c) the phase angle between voltage and current.

[(a) 93.98Ω (b) 2.128A (c) 57.86^0 leading]

T3.16 An alternating voltage v = 250 sin 800t volts is applied across a series circuit containing a 30 Ω resistor and 50 μF capacitor. Calculate (a) the circuit impedance, (b) the current flowing (c) The p.d. across the resistor, (d) the p.d. across the capacitor, and (e) the phase angle between voltage and current. [(a) 39.05Ω (b) 4.526A (c) 135.8V (d) the voltage across the coil, and (e) the voltage across the capacitor. **[(a) 13.18Ω (b) 15.17A (c) 52.630 lagging (d) 772.1V (e) 603.6V]**

T3.17 A 400Ω resistor is connected in series with a 2358 μF capacitor across a 12V a.c. supply. Determine the supply frequency if the current flowing in the circuit is 24mA. **[225 kHz]**

T3.18 A 40μ*F* capacitor in series with a coil of resistance 8Ω and inductance 80mH is connected to a 200V, 100 Hz supply. Calculate (a) the circuit impedanc, (b) the current flowing, (c) the phase angle between voltage and current, (d) the voltage across the coil, and (e) the voltage across the capacitor.

[(a) 13.18Ω (b) 15.17A (c) 52.63^0 lagging (d) 772.1V (e) 603.6V]

T.3.18 Find the values of resistance R and inductance L in the circuit of Figure T3.3

[R = 131Ω, L = 0.545 H]

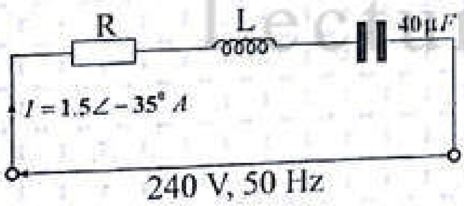


Fig T 3.3

T 3.19 Three impedances are connected in series across a 100V, 2 kHz supply. The impedance comprise (i) an inductance of 0.45 mH and 2Ω resistnce, (ii) an nductance of 570μ*H* and 5Ω resistance, and (iii) a capacitor of capacitance 10μ*F* and resistance 3Ω. Assuming no mutual inductive effects between the two inductances calculate (a) the circuit impedance, (b) the circuit current, (c) the circuit phase angle and (d) the voltage across each impedance

[(a) 11.12Ω (b) 8.99A (c) 25.920 lagging (d) 53.92 V, 78.53 V, 76.64 V]

T 3.20 For the circuit shown in Figure T3.4 determines the voltages V_1 and V_2 if the supply frequency is 1 kHz. Draw the phasor diagram and hence determine the supply voltage V and the circuit phase angle.

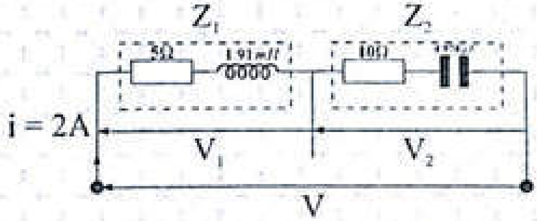


Fig T 3.4

[V_1=26.0V, V_2=67.05V, V=50V, 53.14^0 leading]

T 3.21 An impedance 15 + j20Ω is connected across a 125 V, 60 Hz source. Find (a) the instantaneous current through the load, (b) the instantaneous power, and (c) the average active and reactive powers.

[(a) 7.07 sin (377t-53.1^0) A; (b) 1250 sin 377t sin (377t - 53.1^0) (W); (c) 375 W, 500 VAR]

T 3.22 A voltage source of 100 V has internal impedance 0.1+j.F capacitance. Determine an equivalent series RC circuit such that the two circuits have the same impedance at an angular frequency of 1000 rad/s. **[R=48Ω, C=27.8 μF]**

T 3.24 An AC circuit with a current excitation is shown in Fig. T3.5. Determine (a) the voltage across the inductance, and (b) the power dissipated in the two resistances.

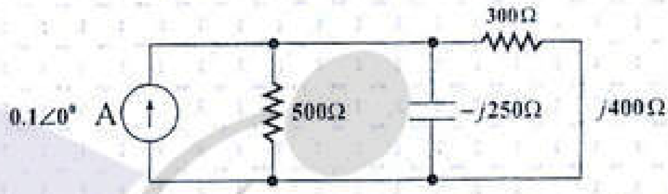


Fig T 3.5

[(a) 20∠0V; (b) 2 W]

T 3.25 A 30Ω resistor is connected in parallel with a pure inductance of 3mH across a 110V, 2kHz supply. Calculate (a) the current in each branch, (b) the circuit, (c) the circuit phase angle, (d) the circuit impedance, (e) the power consumed, and (f) the circuit power factor.

[(a) IR = 3.67A, IL = 2.92A (b) 4.69 A (c) 38.51^0 lagging (d) 23.45Ω (e) 404W (f) 0.782 lagging].

T 3.26 A 40Ω resistance is connected in parallel with a coil of inductance L and negligible resistance across a 200V, 50 Hz supply and the supply current is found to be 8A. Sketch a phasor diagram and determine the inductance of the coil. **[102 mH]**

T. 3.27 A 1500 nF capacitor is connected in parallel with a 16Ω resistor across a 10 V, 10 kHz supply. Calculate (a) the current in each branch, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance, (e) the power consumed, (f) the appacent power, and (g) the circuit power factor. Sketch the phasor diagram.

[(a) IR = 0.625A, IC = 0.943A (b) 1.131 A (c) 56.46^0 leading (d) 8.84Ω (e) 6.25W (f) 11.31 VA (g) 0.553 leading].

T. 3.28 A capacitor C is connected in parallel with a resistance R across a 60 V, 100 Hz supply. The supply current is 0.6A at a power factor of 0.8 leading. Calculate the values of R and C.

[R = 125Ω, C = 9.55μF]

T 3.29 An inductance of 80 mH is connected in parallel with a capacitance of 10μF across a 60V, 100 Hz supply. Determine (a) the branch currents, (b) the supply current, (c) the circuit phase angle, (d) the circuit impedance and (e) the power consumed.

[(a) IC = 0.377A, IL = 1.194A (b) 0.817A (c) 90^0 lagging (d) 73.44Ω (e) 0 W]

T3.30 A coil of resistance 60Ω and inductance 318.4 mH is connected in parallel with a 15μF capacitor across a 200V, 50 Hz supply. Calculate (a) the current in the coil, (b) the current in the capacitor, (c) the supply current and its phase angle, (d) the circuit impedance, (e) the power consumed, (f) the apparent power and (g) the reactive power. Sketch the phasor diagram.

[(a) 1.715A (b) 0.943A (c) 1.028A at 30.88^0 lagging (d) 194.6Ω (e) 176.5W (f) 205.6VA (g) 105.5 var]

T. 3.31 A 25 nF capacitor is connected in parallel with a coil of resistance 2 kΩ and inductance 0.20 H across a 100V, 4kHz supply. Determine (a) the current in the coil, (b) the current in the capacitor, (c) the supply current and its phase angle (by drawing a phasor diagram to scale, and also by calculation), (d) the circuit impedance, and (e) the power consumed.

[(a) 18.48 mA (b) 62.83 mA (c) 46.17mA at 81.49^0 leading (d) 2.166 kΩ (e) 0.683W]

T 3.32 Find the resonant frequency of a series AC circuit consisting of a coil of resistance 10Ω and inductance 50mH and capacitance 0.05μF. Find also the current flowing t resonance if the supply voltage is 100V.

[3.183 kHz, 10A]

T 3.33 The current at resonance in a series L-C-R circuit is 0.2 mA. If the applied voltage is 250 mV at a frequency of 100 kHz and the circuit capacitance is 0.04μF, find the circuit resistance and inductance.

[1.25 kΩ, 63.3μH]

T 3.34 A coil of resistance 25Ω and inductance 100mH is connected in series with a capacitance of 0.12μF across a 200V, variable frequency supply. Calculate (a) the resonant frequency, (b) the current at resonance and (c) the factor by which the voltage across the reactance is greater than the supply voltage.

[(a) 1.453 kHz (b) 8A (c) 36.51]

T 3.35 Calculate the inductance which must be connected in series wth a 1000 pF capacitor to give a resonant frequency of 400 kHz.

[0.158 mH]

T 3.36 A series circuit comprises a coil of resistance 20Ω and inductance 2mH and a 500 pF capacitor. Determine the Q-factor of the circuit at resonance. If the supply voltage is 1.5V, what is the voltage across the capacitor ?

[100, 150V]

T 3.37 A 0.15μF capacitor is connected in parallel with a coil of inductance 50mH and unknown resistance R across a 120V, 50 Hz supply. If the circuit has an overall power factor of 1 find (a) the value of R, (b) the current in the coil, and (c) the supply current.

[(a) 4.11 kHz (b) 38.74mA]

T 3.38 A 30μF capacitor is connected in parallel with a coil of inductance 50mH and unknown resistance R across a 120V, 50 Hz supply. If the circuit has an overall power factor of 1 find (a) the value of R, (b) the current in the coil, and (c) the supply current.

[(a) 37.68Ω (b) 2.94A (c) 2.714A]

T 3.39 A coil of resistance 25Ω and inductance 150mH inductance is connected in parallel with a variable capacitance across a 10V, 8 kHz supply. Calculate (a) the capacitance of the capacitor when the supply current is a minimum, (b) the dynamic resistance, and (c) the current at resonance and (d) the Q-factor at resonance. **[(a) 127.2 Hz (b) 600Ω (c) 0.10A (d) 4.80]**

T. 3.40 A coil of resistance 1.5 kΩ and 0.25H inductance is connected in parallel with a variable capacitance across a 10V, 8 kHz supply. Calculate (a) the capacitance of the capacitor when the supply current is a minimum, (b) the dynamic resistance, and (c) the supply current.

[(a) 1561 μF (b) 106.8 kΩ (c) 93.66 μA]

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Basic Electrical Engineering

Topic:

AC Power

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AC Power

Chapter - 5

In a DC circuit the expression for power is very simple and is given by P = VI. However in AC circuit the power is not a steady value as both voltage and current are alternating in nature and they may not be in same phase. Unlike DC power, AC Power is a complex quantity which comprises of both real (active) and imaginary (reactive) power.

5.1 Power in AC circuits

In an A.C. circuit, the voltage and current are continuously changing, so power value changes with time. The average value of a varying power (p) is $P_{av} = \frac{1}{T}\int_0^T p \cdot dt = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\theta$. This average power is called as ***active power or true power.***

When electric energy is delivered from A.C. supply to resistive loads then the energy is dissipated in the same way as D.C. dissipates energy in a resistor. The power that gives rise to energy dissipation in a resistor is called active power (i.e. I^2 R, where I is the rms value of current that flows through resistor R). For pure reactive loads (inductive /capacitive loads), the energy is delivered to the load and then that energy returned to the source. The power describing the rate of energy moving in and out of a reactance is called reactive power. If a circuit containing both resistance and reactance then there is a mixture of active and reactive powers.

5.1.1 Instantaneous and Average Power

In an A.C. circuit the power at any instant is called *instantaneous power* It is equal to the product of the values of voltage and current at that instant. In an A.C. circuit, Let the instantaneous voltage and current are, $v = V_m \sin\omega t$ and $i = I_m \sin(\omega t - \phi)$.

$\therefore$ The instantaneous power is given by,

$$p = vi = V_m \sin\omega t.\ \ I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin\omega t.\ \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m \left[\cos\phi - \cos(2\omega t - \phi)\right] \quad \left[\therefore \cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \frac{1}{2} V_m I_m \cos\phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$$

The second term in right hand side of the above equation contains a double frequency (2ω) term, So the magnitude of the average value of this term is zero. It is because average of a sinusoidal quantity of double frequency over a complete cycle is zero. Thus the instantaneous power.

$$p = \frac{1}{2} V_m I_m \cos\phi - 0 = \frac{1}{2} V_m I_m \cos\phi$$

This instantaneous power is the average power in the A.C. circuit.

$\therefore$ Average power in A.C. circuit is given by

$$P_{av} = \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = V\, I \cos\phi$$

where $V = \frac{V_m}{\sqrt{2}} = r.m.s\ voltage\ of\ A.C.$

$I = \frac{I_m}{\sqrt{2}} = r.m.s\ current\ of\ A.C.$

$\phi = angle\ between\ V\ and\ I$

5.1.2 Power in Pure resistance

In a purely resistive A.C. circuit the voltage and current are in same phase. Let at any instant voltage and current are,

$v = V_m \sin\omega t$ and $i = I_m \sin\omega t$

Instantaneous power $p = vi = V_m \sin\omega t.\ I_m \sin\omega t$

$$= V_m \sin\theta.\ I_m \sin\theta \quad = V_m I_m \sin^2\theta = \frac{1}{2} V_m I_m (1 - \cos 2\theta) \qquad (\because \theta = \omega t)$$

$\therefore$ Average power, P = average of p over one cycle

$$= \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} V_m I_m (1 - \cos 2\theta).d\theta$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 1 \cdot d\theta - \frac{V_m I_m}{4\pi}\int_0^{2\pi} \cos 2\theta \cdot d\theta$$

$$= \frac{V_m I_m}{4\pi}[\theta]_0^{2\pi} - \frac{V_m I_m}{4\pi}\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{V_m I_m}{4}[2\pi - 0] - 0$$

$$= \frac{V_m I_m}{2} \qquad \begin{Bmatrix} \because V = \frac{V_m}{\sqrt{2}} = r.m.s\ voltage\ of\ A.C \\ I = \frac{I_m}{\sqrt{2}} = r.m.s\ current\ of\ A.C. \end{Bmatrix}$$

$$= \frac{\sqrt{2}V \cdot \sqrt{2}I}{2} = VI$$

The power wave form is shown in fig 5.1 From the power wave form it is seen that the power P remains positive throughout the cycle. It is due to the fact that the voltage and current are in same phase. Even when both voltage and current are negative their product is still positive. This shows that direction of power flow is from source to load resistance R. The energy received by R is called active energy. It is consumed in R and appears in the form of heat. The rate of this consumption is the active power.

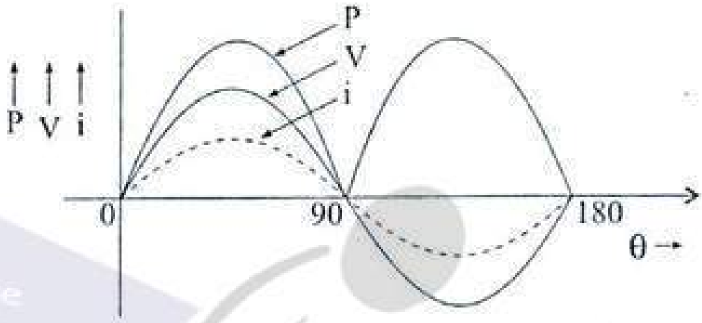

Fig 5.1

5.1.3 Power in Pure inductance

In pure inductive A.C. circuit current lags the voltage by 90^0. Let at any instant the voltage and current in the circuit are $v = V_m \sin \omega t$ and $i = I_m \sin(\omega t - 90)$

Instantaneous power p

$$= V_m \sin\theta \cdot I_m \sin(\theta - 90) \quad = V_m I_m \sin\theta \cdot \sin(\theta - 90) \quad = -V_m I_m \sin\theta \cdot \cos\theta$$

$$= \frac{-V_m I_m}{2}\sin 2\theta \qquad (\because \theta = \omega t)$$

∴ Average power P = average of P over one cycle

$$= \frac{1}{2\pi}\int_0^{2\pi} p \cdot d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{-V_m I_m}{2}\sin 2\theta \cdot d\theta$$

$$= \frac{-V_m I_m}{4\pi} \int_0^{2\pi} \sin 2\theta \cdot d\theta = \frac{-V_m I_m}{4\pi} \left[\frac{-\cos 2\theta}{2} \right]_0^{2\pi}$$

$$= \frac{-V_m I_m}{4\pi} \left[\frac{-\cos 4\pi}{2} + \frac{\cos 0}{2} \right]$$

$$= \frac{-V_m I_m}{4\pi} \left[\frac{-1}{2} + \frac{1}{2} \right] = 0$$

Thus in a purely inductive A.C. circuit the average power over a complete cycle is zero. The power wave form may be plotted by multiplying at every instant the values of voltage and current obtained from their waveforms. The power wave form is shown in fig 5.2.

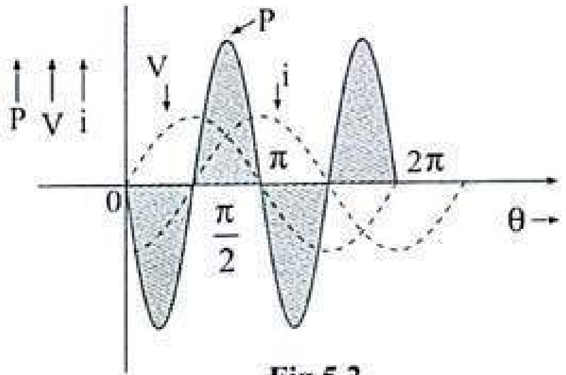

Fig 5.2

5.1.4 Power in Pure capacitance

In a purely capacitive A.C. circuit the current leads the voltage by 90^0. Let at any instant the voltage and current in the circuit are $v = V_m \sin \omega t$ and $i = I_m \sin(\omega t + 90)$.

Instantaneous power $P = vi = V_m \sin \omega t . I_m \sin(\omega t + 90)$

$$= V_m \sin\theta \cdot I_m \sin(\theta + 90) \qquad \{\because \theta = \omega t\}$$

$$= V_m I_m \sin\theta \cdot \sin(\theta + 90)$$

$$= V_m I_m \sin\theta \cdot \cos\theta$$

$$= \frac{V_m I_m}{2} \sin 2\theta$$

$\therefore$ Average power, $P = \frac{1}{2\pi} \int_0^{2\pi} vi \cdot d\theta$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta \cdot d\theta$$

$$= \frac{1}{2\pi} \cdot \frac{V_m I_m}{2} \left[\frac{-\cos 2\theta}{2} \right]_0^{2\pi}$$

$$= \frac{V_m I_m}{4\pi} \left[\frac{-\cos 4\pi}{2} + \frac{\cos 0}{2} \right]$$

$$= \frac{V_m I_m}{4\pi} \left[\frac{-1}{2} + \frac{1}{2} \right] = 0$$

Thus in a purely capacitive A.C. circuit the active power over a complete cycle is zero. The power wave form is shown in fig 5.3.

From fig 5.3 it is clear that positive power is equal to negative power over one cycle. Hence net power absorbed in a pure capacitor is zero.

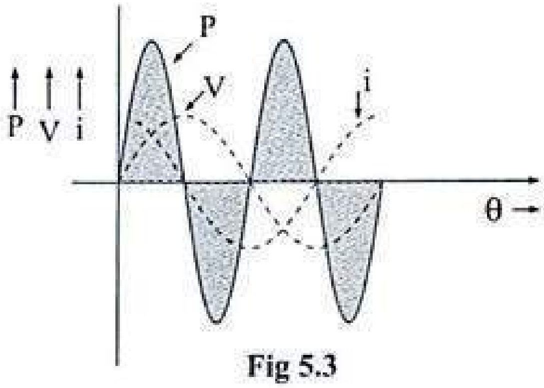

Fig 5.3

5.2 Reactive Power

Reactive power generates from reactive elements (i.e. inductance or capacitance). The product of r.m.s. values of voltage and current with the sine of the angle between them is called the ***reactive power*** in A.C. circuit. It is represented by symbol Q and its unit is VAR.

$$Q = VI\sin\phi$$

Reactive power in a purely inductive circuit $\quad Q_L = V_L I = I^2 X_L = \dfrac{V_L^2}{X_L}$

Reactive power in a purely capacitive circuit $\quad Q_C = V_C I = I^2 X_C = \dfrac{V_C^2}{X_C}$

The reactive power of inductance (Q_L) is positive while that of capacitance (Q_C) in negative. It is because reactance of inductance in positive (i.e. jX_L) and reactance of capacitance is negative (i.e. $-jX_C$). We thus say that an inductance absorbs reactive power while a capacitance injects reactive power. In inductance, reactive power is utilized to develop the flux while in the capacitance, the reactive power's function is to store charge. The other names of reactive power are ***wattless power*** and ***quadrature power.***

5.3 Complex Power

The product of rms values of voltage and current in a circuit is called ***apparent power*** or ***complex power.*** It is represented by S and its unit is VA.

$$S = VI$$

$$= (IZ)\, I \qquad \{\because V = IZ\}$$

$$= I^2 Z$$

In complex form,

$$S = P + jQ\text{, for inductive circuit.}$$

$$S = P - jQ\text{, for capacitive circuit.}$$

Where P is ***active power*** (or true power) and Q is ***reactive power.***

Magnitude of $S = \sqrt{P^2 + Q^2}$

Complex power (s) can be obtained from the product VI*.

$\therefore \quad S = VI^*$

where I* = conjugate current.

5.4. Power triangle

It is the geometrical representation of the apparent power, active power and reactive power. In an inductive load the impedance triangle is shown in fig.5.4 (a) Multiplying each side of the impedance triangle by I^2 we get the power triangle as shown in fig.5.4 (b).

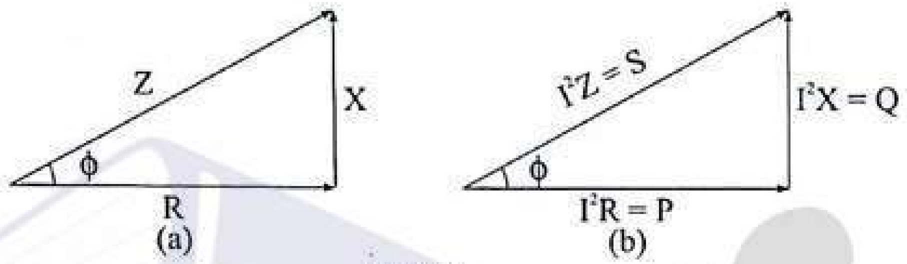

Fig 5.4

From figure, $P = S\cos\phi$ and $Q = S\sin\phi$ Also $\phi = \tan^{-1}\left(\frac{Q}{P}\right)$

5.5 Power factor

The ratio of the active power to the apparent power in an A.C. circuit is called as power factor of the circuit.

$$\text{Power factor} = \frac{active\ power\,(p)}{apparent\ power\,(s)} = \frac{VI\cos\phi}{VI} = \cos\phi$$

The power factor of an A.C. circuit is also equal to the cosine of the phase angle between the applied voltage and the circuit current.

A circuit in which the current lags the voltage (i.e. in inductive circuit) is said to have a lagging power factor. Similarly a circuit in which the current leads the voltage (i e in capacitive circuit) is said to have a leading power factor.

From impedance triangle, power factor is also defined as the ratio of resistance (R) to impedance (Z) of the circuit.

$$\therefore \quad \text{Power factor} = \cos\phi = \frac{active\ power}{apparent\ power} = \frac{R}{Z}$$

For a purely resistive circuit, power factor = cos 0 = 1

For a purely inductive circuit, power factor = cos 90 = 0

For a purely capacitive circuit, power factor = cos (-90) = 0

The power factor can never greater than unity since the maximum value of cos ϕ is unity. Power consumed in a purely resistive circuit = $= VI\cos 0 = VI$

Power consumed in a purely inductive circuit = VI cos 90 = 0

Power consumed in a purely capacitive circuit = VI cos (-90) = 0

Hence, we can conclude that the power is consumed only in the resistor and there is no power consumption in either pure inductor or pure capacitor.

Example 5.1 : Two impedances Z_A and Z_B take the following currents : $I_A = 10\angle 65^0 A$,. $I_B = 8\angle -25^0 A$. Supply voltage being 24 V taking it to be the reference phasor, find the amount of complex power drawn from the supply.

Solution :

As currents I_A and I_B are different, So Z_A and Z_B are connected in parallel.

$$I_A = 10\angle 65^0 A = (4\cdot 226 + j9\cdot 06)A$$

$$I_B = 8\angle -25^0 A = (7\cdot 25 - j3\cdot 38)A$$

$\therefore$ Total current in the circuit, $I = I_A + I_B$

$$= 4\cdot 226 + j9\cdot 06 + 7\cdot 25 - j3\cdot 38$$

$$= (11\cdot 476 + j5\cdot 68)A = 12\cdot 8\angle 26\cdot 33 A$$

$$\therefore \quad I^* = (11\cdot 476 - j5\cdot 68)A = 12\cdot 8\angle -26\cdot 33 A$$

Given supply voltage taken as reference phasor.

$$\therefore \quad V = 24\angle 0$$

Thus complex power, $S = VI^*$

$$= 24\angle 0 \times 12\angle -26\cdot 33$$

$$= 288\angle -26\cdot 33$$

$$= (258\cdot 12 - j127\cdot 72)VA$$

Example 5.2 : The voltages across two series connected circuit elements are $V_1 = 50\sin\omega t$ and $V_2 = 20\sin(\omega t - 45)V$. If the circuit current is (2+j4) A, find the complex power of the circuit.

Solution :

$$V_1 = \frac{50}{\sqrt{2}}\angle 0^0 = (35\cdot 36 + j0)V$$

$$V_2 = \frac{20}{\sqrt{2}}\angle -45 = (9\cdot 998 - j9\cdot 998)V$$

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Net voltage $V = V_1 + V_2 = 35\cdot36 + j0 + 9\cdot998 - j9\cdot998$

$$= 45\cdot358 - j9\cdot998$$

$$= 46\cdot446\angle -12\cdot43$$

The circuit current is $I = (2 + j4)A$

$\therefore \quad I^* = 2 - j4 = 4\cdot47\angle -63\cdot43$ A

Thus the complex power $S = VI^*$

$$= 46\cdot446\angle -12\cdot43 \times 4\cdot47\angle -63\cdot43$$

$$= 207\cdot61\angle -75\cdot86$$

$$= (50\cdot717 - j201\cdot319)VA$$

Example 5.3 : In an A.C. circuit $V = 220\sin(\omega t + 45^0)$ while $i = 2\sin(\omega t - 45^0)$. Find true power.

Solution : $V = \dfrac{220}{\sqrt{2}}\angle 45 = 155\cdot58\angle 45V$

$$I = \frac{2}{\sqrt{2}}\angle -45 = 1\cdot414\angle -45A$$

$$I^* = 1.414\angle 45A$$

Complex Power, $S = VI^* = 155\cdot58\angle 45 \times 1\cdot414\angle 45$

$$= 219\cdot99\angle 90$$

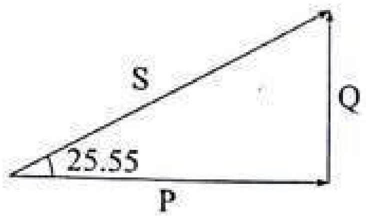
$$= (0 + j219\cdot99)VA$$

Thus true power = 0 watt and reactive power = 219.99 VAR

Example 5.4 : An impedance $Z_B = (2 + j3)\Omega$ is connected in parallel with another impedance $Z_B = (1 - j2)\Omega$ If the input reactive power is 100 VAR, what is the total active power ?

Solution : $Z = \dfrac{Z_A Z_B}{Z_A + Z_B} = \dfrac{(2+j3)(1-j2)}{2+j3+1-j2} = \dfrac{8-j}{3+j} = 2\cdot549\angle -25\cdot55$ ohm.

$\therefore \quad \tan\phi = \dfrac{Q}{p} = \dfrac{\textit{reactive power}}{\textit{active power}}$



$\Rightarrow \textit{Active power} = \dfrac{\textit{reactive power}}{\tan\phi} = \dfrac{100}{\tan 25\cdot55} = 209.18$ watt.

5.6 Power factor Improvement

Power factor is defined as the ratio of active power (p) to apparent power (s).

$$\text{Power factor} = \frac{P}{S}$$

For sinusoidal wave, power factor is the cosine of the phase angle (ϕ) between rms voltage and rms current.

$\therefore$ Power factor = $\cos\phi$

Active power $P = VI\cos\phi$

$$\Rightarrow I = \frac{P}{V\cos\phi}$$

If V is constant, the current I taken by the load varies inversely as the load power factor $\cos\phi$. Thus, a given load takes more current at a low power factor. Low power factor is due to inductive loads.

The current in an inductive load lags behind the voltage. So the power factor is lagging. In order to improve the power factor, some device taking leading current should be connected in parallel with the load. The power factor may be improved by two methods.

(i) By using static capacitors.

(ii) By using synchronous motors.

In this section, we use first method i.e. by static capacitors.

Power factor correction by static capacitors

Let us consider an inductive load connected to an A.C. supply as shown in fig. 5.5.

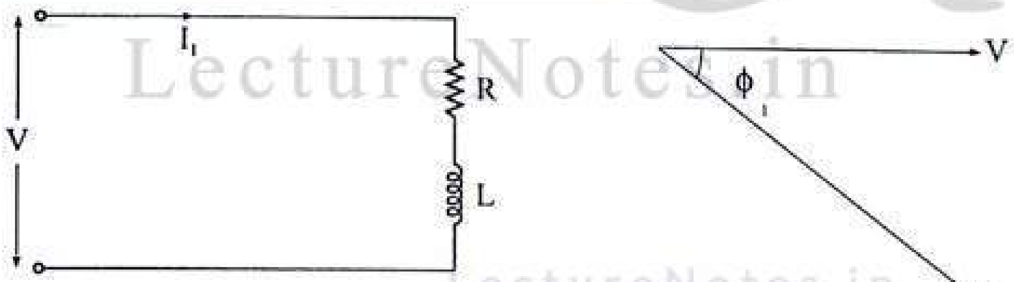


Fig 5.5

Let V = supply voltage

I_1 = load current.

ϕ_1 = phase angle between V and I_1

$\cos\phi_1$ = original power factor.

Let a capacitor C be placed in parallel with the load. It will take a leading current I_C from the supply. The circuit and phasor diagrams are shown in fig. 5.6.

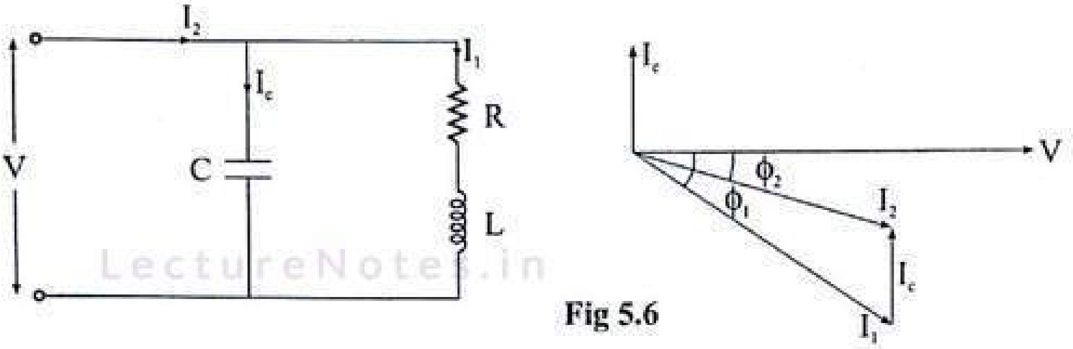


Fig 5.6

The total current I_2 drawn from the supply will be equal to the phasor sum of I_1 and I_C.

$$\therefore \qquad I_2 = I_1 + I_C$$

The phase angle of I_2 is ϕ_2. It is seen from the phasor diagram that ϕ_2 is less than ϕ_1 and hence $\cos\phi_2$ is greater than $\cos\phi_1$. In other words, the power factor is improved from $\cos\phi_1$ to $\cos\phi_2$.

Example 5.5 : Compute the average and instantaneous power dissipated by the load of fig 5.7. $R = 4\Omega$, $L = 8mH$ and $\omega = 377\frac{rad}{sec}$

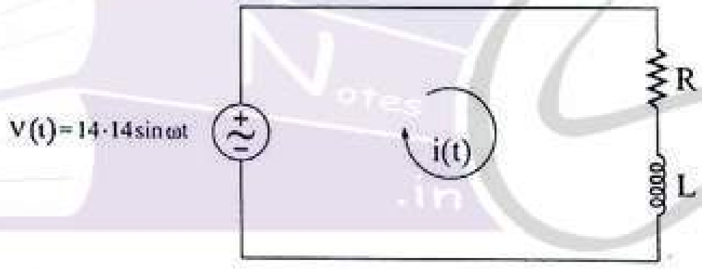


Fig 5.7

Solution : $V(t) = 14\cdot 14\sin\omega t = 14\cdot 14\cos\left(\omega t - \frac{\pi}{2}\right)$

In phasor form the rms voltage will be

$$V_{rms} = \frac{V_m}{\sqrt{2}}\angle -90^0 = \frac{14\cdot 14}{\sqrt{2}}\angle -90^0 = 10\angle -90^0 \text{ volt.}$$

$$Z = R + j\omega L = 4 + j(377)\left(8\times 10^{-3}\right) = 4 + j3 = 5\angle 36\cdot 86^0\,\Omega$$

Phasor form of rms current $I_{rms} = \frac{V_{rms}}{Z}$

$$= \frac{10\angle -90}{5\angle 36\cdot 86^0} = 2\angle\text{-}126\cdot 83^0 A$$

Instantaneous current $i(t) = i_m \cos(377t - 126\cdot83) = 2\sqrt{2}\cos(377t - 126\cdot83)\,A$

Power factor $= \cos\phi = \cos 36\cdot86 = 0\cdot8$

Average power $= V_{rms} I_{rms} \cos\phi$

= 10 x 2 x 0.8 = 16 watt.

Instantaneous Power, $P(t) = V(t)\,i(t)$

$= (14\cdot14\sin 377t)\,2\sqrt{2}\cos\left(377t - 126\cdot83^0\right)$ watt.

Example 5.6 : Compute the average power dissipated by the load of fig. 5.8.

$V_{rms} = 110\angle 0,\quad R_s = 2\Omega, \omega = 377\frac{rad}{sec},\quad R_L = 16\Omega$ and $C = 100\mu F$

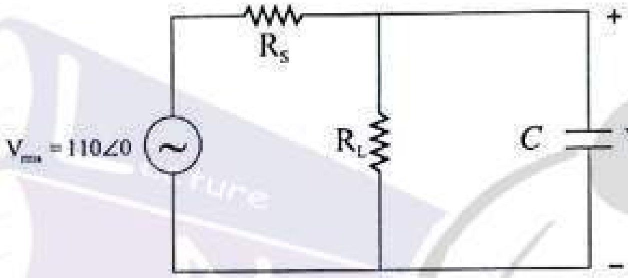


Fig 5.8

Solution : Here load R_L and C are connected in parallel. So impedance of load is,

$$Z_L = \frac{1}{\frac{1}{R_L} + \frac{1}{\frac{1}{j(377)\left(100\times10^{-6}\right)}}}$$

$$= \frac{1}{\frac{1}{16} - \frac{1}{j26\cdot52}}$$

$$= \frac{1}{0\cdot0625 + j0\cdot0377}$$

$$= \frac{1}{0\cdot073\angle 31\cdot09^0}$$

$$= 13\cdot7\angle -31\cdot09^0\,\Omega$$

V_L = Voltage across load.

According to voltage division rule,

$$V_L = 110\angle 0 \times \frac{13\cdot 7\angle -31\cdot 09^0}{2+13\cdot 7\angle -31.09^0} = 97\cdot 6\angle -3\cdot 84^0 \text{ volt.}$$

Average power dissipated by the load is given by,

$$P_{av} = V_L I_L \cos\phi = V_L\left(\frac{V_L}{Z}\right)\cos\phi = \frac{V_L^{\,2}}{Z}\cos\phi = \frac{(97.6)^2}{13\cdot 7}\cos(-31\cdot 09) = 595 \text{ watt.}$$

Example 5.7 : Compute the average power dissipated by the load of fig. 5.9

$$V_{rms} = 110\angle 0 \text{ volt, } R = 10\Omega, \quad L = 0\cdot 05H, \quad C = 470\mu F \text{ and } w = 377\frac{rad}{sec}$$

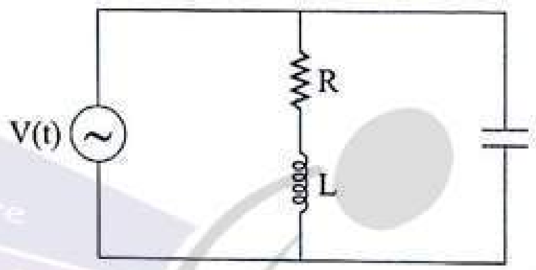


Fig 5.9

Solution : Load impedance

$$Z_L = (R + j\omega L) \| \frac{1}{j\omega c}$$

$$= \frac{(R + j\omega l)\times\frac{1}{j\omega c}}{R + j\omega L + \frac{1}{j\omega c}}$$

$$= 1\cdot 16 - j7\cdot 18 = 7\cdot 27\angle -80\cdot 82\Omega$$

Average Power $P_{av} = \frac{V_{rms}^{\,2}}{Z_L}\cos\phi$

$$= \frac{(110)^2}{7\cdot 27}\cos(-80\cdot 82)$$

$$= 265\cdot 52 \text{ watt.}$$

Example 5.8 : Calculate real and reactive power for the load of fig. 5.10

$V(t) = 100\cos(\omega t + 0\cdot 262)$ volt,

$i(t) = 2\cos(\omega t - 0\cdot 262)\,A$

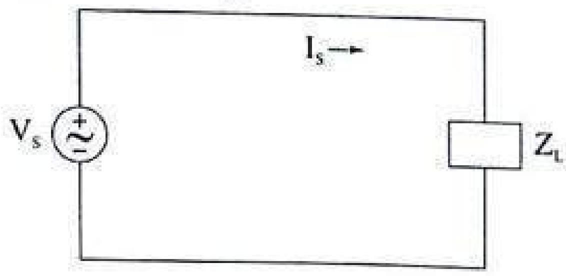


Fig 5.10

Solution : Rms voltage in phasor form is,

$$V_{rms} = \frac{V_m}{\sqrt{2}} \angle\theta = \frac{100}{\sqrt{2}} \angle 0\cdot 262 \text{ volt.}$$

Rms current in phasor form is,

$$I_{rms} = \frac{I_m}{\sqrt{2}} \angle\theta = \frac{2}{\sqrt{2}} \angle -0\cdot 262 A$$

Impedance of the circuit is given by,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{\frac{100}{\sqrt{2}}\angle 0\cdot 262}{\frac{2}{\sqrt{2}}\angle -0\cdot 262} = 50\angle 0\cdot 524\,\Omega$$

$$= 50\angle 30\cdot 03^0\,\Omega$$

Phase difference between V_{rms} and I_{rms} is $\phi = 30\cdot 03^0$

$$\therefore \quad \text{Real power } (P) = |V_{rms}||I_{rms}|\cos\phi = \frac{100}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos 30\cdot 03^0$$

$$= 86.57 \text{ watt.}$$

Reactive Power $(Q) = |V_{rms}||I_{rms}|\sin\phi$

$$= \frac{100}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \sin 30\cdot 03^0$$

$$= 50 \text{ VAR.}$$

Now we apply the definition of complex power to repeat the same calculation.

$$S = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \frac{100}{\sqrt{2}}\angle 0\cdot 262 \times \frac{2}{\sqrt{2}}\angle -(-0\cdot 262)$$

$$= 100\angle 0\cdot 524$$

$$= 100\angle 30\cdot 03^0$$

$$= 86\cdot 6 + j50 \text{ VA}$$

Therefore P = 86.6 watt and Q = 50 VAR

Example 5.9 : Calculate real and reactive power for the load of fig. 5.11

$$V_{rms} = 110\angle 0 \text{ volt,}$$

$$R_s = 2\Omega, \quad \omega = 377\frac{rad}{sec}$$

$$R_L = 5\Omega \text{ and } C = 2000\mu F$$

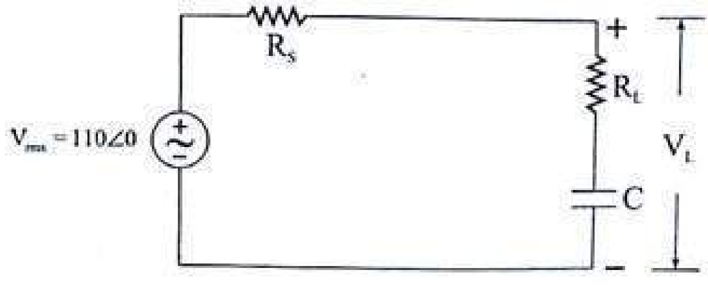


Fig 5.11

Solution : Impedance of load,

$$Z_L = R_L + \frac{1}{j\omega c} = 5 + \frac{1}{j(377)\left(2000\times10^{-6}\right)}$$

$$= 5 - j1\cdot326 = 5\cdot17\angle-14\cdot85^0\,\Omega$$

According to voltage division rule, voltage across load is given by,

$$V_L = 110\angle0 \times \frac{5\cdot17\angle-14\cdot85}{2+5-j1\cdot326} = \frac{568\cdot7\angle-14\cdot85}{7\cdot1244\angle-10\cdot72}$$

$$= 79\cdot82\angle-4\cdot13^0 \text{ volt}$$

Current through load is given by,

$$I_L = \frac{V_L}{Z_L} = \frac{79\cdot82\angle-4\cdot13^0}{5\cdot17\angle-14\cdot85} = 15\cdot439\angle10\cdot72^0\,A$$

Conjugate of $I_L = I_L^* = 15\cdot439\angle-10\cdot72^0\,A$

∴ Complex Power, $S = V_L I_L^* = 79\cdot82\angle-4\cdot13^0 \times 15\cdot439\angle-10\cdot72^0$

$$= 1232\cdot34\angle-14\cdot85^0$$

$$= 1191\cdot17 - j315\cdot835 \text{ VA}$$

Real Power, P = 1191.17 watt

Reactive Power, Q = –315.835 VAR

Example 5.10 : Find the reactive and real power for the load of fig 5.12.

$\omega = 377\frac{rad}{\sec}$, $V_{rms} = 60\angle0$ volt,

$R = 3\Omega$

$jX_L = j9\Omega$ and

$jX_C = -j5\Omega$

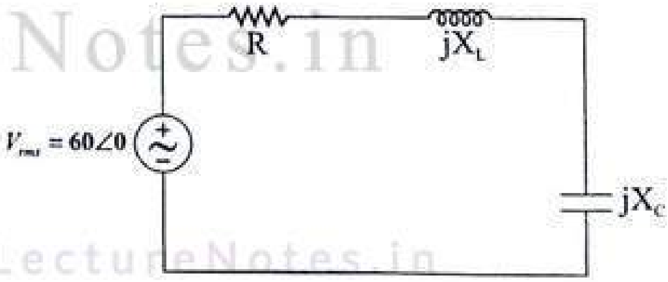


Fig 5.12

Solution : Load impedance $Z_L = R + jX_L - jX_C$

$$= 3 + j9 - j5$$

$$= 3 + j4$$

$$= 5\angle53\cdot13^0\,\Omega$$

Current through load, $I_L = \frac{V}{Z} = \frac{60\angle 0}{5\angle 53\cdot 13} = 12\angle -53\cdot 13^0 A$

Conjugate of $I_L = I_L^* = 12\angle 53\cdot 13^0 A$

Complex Power $S = VI_L^* = 60\angle 0 \times 12\angle 53\cdot 13^0$

$= 720\angle 53\cdot 13^0$

$= 432 + j576$ VA

Real power, P = 432 watt.

Reactive power, Q = 576 VAR.

Total reactive power must be the sum of the reactive powers in each of elements.

We can write $Q = Q_C + Q_L$

$$Q_C = I_L^2 X_C = (12)^2(-5) = -720 \text{ VAR}$$

$$Q_L = I_L^2 X_L = (12)^2(9) = 1296 \text{ VAR}$$

$$\therefore\ Q = Q_L + Q_C = 1296 - 720 = 576 \text{ VAR}$$

Example 5.11 : The heating element in a soldering iron has a resistance of 30Ω. Find the average power dissipated in the soldering iron if it is connected to a voltage source of 117 V rms.

Solution : The power dissipated in the soldering iron is,

$$P = \frac{V^2}{R} = \frac{V_{rms}^2}{R} = \frac{(117)^2}{30} = 456\cdot 3 \text{ watt.}$$

Example 5.12 : A coffee maker has a rated power of 1000 w at 240 V. Find the resistance of the heating element.

$$P = \frac{V_{rms}^2}{R}$$

$$\Rightarrow R = \frac{V_{rms}^2}{P} = \frac{(240)^2}{1000} = 57\cdot 6\Omega$$

Example 5.13 : A current source $i(t)$ is connected to a 50Ω resistor. Find the average power delivered to the resistor, given that $i(t)$ is,

a) 5 cos 50t A

b) 5 cos (50t - 45°) A

c) 5 cos 50t - 2 cos (50t - 0.873) A

d) 5 cos 50t - 2 A.

Solution : The average power can be expressed as,

$$P_{av} = V_{rms} I_{rms} \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = \frac{V_m I_m}{2} \cos\phi$$

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Here, $\phi = 0^0$ (due to resistive circuit)

So $P_{av} = \frac{1}{2}V_m I_m \cos 0 = \frac{1}{2}V_m I_m = \frac{1}{2}(I_m R)I_m = \frac{1}{2}I_m^2 R$

(a) $P_{av} = \frac{1}{2}(5)^2(50) = 625$ watt

(b) $P_{av} = \frac{1}{2}(5)^2(50) = 625$ watt.

(c) By using phasor techniques,

$\mathbf{I} = 5\angle 0^0 - 2\angle -0\cdot 873$

$= 5\angle 0^0 - 2\angle -50\cdot 04^0$ $\left[\because 0\cdot 873\, radian = \frac{180}{\pi}\times 0\cdot 873\, degree\right]$

$= 5 - 1\cdot 2856 + j1\cdot 5321$

$= 3\cdot 7144 + j1\cdot 5321$

$= 4\cdot 0180\angle 22\cdot 41^0\, A$

In time domain form $i(t) = 4\cdot 0180\cos\left(50t + 22\cdot 41^0\right)A$

$\therefore\ P_{av} = \frac{1}{2}(4\cdot 0180)^2(50) = 403\cdot 6$ watt

(d) $i(t) = 5\cos 50t - 2$

$v(t) = Ri(t) = 50[5\cos 50t - 2] = 250\cos 50t - 100$

Instantaneous power $p(t) = v(t)\,i(t)$

$= [250\cos 50t - 100][5\cos 50t - 2]$

$= 1250\cos^2 50t - 1000\cos 50t + 200$

$= 1250\left[\frac{1+\cos 100t}{2}\right] - 1000\cos 50t + 200$

$= 625(1+\cos 100t) - 1000\cos 50t + 200$

$= 625 + 625\cos 100t - 1000\cos 50t + 200$

Therefore average power is,

$P_{av} = 625 + 200 = 825$ watt

Example 5.14 : Find the rms value of each of the following periodic currents,

a) Cos 450t + 2 cos 450t

b) Cos 5t + sin5t

c) Cos 450t+2

d) $\cos 5t + \cos\left(5t + \frac{\pi}{3}\right)$

e) $\cos 200t + \cos 400t$

Solution :

a) $i(t) = \cos 450t + 2\cos 450t = 3\cos 450t$

$$\therefore\ I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2\cdot 1213\,A$$

b) Using phasor techniques,

$$I = \cos 5t + \cos(5t - 90) = 1\angle 0^0 + 1\angle -90^0$$

$$= 1 - j$$

$$= \sqrt{2}\angle -45^0 A$$

$$\therefore\ I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1A$$

c) $I_{rms} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + (2)^2} = 2\cdot 1213\,A$

(Here 2A is D.C. and cos 450t is AC)

d) Using phasor techniques,

$$I = 1\angle 0^0 + 1\angle 60^0 = 1 + 0\cdot 5 + j0\cdot 866 = 1\cdot 732\angle 30^0 A$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1\cdot 732}{\sqrt{2}} = 1\cdot 225\,A$$

e) $I_{rms} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{2\left(\frac{1}{\sqrt{2}}\right)^2} = 1A$

Example 5.15 : A current of 4 A flows when a neon light advertisement is supplied by a 110 V rms power system. The current lags the voltage by 60^0. Find the power dissipated by the circuit and the power factor.

Solution : $P = V_{rms} I_{rms} \cos\phi = (110)(4)\cos 60 = 220\,watt.$

Power factor = cos ϕ = cos 60 = 0.5

Example 5.16 : A residential electric power monitoring system rated for 120 V rms, 60 Hz source registers power consumption of 1.2 KW, with a power factor of 0.8.

Find, a) The rms current.

b) The phase angle

c) The system impedance

d) The system resistance

Solution : a) $P = V_{rms} I_{rms} \cos\phi$

$$I_{rms} = \frac{P}{V_{rms}\cos\phi} = \frac{1200}{(120)(0.8)} = 12.5A$$

b) Power factor $= \cos\phi = 0.8$

$\therefore \quad \phi = \cos^{-1}(0.8) = 36\cdot87^0$

c) The impedance Z is,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{120}{12\cdot5} = 9\cdot6\Omega$$

d) The resistance R is,

$$R = Z\cos\phi = (9\cdot6)(0\cdot8) = 7\cdot68\Omega$$

Example 5.17 : A drilling machine is driven by a single phase induction machine connected to a 110 V rms supply. Assume that the machining operation requires 1KW, that the tool machine has 90 percent efficiency and that the supply current is 14 A rms with a power factor of 0.8. Find the AC machine efficiency.

Solution : $\text{Efficiency} = \frac{\text{output}}{\text{Input}}$

$$\Rightarrow input = \frac{output}{efficiency} = \frac{1000}{0\cdot9} = 1111 \text{ watt.}$$

$$\text{Efficiency of AC machine } (\eta) = \frac{1111}{VI\cos\phi} = \frac{1111}{(110)(14)(0\cdot8)} = 0\cdot9$$

Example 5.18 : For the following numerical values, determine the average power P, the reactive power Q and the complex power S of the circuit shown in fig. 5.13. Note, phasor quantities are rms.

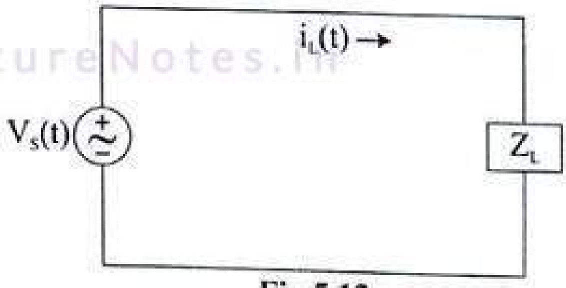

Fig 5.13

a) $V_s(t) = 450\cos 377t$ volt

$i_L(t) = 50\cos(377t - 0\cdot349)A$

b) $V_{rms} = 140\angle 0$ volt

$I_L rms = 5\cdot85\angle{-\pi/6}\, A$

c) $V_s rms = 50\angle 0$ volt

$I_L rms = 19\cdot 2\angle 0\cdot 8A$

d) $V_s rms = 740\angle -\pi/4$ volt

$I_L rms = 10\cdot 8\angle -1\cdot 5A$

Solution : (a) $P = V_{rms} I_{rms} \cos\phi = \frac{450}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \cos 20^0$ =10571.5 watt

$$\left(0\cdot 349\, radian = \frac{180}{\pi} \times 0\cdot 349 \deg ree = 20^0\right)$$

$$Q = V_{rms} I_{rms} \sin\phi = \frac{450}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \sin 20^0 = 3847.72\, VAR$$

In phasor form current $I_{rms} = \frac{50}{\sqrt{2}} \angle -0\cdot 349 = \frac{50}{\sqrt{2}} \angle -20^0$ A

Conjugate, $I^*_{rms} = \frac{50}{\sqrt{2}} \angle 20^0$

$$S = V_{rms} I^*_{rms} = \frac{450}{\sqrt{2}} \cdot \frac{50}{\sqrt{2}} \angle 20^0 = 11250\angle 20^0\, VA$$

b) $P = V_{rms} I_{rms} \cos\phi = (140)(5\cdot 85)\cos\left(-30^0\right) = 709\cdot 3$ watt

$Q = V_{rms} I_{rms} \sin\phi = (140)(5\cdot 85)\sin\left(-30^0\right) = -409\cdot 5$ VAR

$S = V_{rms} I^*_{rms} = 140\angle 0 \times 5\cdot 85\angle \pi/6 = 819\angle 30^0$

c) $P = V_{rms} I_{rms} \cos\phi = (50)(192)\cos 45\cdot 8 = 668\cdot 8$ watt

$$\left(0\cdot 8\, radian = \frac{100}{\pi} \times 0\cdot 8 \deg ree = 45\cdot 8^0\right)$$

$Q = V_{rms} I_{rms} \sin\phi = (50)(19\cdot 2)\sin 45\cdot 8 = 688\cdot 2$ VAR

Conjugate current $= I^* = 19\cdot 2\angle -0\cdot 8 = 19\cdot 2\angle -45\cdot 8^0 A$

$S = V_{rms} I^*_{rms} = 50\angle 0 \times 19\cdot 2\angle -45\cdot 8 = 960\angle -45\cdot 8$ VA

d) $P = V_{rms} I_{rms} \cos\phi = (740)(10\cdot 8)\cos\left(-85\cdot 9^0 + 45^0\right) = 6040\cdot 8$ watt

$$\left(1\cdot 5\, radian = \frac{180}{\pi} \times 1\cdot 5 = 85\cdot 9^0\right)$$

$$Q = V_{rms} I_{rms} \sin\phi = (740)(10\cdot 8)\sin\left(-85\cdot 9^0 + 45^0\right) = -5232\cdot 7 \text{ VAR}$$

$$\text{Conjugate Current} = I^* = 10\cdot 8\angle 1\cdot 5 = 10\cdot 8\angle 85\cdot 9^0$$

$$S = V_{rms} I^*_{rms} = 740\angle -45 \times 10\cdot 8\angle 85\cdot 9^0 = 7992\angle 40\cdot 9^0 VA$$

Example 5.19 : For the circuit of fig.5.14 determine the power factor for the load and state whether it is leading or lagging for the following conditions :

a) $V_s(t) = 780\cos(\omega t + 1\cdot 2)$ volt

$i_L(t) = 90\cos\left(\omega t + \frac{\pi}{2}\right) A$

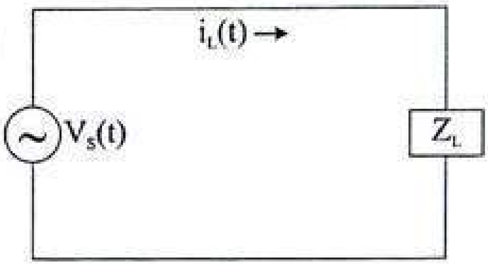

Fig 5.14

b) $V_s(t) = 39\cos\left(\omega t + \frac{\pi}{6}\right)$ volt.

$i_L(t) = 12\cos(\omega t - 0\cdot 185) A$

c) $V_s(t) = 104\cos\omega t$ volt

$i_L(t) = 48\cdot 7\sin(\omega t + 2\cdot 74) A$

d) $Z_L = 12 + j8\Omega$

Solution : Power factor = $\cos\phi$

where ϕ = angle between V_{rms} and $I_{rms.}$

$= \phi_i - \phi_v$

= phase of current - phase of voltage.

(a) $V_s(t) = 780\cos(\omega t + 1\cdot 2) = 780\cos\left(\omega t + 68\cdot 78^0\right)$ volt.

$i_L(t) = 90\cos\left(\omega t + \frac{\pi}{2}\right) A$

$\therefore$ *Power factor* $= \cos\phi = \cos\left(\phi_i - \phi_v\right) = \cos\left(90^0 - 68\cdot 78^0\right) = 0.93$ leading.

(b) $V_s(t) = 39\cos\left(\omega t + \frac{\pi}{6}\right)$ volt.

$i_L(t) = 12\cos(\omega t - 0\cdot 185) = 12\cos\left(\omega t - 10\cdot 6^0\right) A$

$\therefore$ Power factor $= \cos\phi = \cos\left(\phi_2 - \phi_v\right) = \cos\left(-10\cdot 6^0 - 30^0\right) = 0\cdot 759$ logging.

(c) $V_s(t) = 104\cos\omega t$ volt.

$i_L(t) = 48\cdot 7\sin(\omega t + 2\cdot 74) = 48\cdot 7\sin\left(\omega t + 157^0\right)$

$= 48\cdot 7\cos\left(\omega t + 157^0 - 90^0\right) = 48\cdot 7\cos\left(\omega t + 67^0\right)$

$\therefore$ Power factor $= \cos\phi = \cos\left(\phi_i - \phi_v\right) = \cos(67 - 0) = 0391$ *leading*

d) $Z_L = 12 + j8\Omega = 14\cdot 42\angle 33\cdot 7^0\,\Omega$

Power factor $= \cos\phi = \cos 33\cdot 7^0 = 0\cdot 832\, lagging$

Example 5.20 : For the circuit of fig. 5.15 determine whether the load is capacitive or inductive for the circuit shown if

a) pf = 0.48 (lead)

b) pf = 0.17 (lead)

c) $V_s(t) = 18\cos\omega t$

$i_L(t) = 1\cdot 8\sin\omega t$

d) $V_s(t) = 8\cdot 3\cos\left(\omega t - \frac{\pi}{6}\right)$

$i_L(t) = 0\cdot 6\cos\left(\omega t - \frac{\pi}{6}\right)$

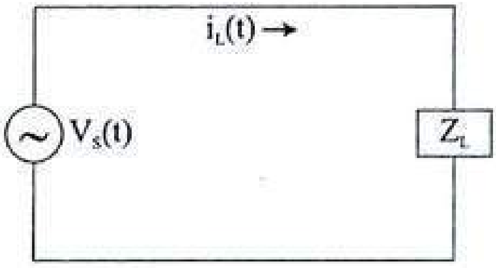

Fig 5.15

Solution : a) capacitive

b) capacitive

c) $i_L(t) = 1\cdot 8\cos\left(\omega t - 90^0\right)$, inductive

d) phase difference is zero. So it is resistive.

Example 5.21 : Find the real and reactive power supplied by the source in the circuit shown in fig 5.16. Repeat if the frequency is increased by a factor 3.

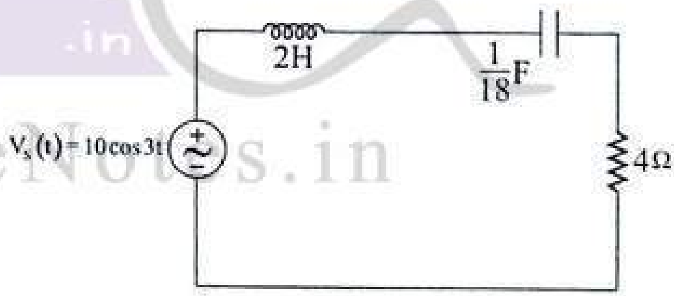

Fig 5.16

Solution : Here $\omega = 3\frac{rad}{\sec}$

(a) Impedance

$$Z = j\omega L - j\frac{1}{\omega c} + R = j6 - j6 + 4 = 4 + j0$$

$V_s(t) = 10\cos 3t$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}$ volt.

Current through the circuit is given by,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{10/\sqrt{2}}{4} = 1 \cdot 77\,A$$

Real Power, $P = (1 \cdot 77)^2 (4) = 12 \cdot 53$ watt

Reactive Power, $Q = (1 \cdot 77)^2 (X) = 0$ $\{\because X = reactamce = 0\}$

(b) In this case $\omega = 9 \frac{rad}{sec}$

Impedance, $Z = j\omega L - j\frac{1}{\omega c} + R$

$$= j(9)(2) - j\frac{1}{(9)\left(\frac{1}{18}\right)} + 4$$

$$= j18 - j2 + 4$$

$$= 4 + j16$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{10/\sqrt{2}\angle 0}{16 \cdot 44\angle 75 \cdot 96^0} = 0 \cdot 42\,A$$

$$P = (0 \cdot 42)^2\, 4 = 0 \cdot 7056 \text{ watt}$$

$$Q = I^2 X = 2 \cdot 82 \text{ VAR}$$

Example 5.22 : The load Z_L in the circuit of fig. 5.17 consists of a 25Ω resistor in series with a $0 \cdot 1 \mu F$ capacitor. Assuming f = 60 Hz, find,

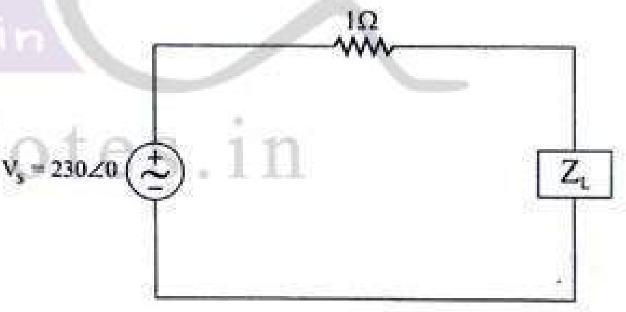

Fig 5.17

a) the source p.f.

b) the rms current I_s

c) the apparent power delivered to load.

d) the apparent power supplied by source.

e) the p.f. of load.

Solution : (a) Total resistance = 25 + 1 = 26Ω

Total impedance $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{(26)^2 + \left(\frac{1}{377 \times 0 \cdot 1 \times 10^{-6}}\right)^2}$$

$$= \sqrt{676 + 7 \cdot 036 \times 10^8}$$

$= \sqrt{703600676}$

$= 26525 \cdot 472\Omega$

$p.f = \frac{R}{Z} = \frac{26}{26525 \cdot 472} = 9 \cdot 8 \times 10^{-4} (lead)$

(b) R.M.S current $I_s = \frac{V_{rms}}{Z} = \frac{230}{26525 \cdot 472\angle 90^0}$

$= 8 \cdot 67 \times 10^{-3} \angle -90^0 A$

(c) Voltage across load $V_L = I_S Z = 8 \cdot 67 \times 10^{-3} \angle -90^0 \times 26525 \cdot 472\angle 90^0$

$= 229 \cdot 975\angle 0^0$ volt.

Apparent Power $S = V_L I_s^* = 229 \cdot 975\angle 0^0 \times 8 \cdot 67 \times 10^{-3} \angle 90^0$

$= 1 \cdot 9938 VA$

(d) Apparent Power of source $= V_s I_s^* = 230\angle 0^0 \times 8 \cdot 67 \times 10^{-3} \angle 90^0$

$= 1 \cdot 9941\angle 90^0$ VA

(e) P.f of load $= \frac{R_{load}}{Z_{load}} = \frac{25}{26525 \cdot 472} = 0 \cdot 00094$

Example 5.23 : The load Z_L in the circuit of fig 5.18 consists of a 25Ω resistor in series with a 0.1H inductor. Assuming f = 60HZ, calculate the following.

a) the apparent power supplied by source.

b) the apparent power delivered to the load.

c) the power factor of the load.

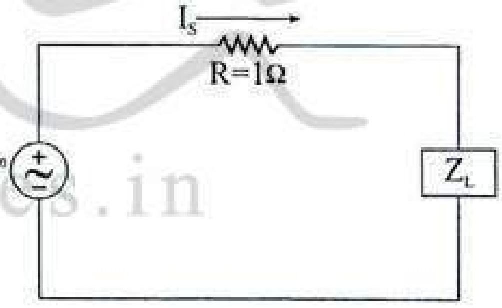

Fig 5.18

Solution : Impedance of load

$Z_L = R + j\omega L = 25 + j(377)(0 \cdot 1)$

$= 25 + j37 \cdot 7 = 45 \cdot 236\angle 56 \cdot 45^0 \Omega$

Impedance of the circuit $Z = 26 + j37 \cdot 7$

$= 45 \cdot 796\angle 55 \cdot 407^0 \Omega$

R.M.S value of current $I_s = \frac{V_s}{Z} = \frac{230\angle 0^0}{45 \cdot 796\angle 55 \cdot 407^0}$

$= 5\angle -55 \cdot 407^0 A$

(a) Apparent power supplied by source is,

$S = V_s I_s^* = 230\angle 0^0 \times 5\angle 55 \cdot 407^0$

$= 1150\angle 55 \cdot 407^0 VA$

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(b) Apparent power delivered to the load is,

$$S = V_L I_s^* = (I_s Z_L)(I_s^*)$$

$$= (5\angle -55\cdot407^0 \times 45\cdot796\angle 55\cdot407^0) \times 5\angle 55\cdot407^0$$

$$= 1144\cdot9\angle 55\cdot407^0 \text{ VA}$$

(c) $\text{P.f of load} = \dfrac{R_{load}}{Z_{load}} = \dfrac{25}{45\cdot236} = 0\cdot55$

Example 5.24 : Calculate the apparent power, real power and reactive power for the circuit shown in fig. 5.19. Draw the power triangle.

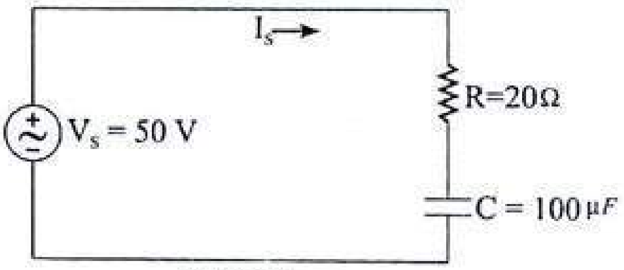


Fig 5.19

Solution : Impedance of circuit is given by,

$$Z = R - j\frac{1}{\omega c}$$

$$= 20 - j\frac{1}{377 \times 100 \times 10^{-6}}$$

$$= 20 - j26\cdot525$$

$$= 33\cdot22\angle 52\cdot983^0 \Omega$$

RMS current $I_s = \dfrac{V_{rms}}{Z} = \dfrac{50\angle 0}{33\cdot22\angle 52\cdot983^0} = 1\cdot505\angle -52\cdot98^0 A$

Apparent power $S = V_s I_s^* = 50\angle 0 \times 1\cdot505\angle 52\cdot98^0$

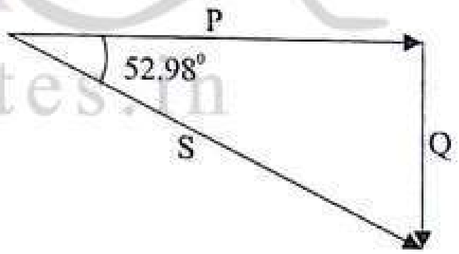
$$= 75\cdot25\angle 52\cdot98^0 VA$$

Real Power $P = S\cos\phi = 75\cdot25\cos 52\cdot983^0$

$$= 45\cdot3044 \text{ watt}$$

Reactive Power $Q = S\sin\phi = 75\cdot25\sin 52\cdot983^0$

$$= 60\cdot08 \text{ VAR}$$



Example 5.25 : Calculate the apparent power, real power and reactive power for the circuit shown in fig. 5.20. For two cases f = 0 HZ (DC) and f = 50 HZ.

Solution : (i) For the frequency of 0 Hz,

apparent power = 0

real power = 0

reactive power = 0

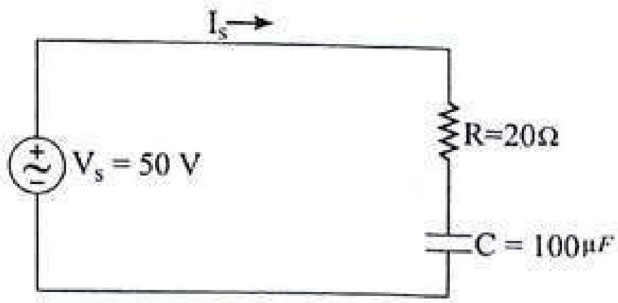


Fig 5.20

(ii) For the frequency of 50 Hz,

impedance of the circuit $Z = R - j\dfrac{1}{\omega c}$

$$= 20 - j\frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 20 - j31 \cdot 847\Omega$$

$$= 37 \cdot 6\angle -57 \cdot 87^0\,\Omega$$

RMS Current $\quad I_s = \dfrac{(V_s)_{rms}}{Z}$

$$= \frac{50\angle 0}{37 \cdot 6\angle -57 \cdot 87^0}$$

$$= 1 \cdot 329\angle 57 \cdot 87^0 A$$

Apparent power $\quad S = VI^* = 50\angle 0 \times 1 \cdot 329\angle -57 \cdot 87^0$

$$= 66 \cdot 45\angle -57 \cdot 87^0$$

$$= 35 \cdot 34 - j56 \cdot 27 \text{ VA}$$

Real power = 35.34 watt.

Reactive power = -56.27 VAR.

Example 5.26 : A single phase motor is connected as shown in fig. 5.21 to a 50 Hz network. The capacitor value is chosen to obtain unity power factor. If V = 220V, I = 20A and I_1=25 A. Find the capacitor value.

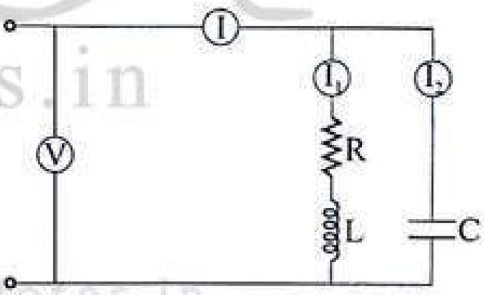


Fig 5.21

Solution : The magnitude of the current I_2 is,

$$I_2 = \sqrt{I_1^2 - I^2} = \sqrt{625 - 400} = 15A$$

Voltage source, $V = I_2 X_C = I_2\left(\dfrac{1}{\omega c}\right)$

$$\Rightarrow C = \frac{I_2}{v\omega} = \frac{15}{(220)(314)} = 217\mu F$$

Example 5.27 : If the voltage and current given below are supplied by a source to a circuit or load, determine

a) the power supplied by the source which is dissipated as heat or work in the circuit (load).

(b) the power stored in reactive components in the circuit (load).

(c) the power factor angle and power factor.

$V_{rms} = 7\angle 0\cdot 873$ volt, $I_{rms} = 13\angle -0\cdot 349 A$

Solution : Given $V_{rms} = 7\angle 0\cdot 873 = 7\angle 50^0$ volt.

$I_{rms} = 13\angle -0\cdot 349 = 13\angle -20^0 A$

a) Apparent Power $S = V_{rms} I_{rms}^*$

$$= 7\angle 50^0 \times 13\angle 20^0$$

$$= 91\angle 70^0 VA$$

$$= 31\cdot 12 + j85\cdot 51 VA$$

∴ Active Power P = 31.12 watt.

b) Reactive Power Q = 85.51 VAR

c) $\phi = \phi_I - \phi_V = -20 - 50 = -70^0$

Power factor = $\cos\phi = \cos\left(-70^0\right) = 0\cdot 342(Lag)$

The load is inductive.

Example 5.28 : Determine the time - average total power, the real power dissipated and the reactive power stored in each of the impedances in the circuit shown in fig. 5.22 if $V_{s1} = 170\angle 0$ volt, $V_{s2} = 170\angle \pi/2$ volt, $\omega = 377 \frac{rad}{sec}$, $Z_1 = 0\cdot 7\angle \pi/6\,\Omega$, $Z_2 = 1\cdot 5\angle 0\cdot 105\Omega$ and $Z_3 = 0\cdot 3 + j0\cdot 4\Omega$

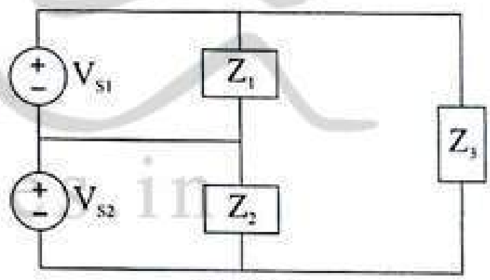


Fig 5.22

Solution : Apparent Power $S = VI^* = V\left(\frac{V}{Z^*}\right) = \frac{V^2}{Z^*}$

(i) So for impedance Z_1 apparent power is,

$$S_1 = \frac{(V_{s1})^2}{Z_1^*} = \frac{\left(\frac{170}{\sqrt{2}}\right)^2}{0\cdot 7\angle -30^0} = 20\cdot 643\angle 30^0$$

$$= 17\cdot 88 + j10\cdot 32 \text{ KVA}$$

∴ $P_1 = 17\cdot 88$ kw

$Q_1 = 1032$ KVAR

(ii) Similarly $S_2 = \frac{(V_{s2})^2}{Z_2^*} = \frac{\left(170/\sqrt{2}\right)^2}{1\cdot5\angle-7^0} = 9\cdot56 + j1\cdot17KVA = P_2 + jQ_2$

(iii) $S_3 = \frac{(V_{s1}+V_{s2})^2}{Z_3^*} = 34\cdot68 - j46\cdot24KVA = P_3 + jQ_3$

5.7 Three phase Circuits

The generator producing single phase supply called single phase generator and it has only one armature winding. But if the generator is arranged to have three separate windings displaced from each other by equal electrical angles then it is called three phase generator. A three-phase generator has three separate but identical windings that are 120^0 electrical apart and rotate in a common magnetic field. It produces three voltages of same magnitude and frequency but displaced 120^0 electrical from one another.

5.7.1 Generation of a three phase supply

In three phase generator (alternator), three coils are stationary and the field rotates as shown in fig. 5.23. The three identical coils A, B and C are symmetrically placed in such a way that the emfs induced in them are displaced 120 electrical degrees from one another. As the coils are identical and are subjected to the same rotating field, so the emfs induced in them will be of same magnitude and frequency.

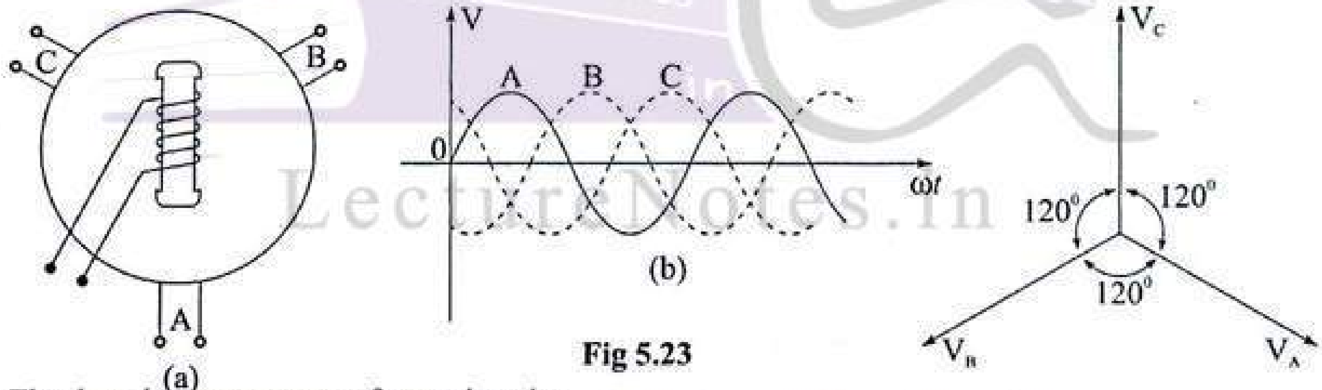


Fig 5.23

The three instantaneous emfs are given by

$$V_A = V_m \sin\omega t = \sqrt{2}V \sin\omega t$$

$$V_B = V_m \sin\left(\omega t - 120^0\right) = \sqrt{2}V \sin\left(\omega t - 120^0\right)$$

$$V_C = V_m \sin\left(\omega t - 240^0\right) = \sqrt{2}V \sin\left(\omega t - 240^0\right)$$

where ω = angular speed of rotor.

and V = r.m.s. value of emf.

In polar form, the three phase rms voltage can be written as,

$$\mathbf{V}_A = V\angle 0$$

$$\mathbf{V}_B = V\angle -120$$

$$\mathbf{V}_C = V\angle -240$$

These three emfs have same frequency f. This frequency is related to the speed of the rotor N and number of poles P of the machine and may be expressed as,

$$f = \frac{PN}{120}$$

Fig. 5.23 (b) shows the wave diagram of three emfs and fig. 5.23 (c) shows the phasor diagram.

5.7.2 Phase sequence

The order in which the voltages in three phases (or coils) reach their maximum positive values is called phase sequence. This is determined by the direction of rotation of alternator. In fig.5.24 (a), the voltage in coil A attains maximum positive value first, next coil B and then coil C. Hence the phase sequence is called A-B-C or positive sequence. If the direction of rotation of the alternator is reversed, then the order in which the three phases attain their maximum positive values would be A-C-B. or negative sequence as shown in fig.5.24 (b).

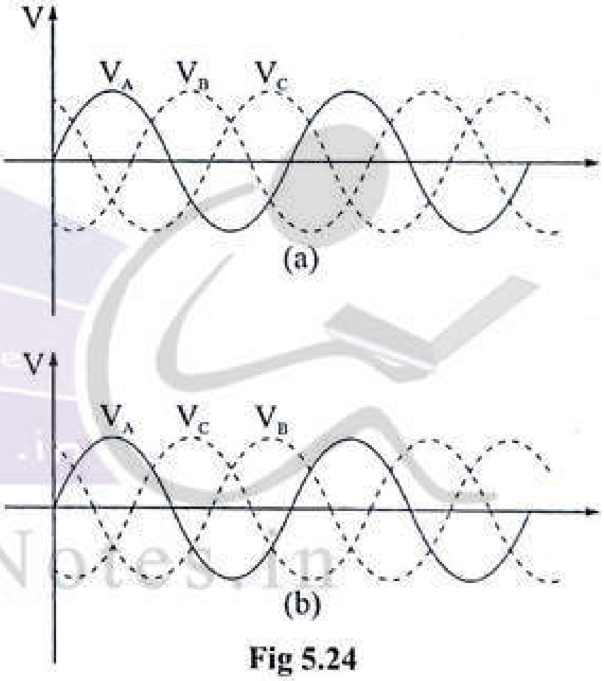


Fig 5.24

5.7.3 Inter Connection of three phases

In three phase generator (or alternator), there are the three coils or phases. Each phase has two terminals (i.e. start and finish). The terminal where current leaves the coil (or phase) called as starting terminal or simply start. The other terminal where current enters the coil is called as finishing terminal or simply finish. If a separate load is connected across each phase as shown in fig. 5.25, then six conductors are required to transmit power. This will make the whole system complicated and expensive. Hence three phases are generally interconnected which results in substantial shaving of copper. The general methods of interconnection are

(i) Star (Y) connection.

(ii) Delta (Δ) connection.

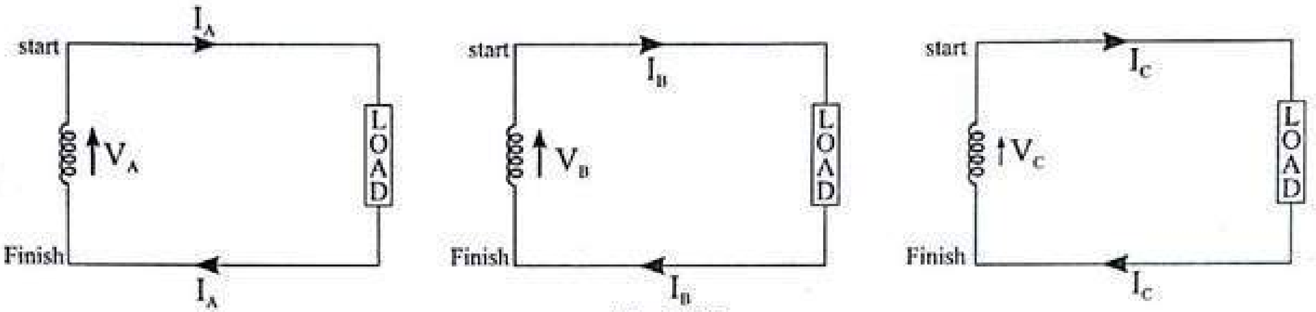


Fig 5.25

(i) Star connection

In this method of inter connection, the similar terminals say, start of three coils or finish of three coils are joined together at a point N as shown in fig. 5.26. The point N is called neutral point. A conductor is connected to N called as neutral conductor. Such an interconnected system is known as four wire, 3 phase system.

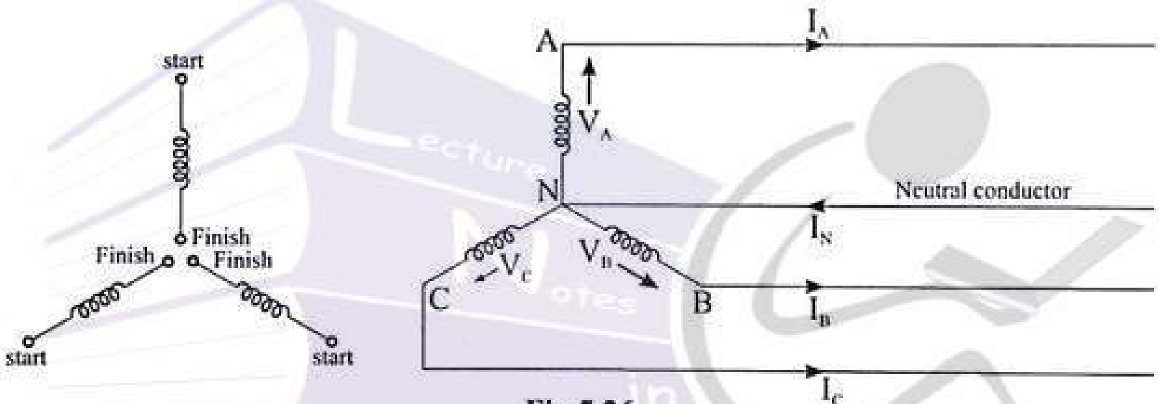


Fig 5.26

(ii) Delta connection

In this method of inter connection, the dissimilar terminals of three phases are joined together i.e. finishing terminal of one phase is connected to starting terminal of other as shown in fig. 5.27. Such an inter connected system is called 3-phase, 3 wire system.

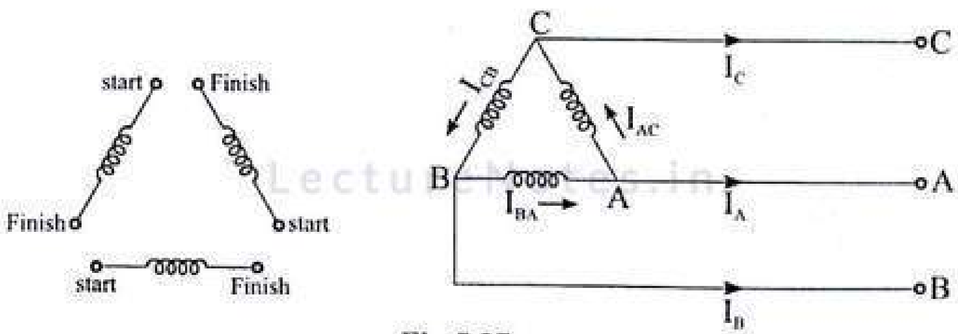
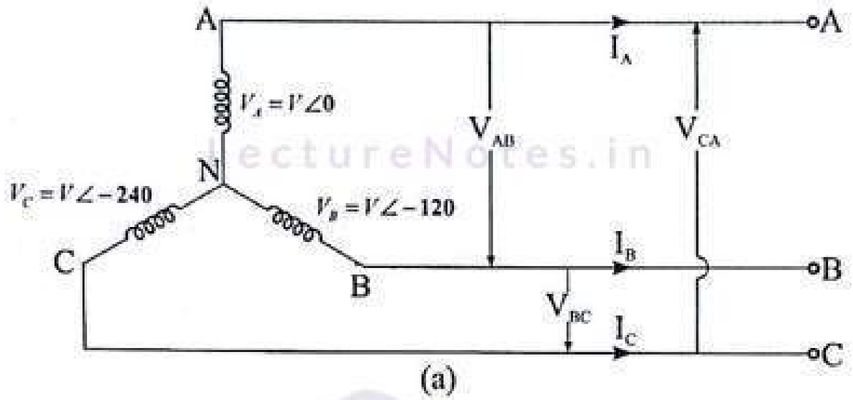


Fig 5.27

5.7.4 Star connected supply

The voltage induced in each coil (or phase) is called phase voltage (V_{ph}) and current in each phase is called phase current. Voltage between any pair of lines is called line voltage (V_L) and current flowing in each line is called line current (I_L)



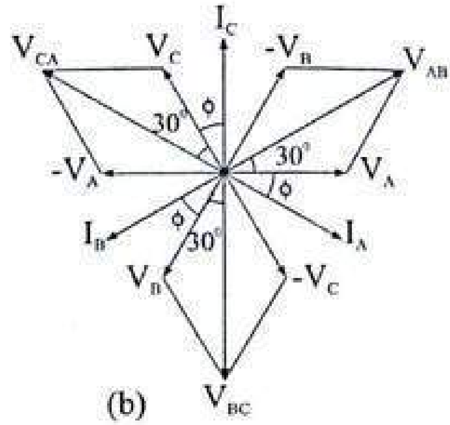


Fig 5.28

Fig. 5.28 (a) shows three phase stat connected supply has three terminals A, B and C called line terminals.

Let V_{AB} = voltage across line A and line B = line voltage = V_L

V_{BC} = voltage across line B and lice C = V_L

V_{CA} = voltage across line C and line A = V_L

V_A = voltage induced in coil A = voltage between A and neutral point N = phase voltage = V_{ph}.

V_B = voltage induced in coil B = voltage between B and N = V_{ph}

V_C = voltage induced in coil C = voltage between C and N = V_{ph}

The relationships between the line and the phase quantities may be derived from circuit diagram and phasor diagram as shown in fig. 5.28 (b). The phase voltages are,

$$\mathbf{V}_A = V\angle 0^0 = V(1+j0)$$

$$\mathbf{V}_B = V\angle -120^0 = V(-0\cdot 5 - j0\cdot 866)$$

$$\mathbf{V}_C = V\angle -240^0 = V(-0\cdot 5 + j0\cdot 866)$$

The line voltage $\mathbf{V}_{AB}$ is the phasor difference of $\mathbf{V}_A$ and $\mathbf{V}_B$.

$$\therefore \quad \mathbf{V}_{AB} = \mathbf{V}_A - \mathbf{V}_B = V\angle 0 - V\angle -120$$

$$\Rightarrow \mathbf{V}_{AB} = V(1+j0) - V(-0\cdot 5 - j0\cdot 866)$$

$$\Rightarrow \mathbf{V}_{AB} = V(1\cdot 5 + j0\cdot 866)$$

$$\Rightarrow \mathbf{V}_{AB} = \sqrt{3}V\angle 30^0 \text{..............................} (1)$$

Similarly $\mathbf{V_{BC}} = \mathbf{V_B} - \mathbf{V_C} = V\angle -120 - V\angle -240$

$= V(-0\cdot 5 - j0\cdot 866) - V(-0\cdot 5 + j0\cdot 866)$

$= \sqrt{3}\ V\ \angle -90^0$ .... (2)

and $\mathbf{V_{CA}} = \mathbf{V_C} - \mathbf{V_A} = V\angle -240 - V\angle 0$

$= V(-0\cdot 5 + j0\cdot 866) - V(1 + j0)$

$= \sqrt{3} V\ \angle 150^0$ .... (3)

It can be seen from equations (1), (2) and (3) that the system of line voltages constitutes a balanced three phase voltage system. The magnitude of line voltages is $\sqrt{3}$ times the magnitude of phase voltages.

∴ Line Voltage = $\sqrt{3} \times$ *phase voltage*

$\Rightarrow V_L = \sqrt{3} V_{ph}$

In this star connected system, the current in a line conductor is identical to that of the phase to which line conductor is connected.

∴ Line current = Phase current

$I_L = I_{ph}$

It will be noted from fig. 5.28 (b)

1) Line voltages are 120^0 apart.

2) Line voltages are 30^0 ahead of their respective phase voltages.

3) The angle between line current and corresponding line voltages is $(30 + \phi)$ with current lagging.

5.7.5 Delta connected supply

In this connection, the line voltage is equal to the phase voltage i.e. $V_L = V_{ph}$. Fig 5.29 (a) shows three phase delta connected supply has three terminals A, B and C called line terminals.

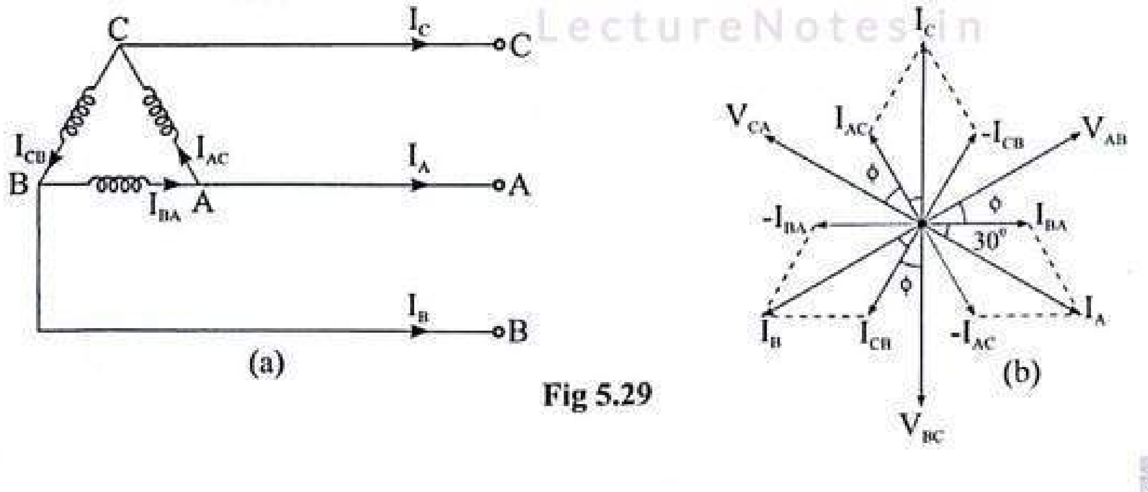


Fig 5.29

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and delivery.

The rms phase voltage phasors of three phases are given by,

$$\mathbf{V_A} = V\angle 0 = V_{ph}$$

$$\mathbf{V_B} = V\angle -120 = V_{ph}$$

$$\mathbf{V_C} = V\angle -240 = V_{ph}$$

The line voltages across the line terminals are $\mathbf{V_{AB}}$, $\mathbf{V_{BC}}$ and $\mathbf{V_{CA}}$. In delta connected supply line voltages are equal to phase voltages.

$$\therefore \quad \mathbf{V}_{AB} = \mathbf{V}_A = V\angle 0$$

$$\mathbf{V}_{BC} = \mathbf{V}_B = V\angle -120$$

and $$\mathbf{V}_{CA} = \mathbf{V}_C = V\angle -240$$

Let, I_{BA}, I_{CB} and I_{AC} are phase currents flowing in three phases. These three phase currents are balanced currents.

From fig.5.29 (b), taking $\mathbf{I}_{BA}$ as reference phasor,

$$\mathbf{I}_{BA} = I_{BA}\angle 0 = I_{ph}\angle 0 = I_{ph}(1 + j0)$$

$$\mathbf{I}_{CB} = I_{CB}\angle -120 = I_{ph}\angle -120 = I_{ph}(-0\cdot 5 - j0\cdot 866)$$

$$\mathbf{I}_{AC} = I_{AC}\angle -240 = I_{ph}\angle -240 = I_{ph}(-0\cdot 5 + j0\cdot 866)$$

Let $\mathbf{I}_A$, $\mathbf{I}_B$ and $\mathbf{I}_C$ are line currents.

Applying KCL at node A, we get

$$\mathbf{I}_{BA} = \mathbf{I}_A + \mathbf{I}_{AC}$$

$$\Rightarrow \mathbf{I}_A = \mathbf{I}_{BA} - \mathbf{I}_{AC}$$

$$\Rightarrow \mathbf{I}_A = I_{Ph}(1 + j0) - I_{ph}(-0\cdot 5 - j0\cdot 866)$$

$$\Rightarrow \mathbf{I}_A = I_{ph}(1\cdot 5 - j0\cdot 866)$$

$$\Rightarrow \mathbf{I_A} = \sqrt{3}\, I_{ph}\angle -30 \quad \text{.................. (1)}$$

Similarly $\mathbf{I_B} = \sqrt{3}\, I_{ph}\angle -150^0$ .... (2) and $\mathbf{I_C} = \sqrt{3}\, I_{ph}\angle 90^0$ .... (3)

It can be seen from equations (1), (2) and (3) that magnitude of line currents is $\sqrt{3}$ times the magnitude of phase currents.

$$\therefore \quad \text{Line current} = \sqrt{3} \times \text{phase current}$$

$$\Rightarrow I_L = \sqrt{3}\, I_{ph}$$

It will be noted from fig. 5.29 (b),

1) Line currents are 120^0 apart.

2) Line currents are 30^0 behind the respective phase currents.

3) The angle between line currents and corresponding line voltages is $(30+\phi)$ with the current lagging.

Note :

* A balanced system is one in which the voltages in three phases are equal in magnitude and differ in phase from one another by 120^0 angles. The currents in three phases are equal in magnitude and also differ in phase from one another by 120^0 angles.

* An unbalanced three phase system is one in which the phases are not balanced. This may happen due to unequal magnitudes of the phase quantities, unequal phase differences between the phase quantities or both. This may also be caused by unbalanced loading, unbalanced supply voltage or both.

* A three phase balanced load is that in which the loads connected across three phases are identical.

5.7.6 Power in Three Phase system

Power in a single phase is $p = V_{ph} I_{ph} \cos\phi$

Power in three phase is $P = 3V_{ph} I_{ph} \cos\phi$

where ϕ is the angle between phase voltage (V_{ph}) and phase current (I_{ph})

(i) In a star connected system, the line voltage $V_L = \sqrt{3}V_{ph}$ and $I_L = I_{ph}$

So power in three phase star connected system is given by,

$$P = 3V_{ph} I_{ph} \cos\phi$$

$$= 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

(ii) In a delta connected system, $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$

So power in three phase delta connected system is given by,

$$P = 3V_{ph} I_{ph} \cos\phi$$

$$= 3V_L \cdot \frac{I_L}{\sqrt{3}} . \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

Hence, it can be concluded that power in a balanced three phase network (star or delta) is given by $P = \sqrt{3} V_L I_L \cos\phi$

where ϕ is the angle between V_{ph} and I_{ph} not between V_L and I_L

5.7.7 Reactive and Apparent Power

Reactive power in a single phase is $= V_{ph} I_{ph} \sin\phi$

Reactive power in three phase is $Q = 3V_{ph} I_{ph} \sin\phi$

where ϕ is the angle between V_{ph} and I_{ph}

(i) For star connection, $V_L = \sqrt{3} V_{ph}$ and $I_L = I_{ph}$

Reactive Power $Q = 3V_{ph} I_{ph} \sin\phi = 3\frac{V_L}{\sqrt{3}} I_L \sin\phi = \sqrt{3} V_L I_L \sin\phi$

(ii) For delta connection, $V_L = V_{ph}$ and $I_L = \sqrt{3} I_{ph}$

Reactive Power $Q = 3V_{ph} I_{ph} \sin\phi = 3V_L \cdot \frac{I_L}{\sqrt{3}} \sin\phi = \sqrt{3} V_L I_L \sin\phi$

Hence reactive power in a balanced three phase network (star or delta) is given by,

$Q = \sqrt{3} V_L I_L \sin\phi$

For a balanced system, the apparent power is given by, $S = 3V_{ph} I_{ph} = \sqrt{3} V_L I_L$

It may be observed that in a balanced three phase system,

$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3} V_L I_L \cos\phi = S \cos\phi =$ *active power*

$Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3} V_L I_L \sin\phi =$ *reactive power.*

$\therefore \quad S = \sqrt{P^2 + Q^2} =$ *apparent power*

In complex form, $\mathbf{S} = \mathbf{P} + jQ$

5.7.8 Balanced star loads

Consider a star connected supply with balanced phase voltages $\mathbf{V}_A$, $\mathbf{V}_B$ and $\mathbf{V}_C$ and the balanced line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$ and $\mathbf{V}_{CA}$. This star connected supply is connected to star connected balanced load, with each limb having an impedance $\mathbf{Z} = Z\angle\theta$ as shown in fig. 5.30.

Let $\mathbf{I}_A$, $\mathbf{I}_B$ and $\mathbf{I}_C$ are three line currents.

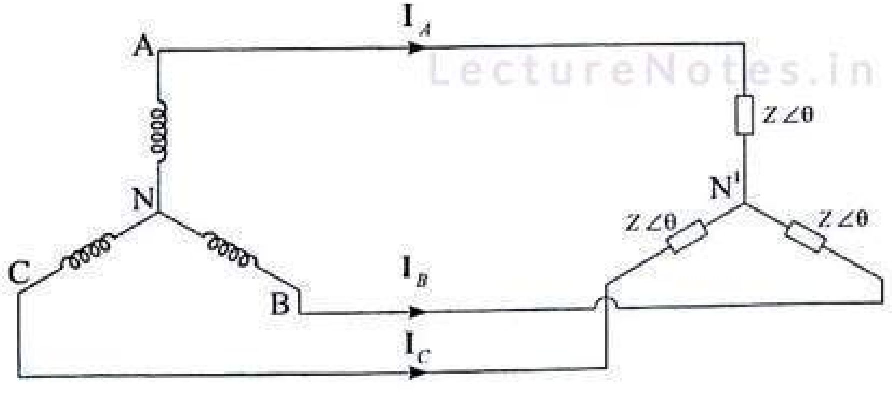

Fig 5.30

We have $\mathbf{V}_A = V_{ph}\angle 0$ = phase voltage of A.

$\mathbf{V}_B = V_{ph}\angle -120$ = phase voltage of B.

$\mathbf{V}_C = V_{ph}\angle -240$ = phase voltage of C.

As three phase supply voltage is balanced,

So $\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C = 0$

Also $\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$

and $V_N = V_{N'}$

Line current $\mathbf{I}_A = \dfrac{\mathbf{V}_A}{\mathbf{Z}} = \dfrac{V_{ph}\angle 0}{Z\angle\theta} = \dfrac{V_{ph}}{Z}\angle -\theta,$

Similarly line current $\mathbf{I}_B = \dfrac{\mathbf{V}_B}{\mathbf{Z}} = \dfrac{V_{ph}\angle -120}{Z\angle\theta} = \dfrac{V_{ph}}{Z}\angle -120-\theta$

and $\mathbf{I}_C = \dfrac{\mathbf{V}_C}{\mathbf{Z}} = \dfrac{V_{ph}\angle -240}{Z\angle\theta} = \dfrac{V_{ph}}{Z}\angle -240-\theta$

5.7.9 Balance delta loads

Consider a delta connected supply connected to a balanced delta connected load as shown in fig. 5.31.

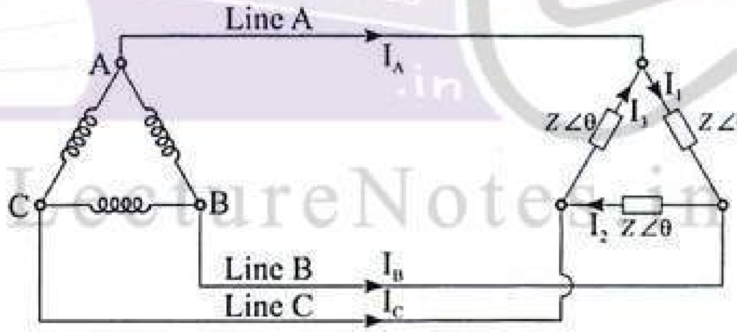

Fig 5.31

Since the phase coils are connected between lines, the phase voltage is equal to the line voltage. Hence magnitude of line voltage = magnitude of phase voltage.

Let, $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$ and $\mathbf{V}_{CA}$ are three line voltages.

$\therefore \quad |\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_L = V_{ph}$

we have $\mathbf{V}_{AB} = V_{AB}\angle 0 = V_{ph}\angle 0$

$\mathbf{V}_{BC} = V_{BC}\angle -120 = V_{ph}\angle -120$

$\mathbf{V}_{CA} = V_{CA}\angle -240 = V_{ph}\angle -240$

At load the phase current $\mathbf{I}_1 = \dfrac{V_{AB}\angle 0}{Z\angle\theta} = \dfrac{V_{ph}}{Z}\angle -\theta$

$$\mathbf{I}_2 = \frac{V_{BC}\angle -120}{Z\angle\theta} = \frac{V_{ph}\angle -120}{Z\angle\theta} = \frac{V_{ph}}{Z}\angle -120-\theta$$

$$\mathbf{I}_3 = \frac{V_{CA}\angle -240}{Z\angle\theta} = \frac{V_{ph}\angle -240}{Z\angle\theta} = \frac{V_{ph}}{Z}\angle -240-\theta$$

The line currents are, $\mathbf{I}_A = \mathbf{I}_1 - \mathbf{I}_3 = \sqrt{3}\dfrac{V_{ph}}{Z}\angle -30-\theta = \sqrt{3}\,I_{ph}\angle -30-\theta$

$$\mathbf{I}_B = \mathbf{I}_2 - \mathbf{I}_1 = \sqrt{3}\frac{V_{ph}}{Z}\angle -150-\theta = \sqrt{3}\,I_{ph}\angle -150-\theta$$

and $\mathbf{I}_C = \mathbf{I}_3 - \mathbf{I}_2 = \sqrt{3}\dfrac{V_{ph}}{Z}\angle 90-\theta = \sqrt{3}\,I_{ph}\angle 90-\theta$

where $I_{ph} = \dfrac{V_{ph}}{Z}$

Example 5.29 : The magnitude of the phase voltage of a three phase wye system is 220V rms. Express each phase and line voltage in both polar and rectangular co-ordinates.

Solution : The phase voltages in polar form are,

$$V_a = 220\angle 0^0 V,\ V_b = 220\angle -120^0\ V\ \ and\ V_c = 220\angle -240^0$$

The rectrangular forms, are,

$$V_a = 220\,volt,\ \ V_b = -110 - j190\cdot 52\,volt,\ V_c = -110 + j190\cdot 5V$$

The line voltages in polar form are,

$$V_{ab} = \sqrt{3}V_a\angle 30^0 = \sqrt{3}\times 220\angle 30^0 = 380\angle 30^0 \text{ volt.}$$

$$V_{bc} = \sqrt{3}V_b\angle -90^0 = \sqrt{3}\times 220\angle -90^0 = 380\angle -90^0 \text{ volt.}$$

$$V_{ca} = \sqrt{3}V_c\angle 150^0 = \sqrt{3}\times 220\angle 150 = 380\angle 150^0 \text{ volt.}$$

The line voltages in rectangular form are,

$$V_{ab} = 329 + j190 \text{ volt.}$$

$$V_{bc} = -j380 \text{ volt.}$$

$$V_{ca} = -329 + j190 \text{ volt.}$$

Example 5.30 : The phase currents in a four-wire wye connected load are as follows.

$$I_a = 10\angle 0A,\qquad I_b = 12\angle\frac{5\pi}{6},\qquad I_c = 8\angle 2\cdot 88$$

Determine the current in the neutral wire.

Solution : Current in neutral wire is,

$$I_\eta = I_a + I_b + I_c = 10\angle 0^0 + 12\angle 150^0 + 8\angle 165^0 = 15\cdot 39\angle 148\cdot 4^0\ A$$

Example 5.31 : For the circuit shown in figure 5.32 we see that each voltage source has a phase difference of $2\pi/3$ in relation to the others.

a) Find V_{RW}, V_{WB} *and* V_{BR} where

$V_{RW} = V_R - V_W$

$V_{WB} = V_W - V_B$

$V_{BR} = V_B - V_R$

b) Repeat part (a) using the calculations

$V_{RW} = V_R\sqrt{3}\angle \frac{-\pi}{6}$

$V_{WB} = V_W\sqrt{3}\angle \frac{-\pi}{6}$

$V_{BR} = V_B\sqrt{3}\angle \frac{-\pi}{6}$

c) Compare the results of part (a) with the results of part (b).

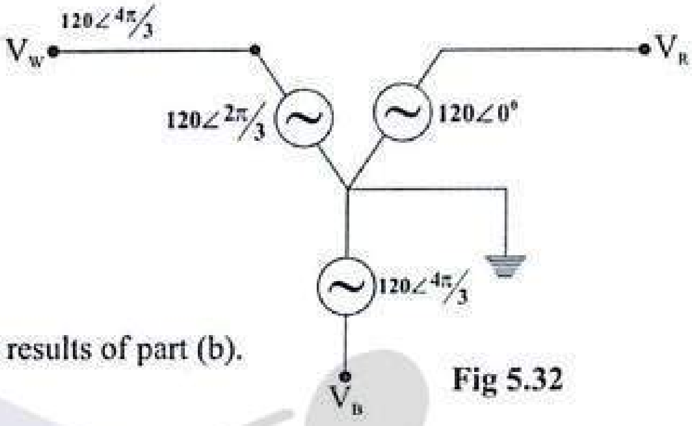

Fig 5.32

Solution :

(a) $V_{RW} = V_R - V_W = 120\angle 0^0 - 120\angle 120^0 = 207\cdot 8\angle -30^0$ volt.

$V_{WB} = V_W - V_B = 120\angle 120^0 - 120\angle 240^0 = 207\cdot 8\angle 90^0$ volt.

$V_{BR} = V_B - V_R = 120\angle 240^0 - 120\angle 0^0 = 207\cdot 8\angle -150^0$ volt.

(b) $V_{RW} = V_R\sqrt{3}\angle -30^0 = 120\sqrt{3}\angle -30^0 = 207\cdot 8\angle -30^0$ volt.

$V_{WB} = V_W\sqrt{3}\angle -30^0 = 120\angle 120^0\sqrt{3}\angle -30^0 = 207\cdot 8\angle 90^0$ volt.

$V_{BR} = V_B\sqrt{3}\angle -30^0 = 120\angle 240^0\sqrt{3}\angle -30^0 = 207\cdot 8\angle 210^0 = 207\cdot 8\angle -150^0$ volt.

(c) The two calculations are identical.

Example 5.32 : For the three phase circuit shown in fig. 5.33, find the current in the neutral wire and real power.

$V_R = 110\angle 0\,volt$

$V_W = 110\angle \frac{2\pi}{3}\,volt$

$V_B = 110\angle \frac{4\pi}{3}\,volt.$

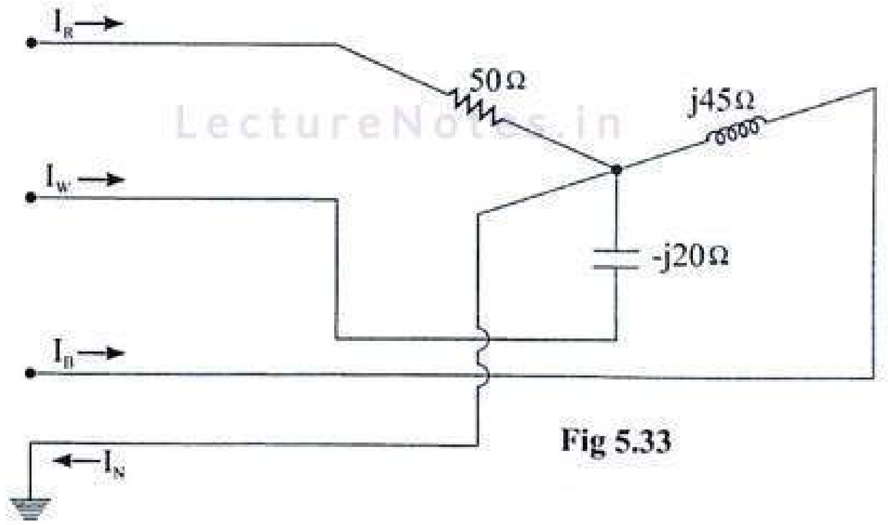

Fig 5.33

Solution : $Z_R = 50\Omega,\quad Z_W = -j20\Omega\quad Z_B = j45\Omega$

a) $$I_R = \frac{V_R}{Z_R} = \frac{110\angle 0^0}{50} = 2\cdot 2\angle 0^0\ A$$

$$I_B = \frac{V_B}{Z_B} = \frac{110\angle 240^0}{j45} = 2\cdot 44\angle 150^0\ A$$

$$I_W = \frac{V_W}{Z_W} = \frac{110\angle 120^0}{-j20} = 5\cdot 5\angle 210^0\ A$$

$$I_N = I_R + I_W + I_B = 2\cdot 2 + 5\cdot 5\angle 210^0 + 2\cdot 44\angle 150^0 = 4\cdot 92\angle -161\cdot 9^0\ A$$

(b) $P = RI_R^2 = 50(2\cdot 2)^2 = 242$ watt.

Example 5.33 : A three phase steel-treatment electrical oven has a phase resistance of 10Ω and is connected at three phase 380 V AC. Compute

(a) the current flowing through the resistors in wye and delta connections.

(b) the power of the oven in wye and delta connections.

Solution : (a) In Y-connection :

$$I_{ph} = \frac{V_{phase}}{R} = \frac{380/\sqrt{3}}{10} = \frac{220}{10} = 22A$$

In Δ - connection

$$I_L = \frac{V_{line}}{R} = \frac{380}{30} = 12\cdot 7A$$

(b) In Y - connection :

$$P = \sqrt{3}V_L I_L = \sqrt{3}(380)(22) = 14\cdot 5KW \qquad (\because I_L = I_{ph})$$

In Δ - connection

$$P = \sqrt{3}V_L I_L = \sqrt{3}(380)(12\cdot 7) = 8\cdot 36\ \text{Kw.}$$

Example 5.34 : A naval in-board synchronous generator has an apparent power of 50 KVA and supplies a three - phase network of 380 V. Compute the phase currents, the active powers and the reactive powers if,

(a) the power factor is 0.85

(b) the power factor is 1.

Solution : (a) $S = \sqrt{3} V_L I_L$

$$\Rightarrow I_L = \frac{S}{\sqrt{3} V_L} = \frac{50000}{\sqrt{3}(380)} = 76A$$

$$P = S\cos\phi = (50000)(0{\cdot}85) = 42{\cdot}5\,KW$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{(50)^2 - (42{\cdot}5)^2} = 26{\cdot}3\,KVAR$$

(b) $S = P$

$$\Rightarrow I = 76A$$

$$P = S\cos\phi = (50000)(1) = 50\,KW$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{S^2 - S^2} = 0 \qquad \{\because S = P\}$$

5.8 Residential Wiring

In the previous topics we have discussed about the utilisation of electrical power in the form three phase supply. Large industrial consumers receive power from discos and transcos at high voltage (from 11 KV to 132 KV) depending upon amount of Power they handle. However the common residential electric power service consist of three phase four wire system. The distribution system which is derived from a 11 KV three phase three wire system consist 400 volts.(Line to line).three phase with neutral. The neutral conductor is drawn from the star point in the secondary of distribution transformer. Primary of the distribution transformer is connected in delta which gets supply from three phase three wire 11 KV system.

In Indian system three phases are named as, red (R), blue (B) and yellow (Y) and the neutral is named as N. There are two types of domestic load namely 1-ϕ type and 3-ϕ type. The 1-ϕ loads are fans, flourscent lamps, all domestic appliances such as washing machines, micro ovens, refrigerators etc. The 3-ϕ loads are mostly of high capacity air-conditioners (more than 5 ton) and high rated water pumps etc.

A domestic consumers which have both 1-ϕ and 3-ϕ type of loads get a connection to his premises from the distribution pole with three phases and neutral. The wire that connects the distribution pole and the consumer is known as ***service main.***

Three phase loads are connected to all the three phases while the 1-ϕ loads are connected between any one phase and neutral. The single phase voltage is $\frac{1}{\sqrt{3}}$ times the line voltage. In Indian system as the line to line voltage at distribution system is 400 volts, the single phase supply voltage is 230 volts $\left(i.e.\ {}^{400}\!/\!_{\sqrt{3}}\ volts.\right)$. The connection diagram of a domestic wiring is shown in the fig. 5.34.

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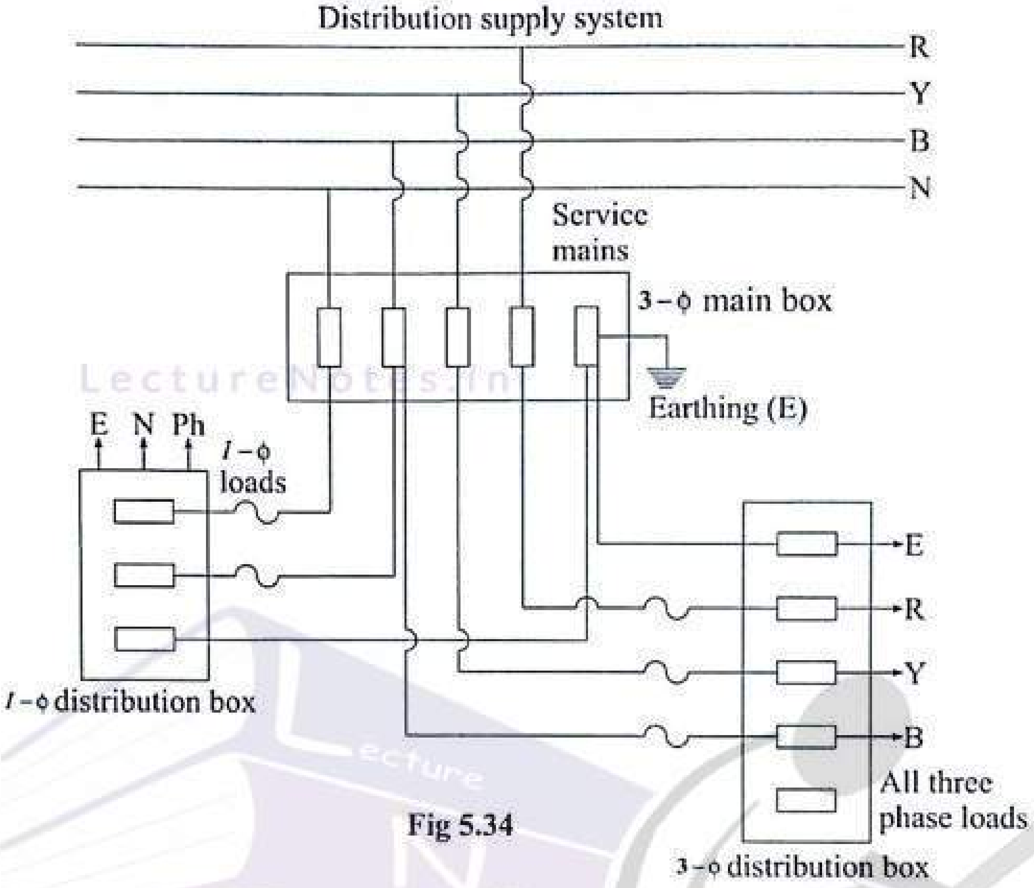


Fig 5.34

5.8.1 Grounding and Safety

Generally the power system is grounded type, it means the whole of the earth behaves like a neutral point (ideally). Hence when we measure the voltage between any phase and solidly grounded point, it gives the phase voltage. A load connected between the two points will cause a current to flow through it, where this current is given by $I_L = \frac{V_{ph}}{R}$, R is the resistance of the load.

This concept creats unsafety while one human being comes in touch with a live conductor (any phase conductor), standing on the ground. This causes a current to flow through his body giving an electric shock, which may be fatal in nature. The current will depend mostly on the body resistance.

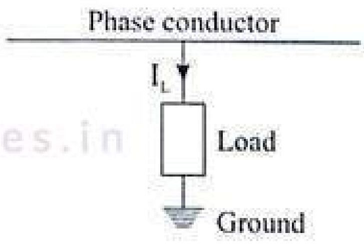


Fig 5.35

It become necessary to give an alternative path to this leakage current, if the appliances are in contact with any live conductors.

To explain this concept, let us consider an arrangement where an appliance is in contact with live conductor. An human being in touch with the appliance standing on the ground.

Let V = voltage between the phase and grounding.

R = resistance of the current carrying path of the human being.

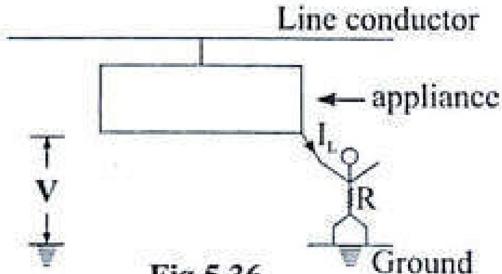


Fig 5.36

The leakage current through the human being is given by $I_L = \frac{V}{R} A$. If $I_L > 40 mA$, it may cause death to the human being.

To protect agaist such electric shock grounding of all appliances is necessary where these are connected to a solidly grounded point known as earth pit. The earth pits are being made such a way that the resistances of the earth pits should be as low as possible. Ideally it should be less than one ohm for industrial system and less than five ohm for domestic system.

To explain how a grounding protects human being from electric shock is shown in fig. 5.37.

Let V = voltage between live conductor and grounded.

R = Body resistance of the human being.

R_E = Earth resistance of earth pit.

I_L = Leakage current through the human being.

I_E = leakage current through the earth pit.

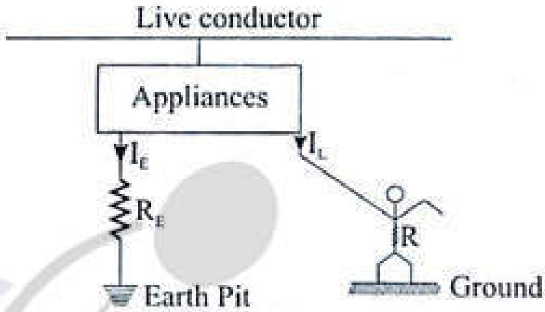


Fig 5.37

Now there are two paths for the leakage current to flow to the ground. They are I_E and I_L.

$$I_E = \frac{V}{R_E}$$

$$I_L = \frac{V}{R}$$

Since $R_E << R$, So $I_E >> I_L$ i.e most of the leakage current flows through the earth pit. Thus protecting the human being from electric shock.

Hence all electrical equipments and appliances are connected to the earth wire drawn from the earth pit. The shocket point available for single phase load consists of phase, neutral and ground as shown in Fig.5.38.

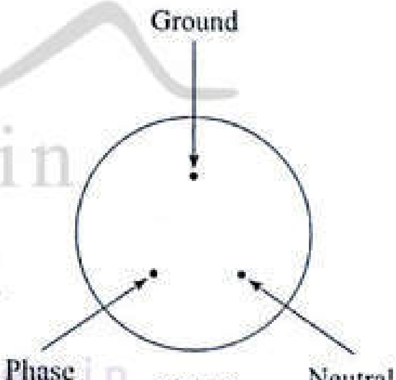


Fig 5.38

5.9 Generation, Transmission and Distribution of Electric Power

Prior to the discovery of Faraday's Laws of electromagnetic discussion, electrical power was available from batteries with limited voltage and current levels. Although complicated in construction, D.C generators were developed first to generate power in bulk. However, due to limitation of the D.C machine to generate voltage beyond few hundred volts, it was not economical to transmit large amount of power over a long distance. For a given amount of power, the current magnitude (I = P/V), hence section of the copper conductor will be large. Thus generation, transmission and distribution of d.c power were restricted to area of few kilometer radius with no interconnections between generating plants. Therefore, area specific generating stations along with its distribution networks had to be used.

5.9.1 A.C generator

A.C power can be generated as a single phase or as a balanced poly-phase system. However, it was found that 3-phase power generation at 50 Hz will be economical and most suitable. Present day three phase generators, used to generate 3-phase power are called *alternators* (synchronous generators). An alternator has a balanced three phase winding on the stator and called the armature. The three coils are so placed in space that there axes are mutually 120° apart as shown in figure 5.39. From the terminals of the armature, 3-phase power is obtained. Rotor houses a field coil and excited by D.C. The field coil produces flux and electromagnetic poles on the rotor surface. If the rotor is driven by an external agency, the flux linkages with three stator coils becomes sinusoidal function of time and sinusoidal voltage is induced in them. However, the induced, voltages in the three coils (or phases) will differ in phase by 120° because the present value of flux linkage with R-phase coil will take place after 120° with Y -phase coil and further 120° after, with B-phase coil. A salient pole alternator has projected poles as shown in Fig.5.39. It has non uniform air gap and is generally used where speed is low. On the other hand a non salient pole alternator has uniform air gap.

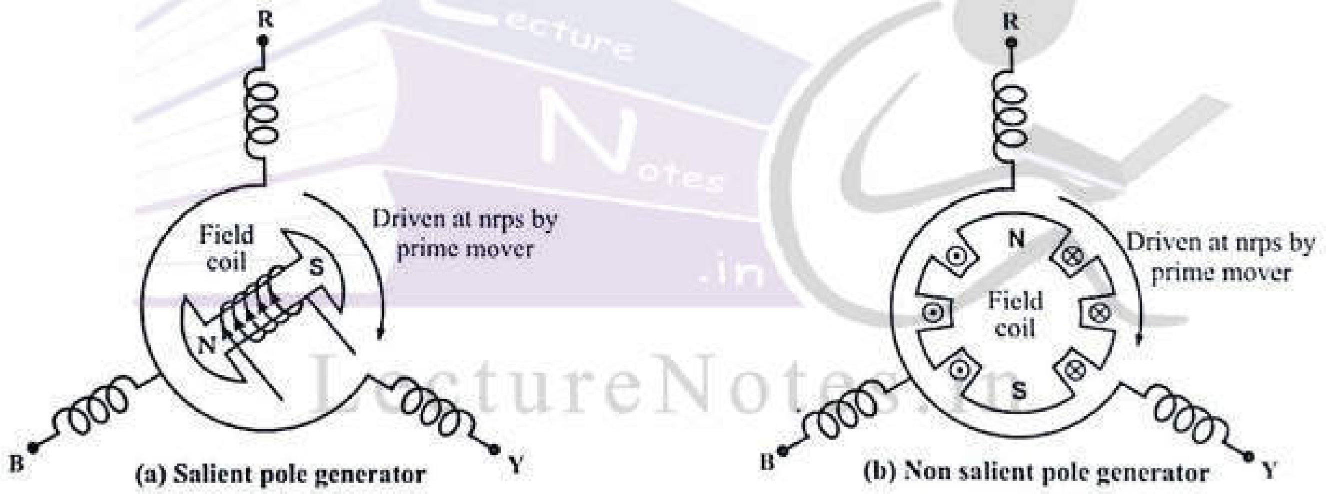


Fig 5.39

5.9.2 Electrical Power Generation from conventional Source

Most of the power generated is from conventional source of energy. They may be from

a. ***Hydel:*** Potential energy of water is converted to mechanical and then to electrical energy

b. ***Thermal:*** Heat energy from burning of coal is used to generate electrical energy

c. ***Neuclear:*** Neuclear energy is used for generation of electricity

In the following section each plant is briefly explained.

5.9.2.1 Thermal plant

We have seen in the previous section that to generate voltage at 50 Hz we have to run the generator at some fixed rpm by some external agency. A turbine is used to rotate the generator. Turbine may be of two types, namely steam turbine and water turbine. In a thermal power station coal is burnt to produce steam which in turn, drives the steam turbine hence the generator (turbo set). In Fig. 5.40 the elementary features of a thermal power plant is shown.

In a thermal power plant coil is burnt to produce high temperature and high pressure steam in a boiler. The steam is passed through a steam turbine to produce rotational motion. The generator, mechanically coupled to the turbine, thus rotates producing electricity. Chemical energy stored in coal after a couple of transformations produces electrical energy at the generator terminals as depicted in the figure. Thus proximity of a generating station nearer to a coal reserve and water sources will be most economical as the cost of transporting coal gets reduced. In our country coal is available in abundance and naturally thermal power plants are most popular. However, these plants pollute the atmosphere because of burning of coals.

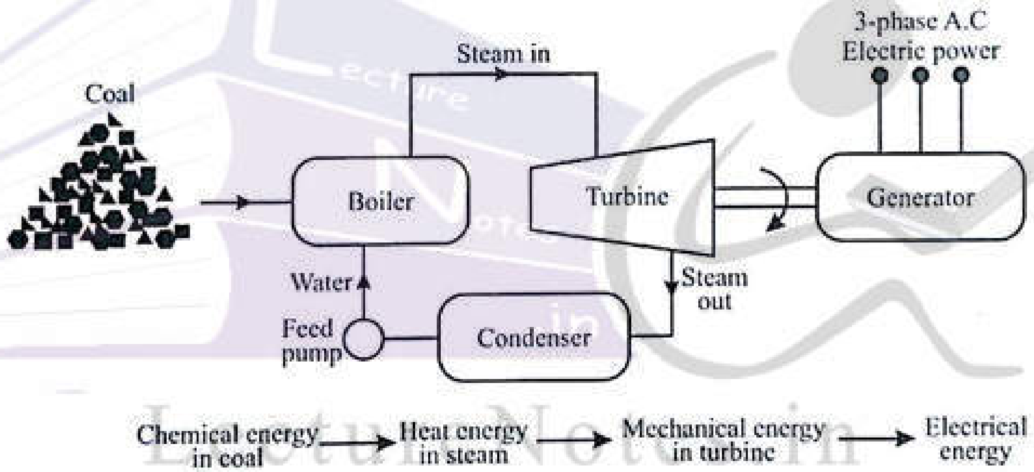


Figure: 5.40 Components of a Thermal Power Plant

Stringent conditions (such as use of more chimney heights along with the compulsory use of electrostatic precipitator) are put by regulatory authorities to see that the effects of pollution are minimized. A large amount of ash is produced every day in a thermal plant and effective handling of the ash adds to the running cost of the plant. Nonetheless 57% of the generation in out country is from thermal plants. The speed of alternator used in thermal plants is 3000 rpm which means 2-pole alternators are used in such plants.

5.9.2.2 Hydro Electric plants

In a hydel power station, water head is used to drive water turbine coupled to the generator as shown in fig. 5.41.Water head may be available in hilly region naturally in the form of water reservoir (lakes etc.) at the hill tops. The potential energy of water can be used to drive the turbo generator set installed at the base of the hills through piping called *pen stock*. Water head may also be created artificially by constructing dams on a suitable river. In contrast to a thermal plant, hydel power plants are eco-friendly, neat and clean as no fuel is to be burnt to produce electricity. While running cost of such plants is low, the initial installation cost is rather high compared to thermal plants due to massive civil construction necessary. Also sites to be selected for such plants depend upon natural availability of water reservoirs at hill tops or availability of suitable Rivers for constructing dams. Water turbines generally operate at low rpm, so number of poles of the alternator is high. For example a 20-pole alternator the rpm of the turbine is only 300 rpm.

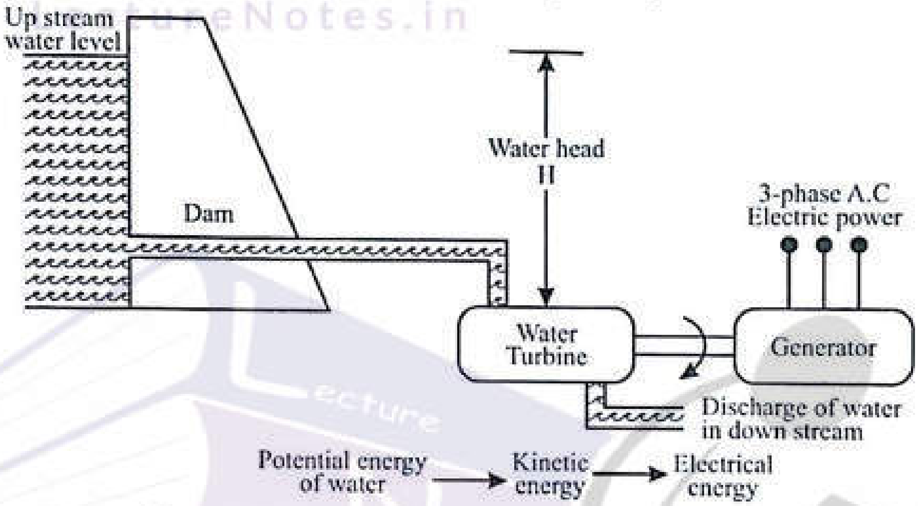


Figure: 5.41 Components of a hydro electric power plant

5.9.2.3 Nuclear plants

As coal reserve is not unlimited, there is natural threat to thermal power plants based on coal. It is estimated that within next 30 to 40 years, coal reserve will exhaust if it is consumed at the present rate. Nuclear power plants are thought to be the solution for bulk power generation. At present the installed capacity of nuclear power plant is about 4300 MW and expected to expand further in our country. The present day atomic power plants work on the principle of nuclear fission of U^{235}. In the natural uranium, U^{235} constitutes only 0.72% and remaining parts is constituted by 99.27% of U^{238} and only about 0.05% of U^{234}. The concentration of U^{235} may be increased to 90% by gas diffusion process to obtain enriched U^{235}. When U^{235} is bombarded by neutrons a lot of heat energy along with additional neutrons are produced. These new neutrons further bombard U^{235} producing more heat and more neutrons. Thus a chain reaction sets up. However this reaction is allowed to take place in a controlled manner inside a closed chamber called *nuclear reactor*. To ensure sustainable chain reaction, moderator and control rods are used. Moderators such as heavy water (deuterium) or very pure carbon C^{12} are used to reduce the speed of neutrons. To control the number neutrons, control rods made of cadmium or boron steel are inserted inside the reactor. The control rods can absorb neutrons. If we want to decrease the number neutrons, the control rods are lowered down further and vice versa. The heat generated inside the reactor is taken out of the chamber with the help of a coolant such as liquid sodium or some gaseous fluids. The coolant gives up the heat to water in heat exchanger to convert it to steam as shown in fig. 5.42. The steam then drives the turbo set and the exhaust steam from the turbine is cooled and fed back to the heat exchanger with the help of water feed pump. Calculation shows that to produce 1000 MW of electrical power in coal based thermal plant, about 6×10^6 Kg of coal is to be burnt daily while for the same amount of power, only about 2.5 Kg of U^{235} is to be used per day in a nuclear power stations.

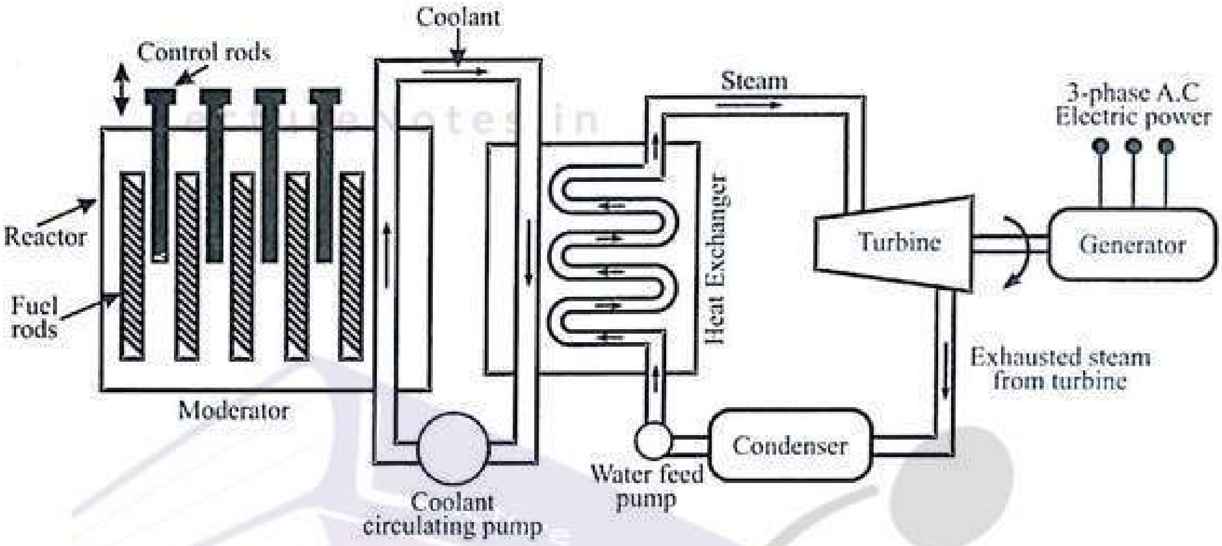


Figure: 5.42 Components of a Nuclear Power Plant

The initial investment required to install a nuclear power station is quite high but running cost is low. Although, nuclear plants produce electricity without causing air pollution, it remains a dormant source of radiation hazards due to leakage in the reactor. Also the used fuel rods are to be carefully handled and disposed off as they still remain radioactive.

The reserve of U^{235} is also limited and can not last longer if its consumption continues at the present rate. Naturally search for alternative fissionable material continues. For example, plutonium (Pu^{239}) and (U^{233}) are fissionable. Although they are not directly available, absorbing neutrons, U^{238} gets converted to fissionable plutonium Pu^{239} in the atomic reactor described above. The used fuel rods can be further processed to extract Pu^{239} from it indirectly increasing the availability of fissionable fuel. Effort is also on to convert thorium into fissionable U^{233} . Incidentally, India has very large reserve of thorium in the world. Total approximate generation capacity and contribution by thermal, hydel and nuclear generation in our country are given below.

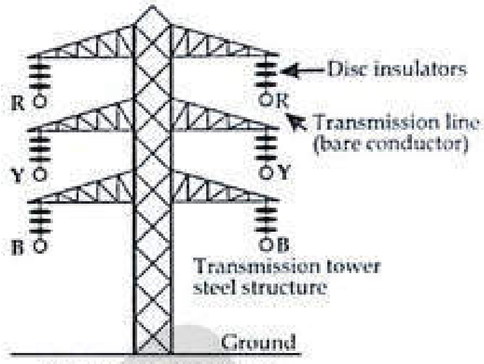
Method of generation	in MW	% contribution
Thermal	77 340	69.4
Hydel	29 800	26.74
Nuclear	2720	3.85
Total generation	1 11 440	- - -

5.9.3 Non conventional sources of energy

The bulk generation of power by thermal, hydel and nuclear plants are called conventional methods for producing electricity. Search for newer avenues for harnessing eco friendly electrical power has already begun to meet the future challenges of meeting growing power demand. Compared to conventional methods, the capacity in terms of MW of each non-conventional plant is rather low, but most of them are eco friendly and self sustainable. Wind power, solar power, MHD generation, fuel cell and power from tidal waves are some of the promising alternative sources of energy for the future.

5.9.4 Transmission of power

The huge amount of power generated in a power station (hundreds of MW) is to be transported over a long distance (hundreds of kilometers) to load centers to cater power to consumers with the help of transmission line and transmission towers as shown in Fig. 5.43.



A transmission Tower

Fig. 5.43

5.9.5 Substations

Substations are the places where the level of voltage undergoes change with the help of transformers. Apart from transformers a substation will house switches (called circuit breakers), meters, relays for protection and other control equipment. Broadly speaking, a big substation will receive power through incoming lines at some voltage (say 400 kV) changes level of voltage (say to 132 kV) using a transformer and then directs it out wards through outgoing lines. Pictorially such a typical power system is shown in Fig.5.44 in a short of block diagram. At the lowest voltage level of 400 V, generally 3-phase, 4-wire system is adopted for domestic connections. The fourth wire is called the neutral wire (N) which is taken out from the common point of the star connected secondary of the 6 kV/400 V distribution transformer.

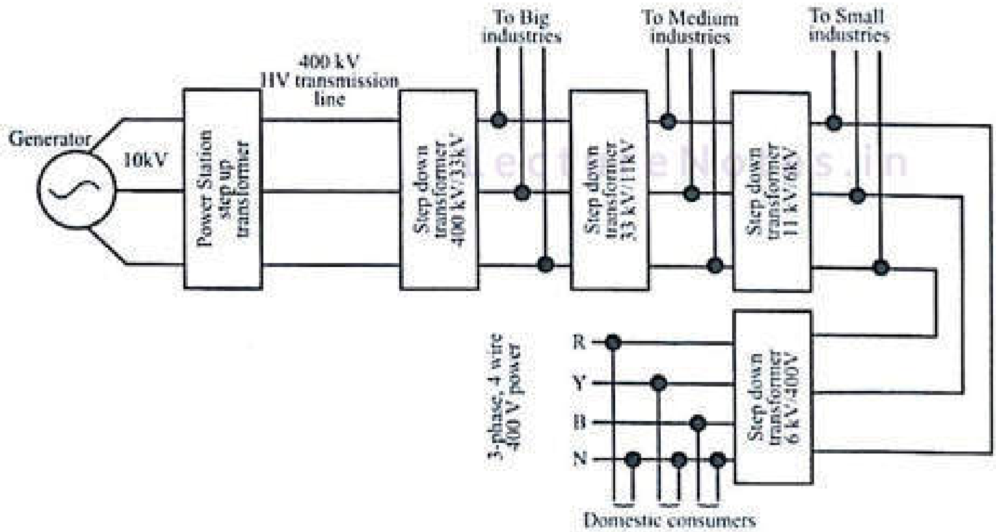


Fig. 5.44 Lay-out of substation

5.9.6 Single line representation of power system

Trying to represent a practical power system where a lot of interconnections between several generating stations involving a large number of transformers using three lines corresponding to R, Y and B phase will become unnecessary clumsy and complicated. To avoid this, a single line along with some symbolical representations for generator, transformers substation buses are used to represent a power system rather neatly. For Example, the system shown in Fig.5.44 with three lines will be simplified to Fig.5.45 using single line.

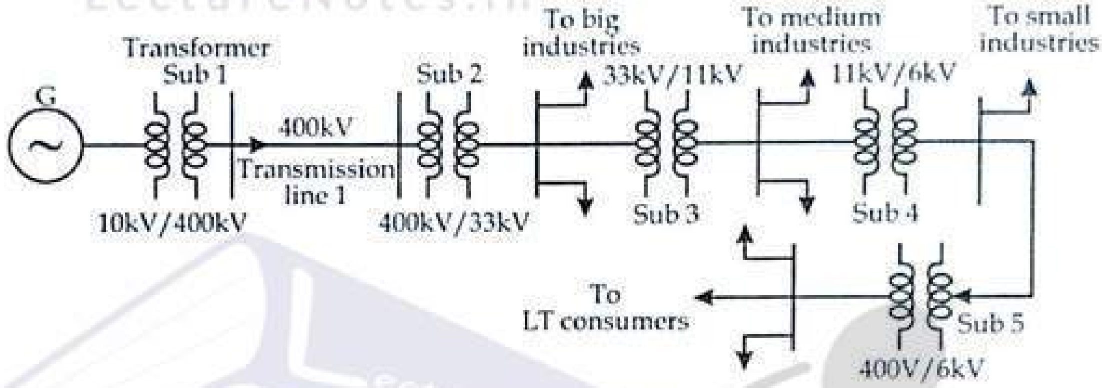


Fig. 5.45 Single line representation of power system

Another example, an interconnected power system is represented in the self explained Fig.5.46 it is hoped that you understand the important features communicated about the system through this figure.

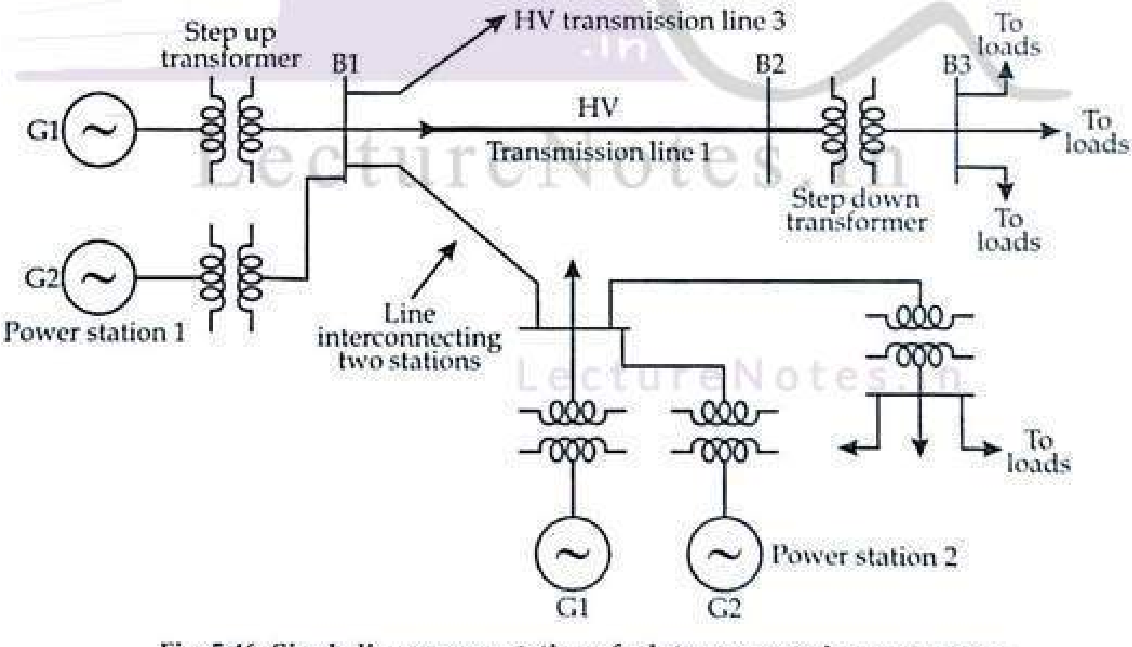


Fig. 5.46 Single line representation of a interconnected power system

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5.9.7 Distribution system

Till now we have learnt how power at somewhat high voltage (say 33 kV) is received in a substation situated near load center (a big city). The loads of a big city are primarily residential complexes, offices, schools, hotels, street lighting etc. These types of consumers are called LT (low tension) consumers. Apart from this there may be medium and small scale industries located in the outskirts of the city. LT consumers are to be supplied with single phase, 220 V, 40 Hz. We shall discuss here how this is achieved in the substation receiving power at 33 kV. The scheme is shown in Fig. 5.47.

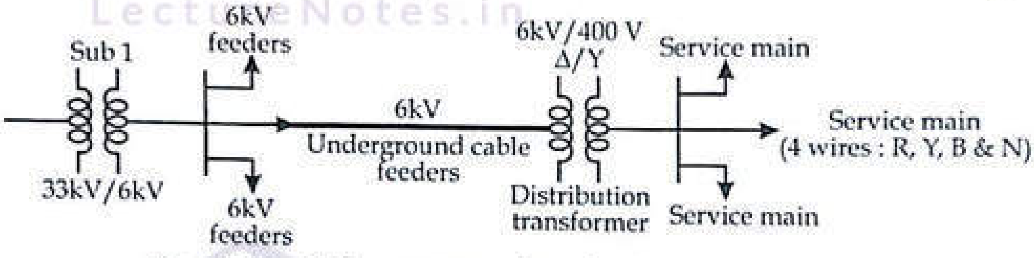


Fig. 5.47 Single line representation of a Distribution system

Power receive at a 33 kV substation is first stepped down to 6 kV and with the help of underground cables (called feeder lines), power flow is directed to different directions of the city. At the last level, step down transformers are used to step down the voltage form 6 kV to 400 V. These transformers are called distribution transformers with 400 V, star connected secondary. You must have noticed such transformers mounted on poles in cities beside the roads. These are called pole mounted substations. From the secondary of these transformers 4 terminals (R, Y, B and N) come out. N is called the neutral and taken out from the common point of star connected secondary. Voltage between any two phases (i.e., R- Y, Y -B and B-R) is 400 V and between any phase and neutral is 230 V. Residential buildings are supplied with single phase 230V, 50Hz. So individual are to be supplied with any one of the phases and neutral. Supply authority tries to see that the loads remain evenly balanced among the phases as far as possible. Which means roughly one third of the consumers will be supplied from R-N, next one third from Y-N and the remaining one third from B-N. The distribution of power from the pole mounted substation can be done either by (1) overhead lines (bare conductors) or by (2) underground cables. Use of overhead lines although cheap, is often accident prone and also theft of power by hooking from the lines takes place. Although costly, in big cities and thickly populated areas underground cables for distribution of power, are used.

BPUT PREVIOUS YEAR QUESTIONS SOLVED

1. The Line voltage of a 3-phase circuit is 400 V with line current of 100 A and power factor of 0.5. Calculate the total power delivered by the 3-phase line. ***(1st semester 2003)***

   Solution : $V_L = 400$ V, $I_L = 100$ A, $\cos\phi = 0\cdot5$

   Total power delivered by 3-phase line is $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3}(400)(100)(0\cdot5)$

   $= 34640$ watt

2. The current flowing in a circuit is leading the applied voltage by 30^0. What is the power factor of the circuit ? ***(1st semester 2003)***

   Solution : Power factor $= \cos\phi = \cos 30 = 0\cdot866$ (lead)

3. Which type of distribution system is used for domestic supply :

   (a) 3-phase, 3-wire

   (b) 3-phase, 4-wire ***(1st semester 2003)***

   Solution : 3-phase, 4 wire

4. What is the basic constructional difference between an energy meter and wattmeter ? ***(1st semester 2003)***

   Solution :

Watt meter	Energy meter
1. Scale with Pointer.	1. Register for recording energy consume for specified period.
2. Damping torque, control torque and Deflecting torque required.	2. Control torque and damping torque replace with braking torque by providing two sets of brake magnets.
3. Indicating instrument	3. Integrating instrument.

5. Find an expression for the current and calculate the power, when a voltage represented by $V = 100\sin\pi t$ is applied to a coil having $R = 50\Omega$ and $L = 0\cdot159H$ ***(1st semester 2003)***

   Solution : $V = 100\sin\pi t$, $R = 50\Omega$, $L = 0\cdot159H$

$$V_{rms} = \frac{100}{\sqrt{2}} = 70\cdot721 \text{ volt.}$$

$\omega = \pi \frac{rad}{sec}$

$X_L = \omega L = 0 \cdot 499\,\Omega$

$Z = R + jX_L = 50 + j0 \cdot 499 = 50\angle 0 \cdot 57^0\ \Omega$

Current flowing through the coil is, $I = \frac{V}{Z}$

$\Rightarrow I = \frac{70 \cdot 721\angle 0^0}{50\angle 0 \cdot 57^0} = 1 \cdot 414\angle -0 \cdot 57^0\ A$

Power $= VI\cos\phi = 70 \cdot 721 \times 1 \cdot 414 \times \cos 0 \cdot 57$

$= 99 \cdot 9945 \approx 100$ watt

Expression for current is $i = I_m \sin(\omega t \pm \phi) = 1 \cdot 414\sqrt{2}\sin(\pi t - 0 \cdot 57^0)$

6. Three coils each having a resistance of 30Ω and impedance of 50Ω are connected in delta across a 400 V, 50 Hz, 3-phase supply. Find (i) the Line current (ii) the Power factor (iii) the active power absorbed. ***(1st semester 2003)***

Solution : $V_L = V_{ph} = 400V$, f = 50 Hz, $Z_{ph} = 50\Omega$ $R = 30\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{50} = 8A$

(i) $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 8 = 13 \cdot 856A$

(ii) Power factor $= \cos\phi = \frac{R}{Z} = 0 \cdot 6$

(iii) Active Power $= \sqrt{3}\, V_L\, I_L \cos\phi$

$= \sqrt{3}(400)(13 \cdot 856)(0 \cdot 6) = 5759 \cdot 66$ watt

7. What is the average power dissipated in 10Ω resistor, if the voltage across it

$V(t) = 5 + 3\cos(t + 10) + \cos(2t + 30)$ ***(1st semester 2004)***

Solution : Average Power dissipated $= \frac{(5)^2}{10} + \frac{\left(3/\sqrt{2}\right)^2}{10} + \frac{\left(1/\sqrt{2}\right)^2}{10} = 3$ watt

8. A 200 μF capacitor is connected to a 240 volt, 60 Hz source. What is the reactive VARs it draws ? ***(1st semester 2004)***

Solution : $C = 200 \times 10^{-6}\ F$, f = 60 Hz, V = 240 volts,

$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(60)(200 \times 10^{-6})} = 13 \cdot 27\Omega$

Reactive Power $= \dfrac{V^2}{X_C} = \dfrac{(240)^2}{13 \cdot 27} = 4340 \cdot 61 VARs$

9. A 2-element series circuit is connected across an AC source, whose voltage $V(t) = 230\sqrt{2}\sin(314t + 30^0)$. The current in the circuit is found to be $i(t) = 10\sqrt{2}\cos(314t - 35^0)$

(i) determine the parameters of the circuit and power factor.

(ii) the average active and reactive power in the circuit. *(1st semester 2004)*

Solution : $Z = \dfrac{V_M}{I_M} = \dfrac{230\sqrt{2}}{10\sqrt{2}} = 23\,\Omega$

(i) Power factor $= \cos\phi = \cos 65 = 0 \cdot 422$

Resistance, $R = Z\cos\phi = 9 \cdot 706\,\Omega$

Inductive reactance $X_L = Z\sin\phi = 20 \cdot 84\Omega$

(ii) Active Power $= VI\cos\phi = \dfrac{230\sqrt{2}}{\sqrt{2}} \cdot \dfrac{10\sqrt{2}}{\sqrt{2}} \cdot \cos 65 = 972$ watt.

Reactive Power $= VI\sin\phi = 2084 \cdot 5 VAR$

10. A choke is connected in series with a $18\mu F$ capacitor with a constant 240 volt supply voltage, the maximum current drawn by the circuit at 60 Hz supply frequency is 60 A. Find (i) the resistance and reactance of the choke. (ii) the voltage across the capacitor.

(1st semester 2004)

Solution : $C = 18 \times 10^{-6}\, F$, V = 240 volts.

(i) Under resonance condition, current is maximum.

Maximum Current, $I = \dfrac{V}{R} = 60$ A

$\Rightarrow R = \dfrac{V}{60} = \dfrac{240}{60} = 4\Omega$

Resonant frequency = 60 Hz.

$\Rightarrow \dfrac{1}{2\pi\sqrt{LC}} = 60$

$\Rightarrow L = 0 \cdot 391 H$

Inductive reactance $= X_L = 2\pi f L = 2\pi \times 60 \times 0 \cdot 391 = 147 \cdot 3\Omega$

(ii) Voltage across the capacitor is $V_C = IX_C$

$\Rightarrow V_C = I \cdot \dfrac{1}{\omega c} = 60 \times \dfrac{1}{2\pi(60)(18 \times 10^{-6})} = 8846 \cdot 4$ volts.

11. A 3-phase balanced system has a line voltage of 202 V, rms and feeds a delta connected load with a phase impedance of $Z_{ph} = 25\angle 60^0\ \Omega$. Find the line current and power supplied to the load.

(1st semester 2004)

Solution : $V_L = 202$ volts. $Z_{ph} = 25\angle 60^0\ \Omega$

$$V_{ph} = 202 \text{ volts.}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = 8\cdot 08A$$

$$I_L = \sqrt{3}\, I_{ph} = 13\cdot 994A$$

$$P = \sqrt{3} V_L\, I_L \cos\phi = \sqrt{3}(202)(13\cdot 994)\cos 60 = 2448 \text{ watt.}$$

12. A 3-phase delta connected balanced load each phase having resistance of 200 ohm is supplied by 3-phase star connected voltage source of 200 V per phase. Find the total active power consumed by the load.

(2nd semester 2004)

Solution : Supply voltage per phase $V_{ph} = 200V$

Supply line voltage $V_L = \sqrt{3} V_{ph} = \sqrt{3}(200) = 346\cdot 4V$

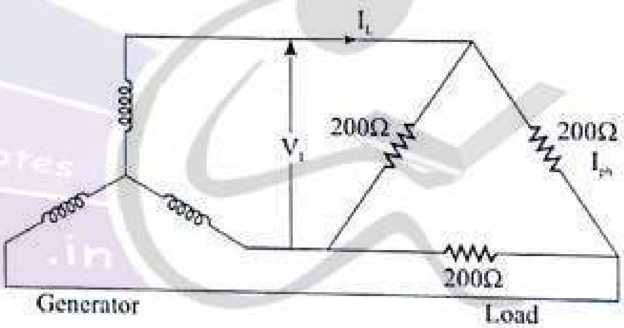
Load is delta connected.

So $V_L = V_{ph} = 346\cdot 4V$

$R_{ph} = 200\Omega$

$$I_{ph} = \frac{V_{ph}}{R_{ph}} = \frac{346\cdot 4}{200} = 1\cdot 732A$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\times 1\cdot 732 = 2\cdot 999A \approx 3A$$



Active power consumed by load $= \sqrt{3} V_L\, I_L \cos\phi = \sqrt{3}(346\cdot 4)(3)(1) = 1799\cdot 89$ watt.

13. Which type of generation is preferred for pollution free environment ? *(2nd semester 2004)*.

Solution : Under conventional method hydro electric generation is preferred and under non-conventional method solar, wind, geothermal etc. are preferred for pollution free environment.

14. Name four industrial application of electric energy. *(2nd semester 2004)*

Solution : (i) Electric traction, (ii) Heating, (iii) Metal refining (Electrolysis),

(iv) Illumination.

15. When a voltage of 100 V at 50 Hz is applied to a coil A having resistance R_A and industance L_A, the current taken and power consumed are 8A and 120W respectively. When applied to a coil B having resistance R_B and inductance L_B the current taken and power consumed are 10A and 500w respectively. What current and Power will be taken when 100V is applied across these two coils connected in series ?

(2nd semester 2004)

Solution : (i) For coil A,

V = 100 volts, I = 8A, f = 50 Hz, P = 120 watt

$$Z_A = \frac{100}{8} = 12\cdot5\Omega$$

$$\cos\phi_A = \frac{P}{VI} = \frac{120}{100\times8} = 0\cdot15$$

$$R_A = Z_A\cos\phi_A = 12\cdot5\times0\cdot15 = 1\cdot875\Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(12\cdot5)^2 - (1\cdot875)^2} = 12\cdot35\Omega$$

(ii) For coil B,

V = 100 volts, I = 10A, P = 500 watt

$$Z_B = \frac{V}{I} = \frac{100}{10} = 10\Omega$$

$$\cos\phi_B = \frac{P}{VI} = \frac{500}{100\times10} = 0\cdot5$$

$$R_B = Z_B\cos\phi_B = 10\times0\cdot5 = 5\Omega,\ X_B = \sqrt{Z_B^2 - R_B^2} = 8\cdot66\Omega$$

When these two coils A and B are connected in series then total resistance $R = R_A + R_B = 6\cdot875\Omega$ and total reactance, $X = X_A + X_B = 21\Omega$

$\therefore$ Impedance $Z = \sqrt{R^2 + X^2} = 22\cdot106\Omega$

Current flows through the circuit, $I = \dfrac{V}{Z} = \dfrac{100}{22\cdot10} = 4\cdot524A$

Power taken, $P = I^2R = (4\cdot524)^2\,(6\cdot875) = 140\cdot683$ watt.

16. Three identical impedances are connected in start across a 440 volts 3-phase, 50 Hz supply. The Line current is 40A and the p.f. is 0.8 leading. Find the value of resistance and capacitance in each phase. ***(2nd semester 2004)***

Solution : $I_L = 40A$, $V_L = 440$ volts, f = 50 Hz, p.f. = 0.8

$$V_{ph} = \frac{V_L}{\sqrt3} = \frac{440}{\sqrt3} = 254\cdot04\text{ volts},\ I_{ph} = I_L = 40A$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254\cdot04}{40} = 6\cdot351\Omega$$

$$R_{ph} = Z_{ph}\cos\phi = 6\cdot351\times0\cdot8 = 5\cdot08\Omega$$

$$X_{ph} = Z_{ph}\sin\phi = 6\cdot351\times0\cdot6 = 3\cdot81\Omega$$

Capacitance $C = \dfrac{1}{2\pi f\,X_{ph}} = 835\cdot8\mu F$

17. A resistance R and inductance L = 0.01 H and a capacitance C are in series. When an alternating voltage $V = 400\sin(3000t - 20^0)$ is applied to the series combination, the current flowing is $10\sqrt{2}\sin(3000t - 65^0)$ *(Supplementary Exam-04)*

(i) Find the values of 'R' and 'C'

(ii) Find the voltage across 'C'

(iii) Calculate the total power drawn by the circuit.

Solution : L = 0.01 H, $V_m = 400$ volts, $I_m = 10\sqrt{2}$ volts.

(i) $$Z = \frac{V_{rms}}{I_{rms}} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28 \cdot 2885\,\Omega$$

Phase angle $\phi = -20 + 65 = 45^0$ (lag)

Net reactance, X = Z sin θ

$\Rightarrow X_L - X_C = Z\sin\theta = 28\cdot 288\sin 45$

$\Rightarrow X_L - X_C = 20$

$\Rightarrow X_C = X_L - 20 = \omega L - 20 = 3000(0\cdot 01) - 20$

$\Rightarrow X_C = 10$

$\Rightarrow \frac{1}{\omega c} = 10$

$\Rightarrow C = \frac{1}{10\omega} = \frac{1}{10(3000)} = 33\cdot 33\mu F$

$R = Z\cos\theta = 28\cdot 2885\cos 45 = 20\Omega$

(ii) Voltage across 'C' is $V_C = IX_C = 10(10) = 100V$

Instantaneous value of voltage across 'C' is

$V_C = \sqrt{2}V_C\sin(3000t - 155) = 100\sqrt{2}\sin(3000t - 155)$ volt.

(iii) Total power drawn by the circuit $= I^2R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = 2000$ watt.

18. 400 V (line to line) is connected to a star connected load of $3 + j4\Omega$ in each phase. Find the line current and power. *(Supplementary Exam 2004)*

Solution : $V_L = 400$ volts.

$Z_{ph} = 3 + j4 = 5\angle 53\cdot 13^0\ \Omega$

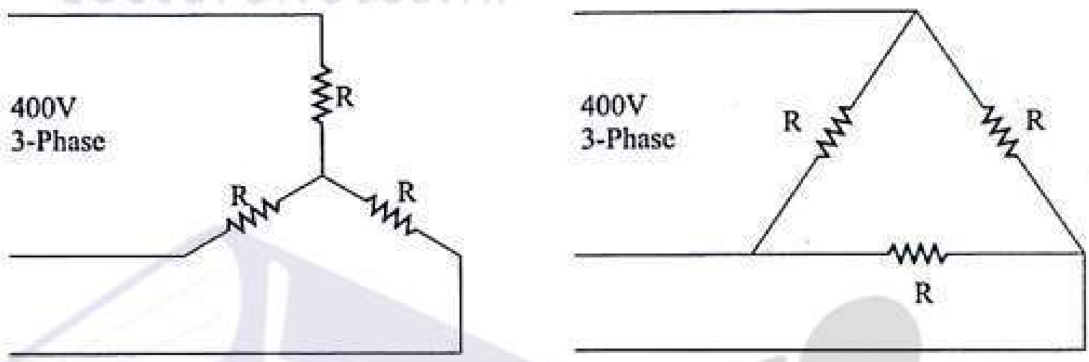
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230 \cdot 94 \text{ volts.}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230 \cdot 94}{5} = 46A$$

$$\therefore I_L = I_{ph} = 46A$$

$$\text{Power} = \sqrt{3} V_L \, I_L \cos\phi = \sqrt{3}(400)(46)\cos 53 \cdot 13 = 19 \cdot 21 KW$$

19.



If power in 1st circuit is P then Prove that power in 2nd circuit is 3P.

(Supplementary Exam 2004)

Solution : In star circuit power $= P = \left(\frac{400}{\sqrt{3}} \Big/ R \right)^2 R$

$$\Rightarrow P = \frac{(400)^2}{3R}$$

In delta circuit power $= P_1 = \left(\frac{400}{R} \right)^2 R = \frac{(400)^2}{R}$

$$\therefore \quad \frac{P_1}{P} = 3 \;\; \Rightarrow P_1 = 3P$$

20. Why 3-phase system is used for generation and transmission of electric power ?

(1st semester 2005)

Solution : (i) To transmit a given amount of power over a given distance less copper and other materials are required in a 3-phase system.

(ii) In polyphase system both 1-phase (low voltage) and 3-phase (high voltage) can be provided from the same source.

(iii) Polyphase motors have uniform torque and self starting.

(iv) For an equal size motor the output of a 3-phase motor is high.

(v) Polyphase generators can easily be synchronized for parallel running.

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21. A circuit consists of voltage $V = 4 + j3$ and impedance $Z = 3 + j4$ ohms. Find the current, active power and reactive power.

(1st semester 2005)

Solution : $V = 4 + j3 = 5\angle 36\cdot 86$ volts.

$Z = 3 + j4 = 5\angle 53\cdot 13$ ohms.

Current $I = \frac{V}{Z} = 1\angle -16\cdot 27^0$ Ampere. $\phi = 53\cdot 13^0$

Active power $= VI\cos\phi = (5)(1)\cos 53\cdot 13 = 3$ watt.

Reactive Power $= VI\sin\phi = (5)(1)\sin 53\cdot 13 = 4$ VAR.

22. A 20 ohm resistance and 30 mH inductance are connected in series and the circuit is fed from a single phase 230 V, 50 Hz AC supply. Calculate (i) inductive reactance (ii) impedance (iii) admittance (iv) current (v) active power.

(1st semester 2005)

Solution : $R = 20\Omega$, $L = 30\times 10^{-3}\ H$ $V = 230V$ $f = 50\,Hz$

(i) Inductive reactance $X_L = 2\pi fL = 9\cdot 42\Omega$

(ii) Impedance $Z = R + jX_L = 20 + j9\cdot 42 = 22\cdot 1\angle 25\cdot 22\,\Omega$

(iii) Admittance, $Y = \frac{1}{Z} = 0\cdot 0452\angle -25\cdot 22\,mho$

(iv) Current, $I = \frac{V}{Z} = 10\cdot 4A$

(v) Active Power $= VI\cos\phi = (230)(10\cdot 4)\cos 25\cdot 22 = 2\cdot 16KW$

23. Two coils of impedance $25\cdot 23\angle 37^0$ ohms and $18\cdot 65\angle 68^0$ ohms are connected in series across single phase 230 V, 50 Hz AC supply. Determine (i) the total impedance (ii) current (iii) active power (iv) reactive power (v) power factor of the circuit.

(1st semester 2005)

Solution : $Z_1 = 25\cdot 23\angle 37^0 = 20\cdot 15 + j15\cdot 18$

$Z_2 = 18\cdot 65\angle 68^0 = 6\cdot 986 + j17\cdot 29$

(i) Total impedance $Z = 27\cdot 136 + j32\cdot 47 = 42\cdot 31\angle 50\cdot 11\Omega$

(ii) Current, $I = \frac{V}{Z} = \frac{230}{42\cdot 31} = 5\cdot 436A$

(iii) $\phi = 50\cdot 11$

Active Power $= VI\cos\phi = (230)(5\cdot 436)\cos 50\cdot 11 = 0\cdot 801KW$

(iv) Reactive Power $= VI\sin\phi = (230)(5\cdot 436)\sin 50\cdot 11 = 959\cdot 31VAR$

(v) Power factor $= \cos\phi = 0\cdot 6413$

24. A star connected three phase load has a resistance of 8 ohms and a capacitive reactance of 10 ohms in each phase. The load is fed from a 3-phase 400 V, 50 Hz balanced supply. Calculate (i) the line current (ii) active and reactive power.

(1st semester 2005)

Solution : $R = 8\Omega$, $X_C = 10\Omega$

$Z_{ph} = R - jX_C = 8 - j10 = 12\cdot8\angle -51\cdot34\Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230\cdot94$ volts.

(i) Line Current = Phase Current $= \frac{V_{ph}}{Z_{ph}} = 18\cdot04A$

(ii) Active Power $= \sqrt{3}V_L I_L \cos\phi = \sqrt{3}(400)(18\cdot04)\cos(-51\cdot34)$
$= 7807\cdot54$ watt.

Reactive Power $= \sqrt{3}V_L I_L \sin\phi = -9759\cdot359$ VAR

25. A circuit has current $I = 4 + j3$ Ampere voltage $V = 6 + j8$ volts. Give the complex form of power. ***(2nd semester 2005)***

Solution : Complex form of power $= VI^*$

Given $V = 6 + j8$ volts.

$I = 4 + j3A$

$I^* = 4 - 3jA$

Complex form of power $= VI^* = (6 + j8)(4 - 3j)$

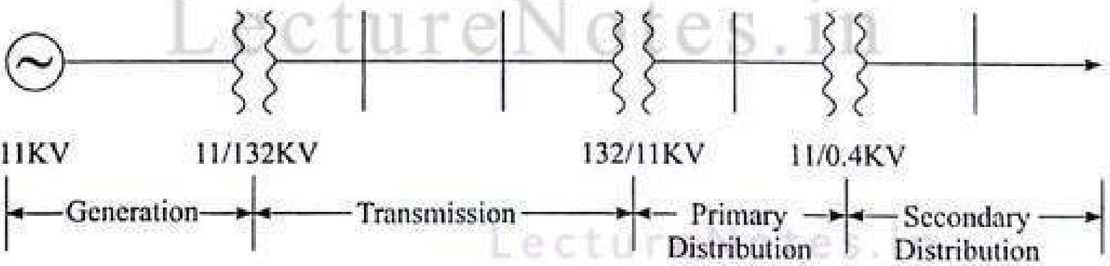
$= 48 + 14j$

Real Power = Active Power = watt

Reactive Power = 14 VAR

26. Name the different sections of power system by which the electric power reaches your home from the generating station. ***(2nd semester 2005)***

Solution :



27. Three impedances of each $(8 + j6)\Omega$ are connected in star and to a 200V, 50 Hz, 3-phase supply. What is the total power consumed? What will be the change in power consumed if the same impedances are connected in delta ? ***(2nd semester 2005)***

Solution : (i) Star-connection :

$Z = 8 + j6 = 10\angle 36\cdot86^0\ \Omega$

$\phi = 36\cdot86^0$

$V_L = 200$ volts.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115 \cdot 47 \text{ volts.}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{115 \cdot 47}{10} = 11 \cdot 547 A$$

$$\therefore \quad I_L = 11 \cdot 547 A$$

Power consumed $= \sqrt{3} V_L I_L \cos\phi = \sqrt{3}(200)(11 \cdot 547)\cos 36 \cdot 86 = 3 \cdot 2 KW$

(ii) Delta - Connection :

$V_L = V_{ph} = 200$ volts.

$Z = 10\angle 36 \cdot 86\Omega$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{200}{10} = 20A$$

$I_L = \sqrt{3} I_{ph} = \sqrt{3}(20) = 34 \cdot 64 A$

Power $= \sqrt{3} V_L I_L \cos\phi = 9 \cdot 6 KW$

28. What are the various domestic used of electricity ? You have a heater rated 250 V, 1500 W. What is the resistance of heating element ? ***(2nd semester 2005)***

Solution : Some of the uses of electrical energy in domestic applications are,

(i) heating : Micro oven, heater, immersion heater, electrical kettle etc.

(ii) Cooling : refrigeration, Air conditioning

(iii) Motoring : Celling / table fan, water pump, vacuum cleaner etc.

The resistance of the heating element is $R = \frac{V^2}{P} = \frac{(250)^2}{1500} = 41 \cdot 66\Omega$

29. 2 impedances of values $(4 + j3)\Omega$ and $(4 - j3)\Omega$ are connected in parallel. Find the impedance and power factor of the parallel circuit. ***(2nd semester 2005)***

Solution : $Z_1 = 4 + j3 = 5\angle 36 \cdot 89\Omega$

$Z_2 = 4 - j3 = 5\angle -36 \cdot 89\Omega$

Total impedance $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$

$$= \frac{5\angle 36 \cdot 89 \times 5\angle -36 \cdot 89}{4 + j3 + 4 - j3}$$

$$= \frac{25\angle 0}{8} = 3 \cdot 125\angle 0^0\,\Omega$$

Power factor = $\cos\phi = \cos 0 = 1$

30. The potential drop across a circuit is represented by $(40+j25)$ volts with reference to the circuit current. The power absorbed by the circuit is 160 volts. Find (i) p.f. (ii) the complex impedance (iii) the magnitude of impedance and (iv) the magnitude of the circuit current.

(1st semester 2006)

Solution : $V = 40 + j25$ volts $= 47\cdot17\angle32^0$ volts, With current as reference vector.

By taking V as reference vector, $V = 47\cdot17\angle0^0$ volts. and $I = I\angle-32^0\ A$

(i) Given Power, P = 160 watt.

$\cos\phi = \cos(-32) = 0\cdot84(lag)$

(ii) Magnitude of circuit current, $I = \dfrac{P}{V\cos\phi} = \dfrac{160}{(47\cdot17)(0\cdot84)} = 4\cdot03A$

(iii) The Complex impedance, $Z = \dfrac{47\cdot17\angle0^0}{4\cdot03\angle-32} = 11\cdot7\angle32^0 = 9\cdot92 + j6\cdot2$ ohms.

(iv) Magnitude of impedance $= 11\cdot7$ ohms

31. Three similar coils having each a resistance of 10 ohms and an inductance of $0\cdot05H$ are connected in delta across a 3-phase, 400 V, 50 Hz supply. Calculate the total power absorbed and the line currents drawn from the source.

(1st semester 2006)

Solution : $R = 10\Omega, L = 0\cdot05H$, f = 50 Hz

$X_L = 2\pi fL = 15\cdot7\Omega$

$Z_{ph} = R + jX_L = 10 + j15\cdot7 = 18\cdot61\angle57\cdot5\Omega$

$V_L = V_{ph} = 400$ volts, $\phi = 57\cdot5^0\ (lag)$

$I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{18\cdot61} = 21\cdot5A$

$I_L = \sqrt{3}\,I_{ph} = 37\cdot227A$

Power absorbed $= \sqrt{3}V_L I_L \cos\phi = \sqrt{3}(400)(37\cdot227)\cos57\cdot5$

$= 13\cdot857\,KW$

32. In a series circuit containing pure resistance and pure inductance, the current and voltage are expressed as :

$i(t) = 5\sin\left(314t + \dfrac{2\pi}{3}\right)$

and $V(t) = 15\sin\left(314t + \dfrac{5\pi}{6}\right)$

(i) What is the impedance of the circuit ?

(ii) What is the value of the resistance ?

(iii) What is the inductance in henries ?

(iv) What is the power drawn and p.f. ?

(2nd semester 2006)

Solution : $i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right) = 5\sin\left(314t + 120^0\right)$

$$V(t) = 15\sin\left(314t + \frac{5\pi}{6}\right) = 15\sin\left(314t + 150^0\right)$$

(i) $Z = \frac{15}{5} = 3\Omega$

(ii) $\phi = 30^0\ (lag)$ $\quad R = Z\cos\phi = 2\cdot 59\Omega$

(iii) $X_L = Z\sin\phi = 1\cdot 5\Omega$

$$L = \frac{1\cdot 5}{2\pi f} = \frac{1\cdot 5}{314} = 4\cdot 77\ mH$$

(iv) $\text{p.f} = \cos\phi = 0\cdot 866$

Power drawn $= VI\cos\phi = \frac{15}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos 30$

= 32.47 watt.

33. Three similar coils having a resistance of 20Ω and an inductance of $0\cdot 05H$ are connected in star to form a 3- phase balanced load. What will be the line currents drawn by this load when connected to a 3-phase, 50 Hz supply with 400 V between lines. *(2nd semester 2006)*

Solution : $R = 20\Omega,\quad L = 0\cdot 05H,\quad f = 50Hz$

$$X_L = 2\pi fL = 15\cdot 7\Omega$$

$$Z_{ph} = R + jX_L = 10 + j15\cdot 7 = 18\cdot 61\angle 57\cdot 5\Omega$$

$$V_L = 400V$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230\cdot 94 \text{ volts}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230\cdot 94}{18\cdot 61} = 12\cdot 40A$$

$$I_L = I_{ph} = 12\cdot 40A$$

34. A balanced star connected load of 10 + j8 ohms per phase is connected to a 3-phase 250 V supply. Find the line current and the p.f. *(1st semester 2007)*

Solution : $V_L = 250V$, $Z_{ph} = 10 + j8 = 12\cdot 8\angle 38\cdot 6$ ohms

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{250}{\sqrt{3}} = 144 \cdot 34 \text{ volts.}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{144 \cdot 34}{12 \cdot 8} = 11 \cdot 276 A$$

$$I_L = I_{ph} = 11 \cdot 276$$

$$\text{p.f} = \cos\phi = \cos 38 \cdot 6 = 0 \cdot 781$$

35. Calculate the power dissipated in a 15 ohm resistance when a voltage of 225+225 sin 314t is applied across it. *(2nd semester 2007)*

Solution : Power dissipated $= \frac{(225)^2}{15} + \frac{\left(225/\sqrt{2}\right)^2}{15} = 5063$ watt.

36. Three similar resistors connected in star draw a line current of 10A from a 3-phase 415 V, 50 Hz balanced supply. What should be the value of the line voltage to obtain the same line current with the resistors connected in delta ? *(2nd semester 2007)*

Solution :

(i) In star :

$$I_L = I_{ph} = 10A$$

$$V_L = 415 \text{ volts.}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239 \cdot 6 \text{ volts.}$$

$$R_{ph} = \frac{V_{ph}}{I_{ph}} = 23 \cdot 96\Omega$$

(ii) In delta :

$$I_L = 10A$$

$$I_{ph} = \frac{10}{\sqrt{3}} = 5 \cdot 77 A$$

$$R_{ph} = 23 \cdot 96\Omega$$

$$V_{ph} = I_{ph} R_{ph} = 13825 \text{ volts.}$$

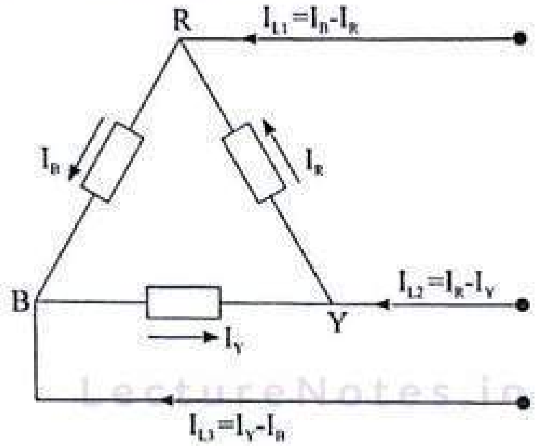
$$V_L = V_{ph} = 138 \cdot 25 \text{ volts.}$$

37. A 3-phase, 3 wire, 240 volts, 50 Hz, RYB system of supply has a delta connected load with $Z_{RY} = Z_{YB} = Z_{BR} = 15\angle -30^0$ ohms. Obtain the three line currents and draw the complete phasor diagram showing the line voltages, phase currents and line currents.

(2nd semester 2007)

Solution : $V_{RY} = 240\angle 0$, $V_{YB} = 240\angle -120^0$, $V_{BR} = 240\angle -240$

$V_{RY} = V_{YB} = V_{BR} =$ phase voltages = Line voltages



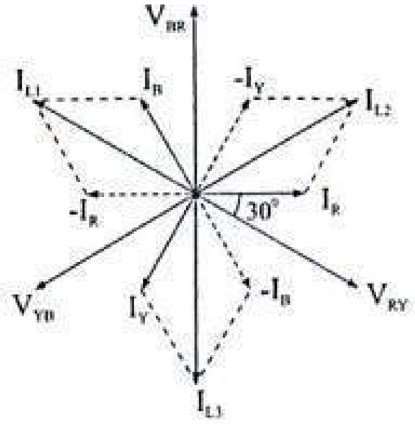


Fig shows the phasor diagram.

$Z_{ph} = 15\angle -30^0 = 12\cdot 99 - j7\cdot 5\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{240}{15} = 16A$

$I_L = \sqrt{3}\, I_{ph} = 27\cdot 712A$

38. A 3-phase balanced delta connected load is connected to a symmetrical 3-phase 440 V balanced supply. The current in each phase is 50 amperes and lags 30 degrees behind the corresponding phase voltage. Find the Line current and the total power.

(1st semester 2008)

Solution : $V_L = V_{ph} = 440$ volts

$I_{ph} = 50A,\ \phi = 30^0$ (lag)

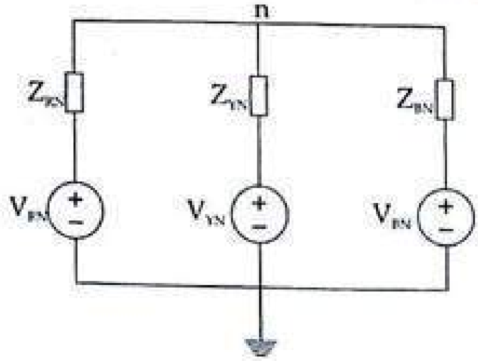
$I_L = \sqrt{3}\, I_{ph} = 50\sqrt{3} = 86\cdot 6A$

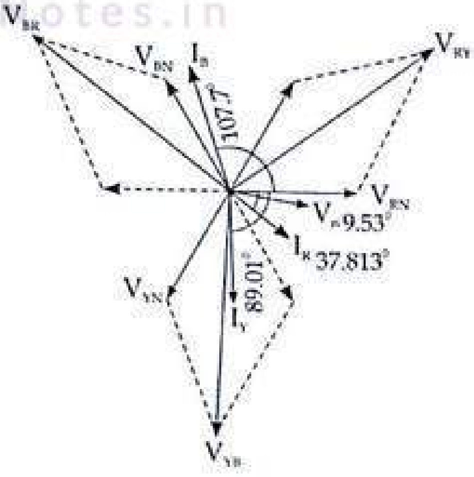
Total Power = $\sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3}(440)(86\cdot 6)\cos 30 = 57\cdot 154\,KW$

39. A 3-phase, 3 wire, 415 volts, 50 Hz, RYB system of balanced supply is connected to a star connected load with $Z_{RN} = 110\angle 40^0\ \Omega, Z_{YN} = 105\angle -50^0\ \Omega$ and $Z_{BN} = 90\angle 30^0\ \Omega$, where 'N' is the neutral point of the star connection. Obtain the three line currents and draw the complete phasor diagram showing the line voltages, Phase voltages and line currents. The supply neutral is earthed.

(1st semester 2008)

Solution :





Given Line voltage $V_L = 415$ volts.

Phase voltage $V_{ph} = \frac{415}{\sqrt{3}} = 239 \cdot 6$ volts.

$V_{RN} = 239 \cdot 6\angle 0°$ volts $\quad V_{YN} = 239 \cdot 6\angle -120°$ volts

$V_{BN} = 239 \cdot 6\angle 120°$ volts.

Let V_n = voltage at node 'n'.

Apply KCL to node n,

$$\frac{V_{RN} - V_n}{Z_{RN}} + \frac{V_{YN} - V_n}{Z_{YN}} + \frac{V_{BN} - V_n}{Z_{BN}} = 0$$

$$\Rightarrow \frac{V_{RN}}{Z_{RN}} + \frac{V_{YN}}{Z_{YN}} + \frac{V_{BN}}{Z_{BN}} = V_n\left[\frac{1}{Z_{RN}} + \frac{1}{Z_{YN}} + \frac{1}{Z_{BN}}\right]$$

$$\Rightarrow V_n = \frac{\frac{V_{RN}}{Z_{RN}} + \frac{V_{YN}}{Z_{YN}} + \frac{V_{BN}}{Z_{BN}}}{\frac{1}{Z_{RN}} + \frac{1}{Z_{YN}} + \frac{1}{Z_{BN}}} = \frac{\frac{239 \cdot 6\angle 0}{110\angle 40°} + \frac{239 \cdot 6\angle -120°}{105\angle -50°} + \frac{239 \cdot 6\angle 120°}{90\angle 30°}}{\frac{1}{110\angle 40°} + \frac{1}{105\angle -50°} + \frac{1}{90\angle 30°}}$$

$\Rightarrow V_n = 113 \cdot 116\angle -9 \cdot 53°$ volts

$= 111 \cdot 55 - j18 \cdot 727$ volts

$$I_R = \frac{V_{RN} - V_n}{Z_{RN}} = \frac{239 \cdot 6\angle 0 - 113 \cdot 116\angle -9 \cdot 53°}{110\angle 40°} = 1 \cdot 147\angle -37 \cdot 813°\ A$$

$$I_y = \frac{V_{YN} - V_n}{Z_{YN}} = \frac{239 \cdot 6\angle -120° - 113 \cdot 116\angle -9 \cdot 53°}{105\angle -50} = 2 \cdot 9433\angle -89 \cdot 01°\ A$$

$$I_B = \frac{V_{BN} - V_n}{Z_{BN}} = \frac{239 \cdot 6\angle 120° - 113 \cdot 116\angle -9 \cdot 53}{90\angle 30} = 3 \cdot 505\angle 107 \cdot 699\ A$$

40. A balanced three phase star connected load of 200 KW takes a leading current of 150 A with a line voltage of 1100 V, 50 Hz. Find the circuit constants (resistance and capacitance) of the load per phase. ***(1st semester 2008)***

Solution : $I_L = I_{ph} = 150A$

$V_L = 1100$ volts, f = 50 Hz $\quad V_{ph} = \frac{V_L}{\sqrt{3}} = 635 \cdot 10$ volts. $\quad Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{635 \cdot 10}{150} = 4 \cdot 234\Omega$

Power $= \sqrt{3} V_L I_L \cos\phi$

$$\Rightarrow \cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{200 \times 10^3}{\sqrt{3} \times 1100 \times 150} = 0 \cdot 7 \text{ (lead)}$$

$R = Z_{ph} \cos\phi = 2 \cdot 963\Omega \quad X_C = Z_{ph} \sin\phi = 3 \cdot 02\Omega \quad \Rightarrow \frac{1}{2\pi fc} = 3 \cdot 02 \Rightarrow C = 1 \cdot 053 \times 10^{-3} F$

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41. Three identical impedances connected in star fashion draw a line current of $5\angle-30^0$ A, when connected across a 400 V, 50 Hz, three phase AC supply. Find the resistance and reactance of impedance per phase. *(1st semester 2009)*

Solution : $I_L = 5\angle-30^0 A$, $V_L = 400$ volts, f = 50 Hz

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230\cdot94V$$

$$I_{ph} = I_L = 5\angle-30^0 A$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230\cdot94}{5\angle-30} = 46\cdot188\angle30^0\ \Omega$$

$$\phi = 30^0\ \text{(lag)}$$

$$R_{ph} = Z_{ph}\cos30 = 40\Omega$$

$$X_{ph} = Z_{ph}\sin30 = 23\cdot09\Omega\ \text{(Inductive)}$$

42. A resistance of 10Ω, an inductor of inductance of 2H and a capacitor of capacitance 100 micro farad are connected to a single phase 230 V AC source of varies frequency. Find (i) current, (ii) p.f (iii) active power consumption corresponding to supply frequencies of 50 Hz and 100 Hz respectively. *(1st semester 2009)*

Solution : $R = 10\Omega$, L = 2H, $C = 10\times10^{-6}\ F$, V = 230 volts

(i) when frequency f = 50 Hz then

$$\omega = 2\pi f = 2\pi(50) = 314\frac{rad}{sec}$$

$$X_L = \omega L = 628\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314\times10\times10^{-6}} = 318\cdot47\Omega$$

$$Z = R + j(X_L - X_C) = 10 + j(628 - 318\cdot47)$$

$$= 10 + j309\cdot53 = 309\cdot69\angle88\cdot15^0\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{309\cdot69\angle88\cdot15} = 0\cdot74\angle-88\cdot15A$$

Similarly when supply frequency is f = 100 Hz then $\omega = 2\pi f = 628\frac{rad}{sec}$

$$X_L = \omega L = 1256\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{628\times10\times10^{-6}} = 159\cdot23\Omega$$

$$Z = R + j(X_L - X_C) = 10 + j(1256 - 159\cdot23)$$

$$= 10 + j1096 \cdot 77 = 1096 \cdot 81 \angle 89 \cdot 477\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{1096 \cdot 81 \angle 89 \cdot 477} = 0 \cdot 2097 \angle 89 \cdot 477\ A$$

(ii) When f = 50 Hz then Power factor = cos 88.15 = 0.0322

When f = 100 Hz then Power factor = cos 89.477 = 9.12×10^{-3}

(iii) Active Power = $VI\cos\phi = (230)(0 \cdot 74)(0 \cdot 0322) = 5 \cdot 48$ watt

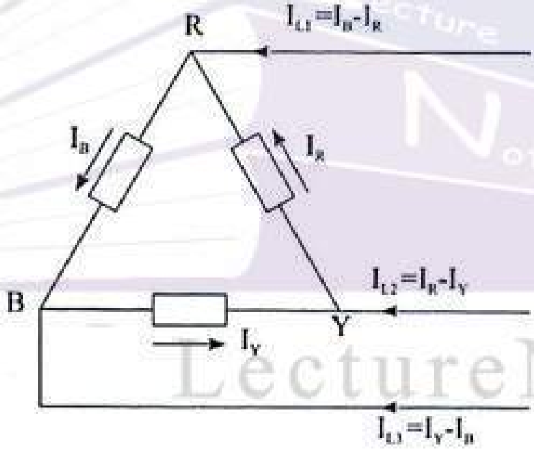
Active Power = $VI\cos\phi = (230)(0 \cdot 2097)\left(9 \cdot 12 \times 10^{-3}\right) = 0 \cdot 439$ watt

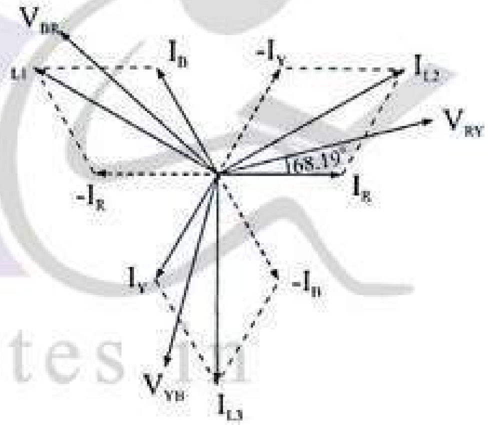
43. A balanced 3-phase delta load has load impedance of $10 + j25$ ohms per phase and is supplied from a balanced 3-phase 400 volts, 50 Hz, AC supply. Draw the phasors indicating the values of

(i) Line voltages, phase voltages and

(ii) Line currents, phase currents. *(1st semester 2009)*

Solution :





(i) $Z_{ph} = 10 + j25 = 26 \cdot 92 \angle 68 \cdot 19$ ohms.

$V_L = V_{ph} = 400$ volts.

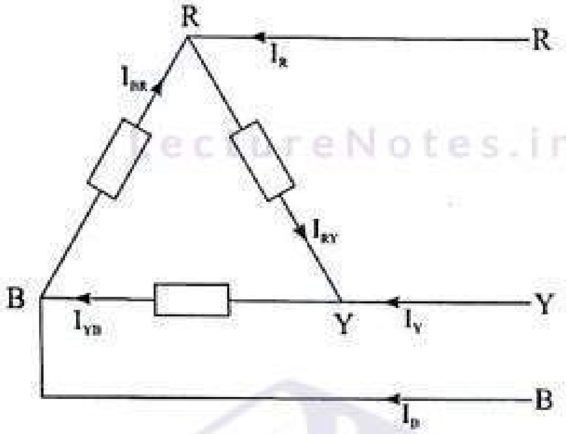
(ii) $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{26 \cdot 92 \angle 68 \cdot 19} = 14 \cdot 85 \angle -68 \cdot 19\ A$

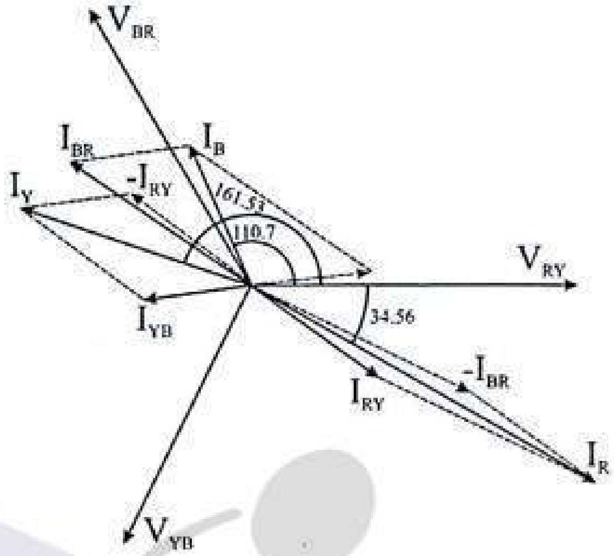
$I_L = \sqrt{3}\, I_{ph} = 25 \cdot 7\ A$

$\phi = 68 \cdot 19$ (lag)

44. A 3-phase, 3 wire, 415 volts, 50 Hz, RYB system of balance supply is connected to a delta connected load with $Z_{RY} = 120\angle 40^0$ ohms, $Z_{YB} = 155\angle 50^0$ ohms and $Z_{BR} = 100\angle -30^0$ ohms. Obtain the 3-line currents and draw the complete phasor diagram showing the line voltages, line currents and phase currents. The supply neutral is earthed.

(2nd semester 2009)





Let,

$I_R = I_y = I_B =$ Line currents.

$I_{RY} = I_{YB} = I_{BR} =$ Phase currents.

$V_{RY} = V_{YB} = V_{BR} =$ Phase voltages = Line voltages.

$V_{RY} = 415\angle 0°$ volts.

$V_{YB} = 415\angle -120°$ volts.

$V_{BR} = 415\angle 120°$ volts.

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{415\angle 0}{120\angle 40} = 3\cdot 458\angle -40° \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{415\angle -120}{155\angle 50} = 2\cdot 677\angle -170° \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{415\angle 120°}{100\angle -30} = 4\cdot 15\angle 150° \text{ A}$$

$$I_R = I_{RY} - I_{BR} = 3\cdot458\angle-40°-4\cdot15\angle150° = 6\cdot234 - j4\cdot295 = 7\cdot57\angle-34\cdot5 \text{ A}$$

$$I_Y = I_{YB} - I_{RY} = 2\cdot677\angle-170°-3\cdot458\angle-40° = 5\cdot556\angle161\cdot53° \text{ A}$$

$$I_B = I_{BR} - I_{YB} = 4\cdot15\angle150°-2\cdot677\angle-170° = 2\cdot71\angle110\cdot7° \text{ A}$$

45. A 3-phase balanced star-connected load is connected to a symmetrical 3-phase 440 volts balanced supply. The current in each phase is 60 amperes and lags 45^0 behind the corresponding phase voltage. Find the phase voltage and the total power.

(2nd semester 2009)

Solution : $V_L = 440V, \quad I_L = I_{ph} = 60A, \phi = 45^0$ (lag)

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254\cdot04 \text{ volts.}$$

$$\text{Power} = \sqrt{3}V_L\, I_L \cos\phi = \sqrt{3}(440)(60)\cos 45 = 32\cdot33\, KW$$

46. A balanced three phase star connected load of 250 KW takes a lagging current of 200 A with a line voltage of 1000V, 50 Hz. Find the circuit constants (resistance and inductance) of the load per phase. ***(2nd semester 2009)***

Solution :

$I_L = I_{ph} = 200A, \qquad V_L = 1000$ volts.

Power $= \sqrt{3}V_L\, I_L \cos\phi = 250\times10^3$ watt.

$$\therefore \qquad \cos\phi = \frac{250\times10^3}{\sqrt{3}V_L\, I_L} = \frac{250\times10^3}{\sqrt{3}(1000)(200)} = 0\cdot721$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1000}{\sqrt{3}} = 577\cdot367 \text{ volts.}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{577\cdot367}{200} = 2\cdot886 \text{ ohms.}$$

$$R_{ph} = Z_{ph}\cos\phi = (2\cdot886)(0\cdot721) = 2\cdot08 \text{ ohms.}$$

$$X_{ph} = Z_{ph}\sin\phi = (2\cdot886)(0\cdot692) = 1\cdot999 \text{ ohms}$$

$$L = \frac{X_{ph}}{2\pi f} = \frac{1\cdot999}{2\pi(50)} = 6\cdot366\times10^{-3} \text{ Henrys.}$$

Do Your Self

T5.1 A voltage v=20 sin cos volts is applied across a pure resistance of 1.5 Ω. Find the power dissipated in the resistor. **[13.33W]**

T5.2 A 50 μF capacitor is connected to a 100V, 200 Hz supply. Determine the true power and the apparent power. **[0, 628.3VA]**

T5.3 A motor takes a current of 10A when supplied from a 250V A.C. supply. Assuming a power factor of 0.75 lagging find the power consumed. Find also the cost of running the motor for 1 week continuously if 1 kWh of electricity costs Rs.3.50. **[1875W, Rs.1102.05]**

[Hint: electrical Energy = $\dfrac{V \times 1 \times \cos\phi \times Time\left(in\ hour\right)}{1000}$

T5.4 A motor takes a current of 12A when supplied frm a 240V AC supply. Assuming a power factor of 0.70 lagging, find the power consumed. **[2.016 kW]**

T5.5 A substation is supplying 200 kVA and 150 kvar. Calculate the corresponding power and power factor. **[132kW, 0.66]**

T5.6 A load takes 50kW at a power factor of 0.8 lagging. Calculate the apparent power and the reactive power. **[62.5 kVA, 37.5 kvar]**

T5.7 A coil of resistance 400Ω and inductance 0.20H is connected to a 75V, 400 Hz supply. Calculate te power dissipated in the coil. **[5.452W]**

T5.8 An 80Ω resistor and a 6μF capacitor are connected in series across a 150 V, 200 Hz supply. Calculate (a) the circuit impedance (b) the current flowing and (c) the power dissipated in the circuit. **[(a) 154.9Ω (b) 0.968A (c) 75W]**

T5.9 The power taken by a series circuit containing resistance and inductance is 240W when connected to a 200V, 50 Hz supply. If the current flowing is 2A find the values of the resistance and inductance. **[60Ω, 255mH]**

T5.10 A circuit consisting of a resistor in series with an inductance takes 210W at a power factor of 0.6 from a 50V, 100 Hz supply. Find (a) the current flowing, (b) the circuit phase angle, (c) the resistance, (d) the impedance and (e) the inductance.

[(a) 7A (b) 53.13^0 lagging (c) 4.286Ω (d) 7.143Ω (e) 9.095 mH]

T5.11 A 200V, 60 Hz supply is applied to a capacitive circuit. The current flowing is 2A and the power dissipated is 150W. Calculate the values of the resistance and capacitance.

[37.5Ω, 28.61 μF]

T5.12 A 415 V alternator is supplying a load of 55kW at a power factor of 0.65 lagging. Calculate (a) the kVA loading and (b) the current taken from the alternator. (c) If the power factor is now raised to unity find the new kVA loading. **[(a) 84.6 kVA (b) 203.9A (c) 84.6 kVA]**

T5.13 A single-phase motor takes 30A at a power factor of 0.65 lagging from a 240V, 50 Hz supply. Determine (a) the current taken by the capacitor connected in parallel to correct the power factor to unity, and (b) the value of the supply current after power factor correction.

[(a) 22.80A (b) 19.50 A]

T5.14 A 20Ω non-reactive resistor is connected in series with a coil of inductance 80mH and negligible resistance. The combined circuit is connected to a 200V, 50 Hz supply. Calculate (a) the reactance of the coil, (b) the impedance of the circuit, (c) the current in the circuit, (d) the power factor of the circuit, (e) the power absorbed by the circuit, (f) the value of a power factor correction capacitor to produce a power factor of unity, and (g) the value of a power factor correction capacitor to produce a power factor of 0.9. **[(a) 25.13Ω (b) $32.12\angle 51.49^0$Ω (c) $6.22\angle{-51.49^0}$ A (d) 0.623 (e) 775.5W (f) 77.56μF (g) 47.67μF]**

T5.15 A motor has an output of 6kW, an efficiency of 75% and a power factor of 0.64 lagging when operated from a 250V, 60 Hz supply. It is required to raise the power factor to 0.925 lagging by connecting a capacitor in parallel with the motor. Determine (a) the current taken by the motor, (b) the supply current after power factor correction, (c) the current taken by the capacitor, (d) the capacitance of the capacitor and (e) the kvar rating of the capacitor.

[(a) 50A (b) 34.59A (c) 25.28A (d) 268.2μF (e) 6.32 kvar]

T5.16 A 200V, 50 Hz single-phase supply feeds the following loads : (i) fluorescent lamps taking a current of 8A at a power factor of 0.9 leading, (ii) incandescent lamps taking a current of 6A at unity power factor, (iii) a motor taking a current of 12A at a power factor of 0.65 lagging. Determine the total current taken from the supply and the overall power factor. Find also the value of a static capacitor connected in parallel with the loads to improve the overall power factor to 0.98 lagging. **[21.74A, 0.966 lagging, 21.74μF]**

T5.17 Two single-phase 60 Hz sinusoidal source generators (with negligible internal impedances) are supplying to a common load of 10 kW at 0.8 power factor lagging. The impedance of the feeder connecting the generator G_1 to the load is 1.4+j1.6Ω, whereas that of the feeder connecting the generator G_2 to the load is 0.8+j1.0Ω. If the generator G1, operating at a terminal voltage of 462 V (rms), supplies 5kW at 0.8 power factor lagging, determine : (a) The voltage at the load terminal voltage of 462 V (rms), supplies 5 kW at 0.8 power factor lagging, determine : (a) The voltage at the load terminals; (b) The terminal voltage of generator G_2 and (c) The real power and the reactive power output of the generator G_2.

[433.8 V (rms), 452.75 V (rms), 10.4 kW, 8 kVAR]

T5.18 An impedance 15+J20Ω is connected across a 125 V, 60 Hz, source. Find (a) the instantaneous current through the load, (b) the instantaneous power, and (c) the average active and reactive powers. **[(a) 7.07 sin $\left(377t - 53.1^\circ\right) A$, (b) 1250 sin 377t sin $\left(377t - 53.1^\circ\right) W$, (c) 375 W, 500 VAR]**

T5.19 A voltage source of 100 V has internal impedance 0.1+j0.1 ohms and supplies a load having that same impedance. Calculate the power absorbed by the load. **[125W]**

T5.20 Three loads, each of resistance 50Ω are connected in star to a 400 V, 3-phase supply. Determine (a) the phase voltage, (b) the phase current and (c) the line current.

[(a) 231V (b) 4.62 A (c) 4.62 A]

T5.21 If the loads in question T5.20 are connected in delta to the same supply determine (a) the phase voltage, (b) the phase current and (c) the line current. **[(a) 400V (b) 8A (c) 13.86 A]**

T5.22 A star-connected load consists of three identical coils, each of inductance 159.2mH and resistance 50Ω. If the supply frequency is 50 Hz and the line current is 3A determine (a) the phase voltage and (b) the line voltage. **[(a) 212 V (b) 367V]**

T5.23 Three identical capacitors are connected (a) in star, (b) in delta to a 400V, 50 Hz, 3-phase supply. If the line current is 12 A determine in each case the capacitance of each of the capacitors. **[(a) 165.4μF (b) 55.13μF]**

T5.24 Three coils each having resistance 6Ω and inductance H are connected (a) in star and (b) in delta, to a 415V, 50 Hz, 3-phase supply. If the line current is 30 A, find for each connection the value of L. **[(a) 16.78mH (b) 73.84mH]**

T5.25 A 400 V, 3-phase, 4 wire, star-connected system supplies three resistive loads of 15 kW, 20 kW and 25 kW in the red, yellow and blue phases respectively. Determine the current flowing in each of the four conductors. **[IR=64.95A, IY=86.60A, IB=108.25A, IN=37.50A]**

T5.26 A 3-phase, star connected alternator delivers a line current of 65A to a balanced delta-connected load at a line voltage of 380V. Calculate (a) the phase voltage of the alternator, (b) the alternator phase current and (c) the load phase current.

[(a)219.4V (b) 65A (c) 37.53A]

T5.27 Three 24μ*F* capacitors are connected in star across a 400V, 50 Hz, 3-phase supply. What value of capacitance must be connected in delta in order to take the same line current? **[8μ*F*]**

T5.28 Determine the total power dissipated by three 20Ω resistors when connected (a) in star and (b) in delta to a 440V, 3-phase supply. **[(a) 9.68kW (b) 29.04 kW]**

T5.29 A balanced delta-connected load has a line voltage of 400V, a line current of 8A and a lagging power factor of 0.94. Draw a complete phasor diagram of the load. What is the total power dissipated by the load? **[5.21 kW]**

T5.30 Three inductive loads, each of resistance 4Ω and reactance 9Ω are connected in delta. When connected to a 3-phase supply the loads consume 1.2kW. Calculate (a) the power factor of the load, (b) the phase current, (c) the line current and (d) the supply voltage.

[(a) 0.406 (b) 10A (c) 17.32A (d) 98.53V]

T5.31 The input voltage, current and power to a motor is measured as 415V, 16.4A and 6kW respectively. Determine the power factor of the system. **[0.509]**

T5.32 A 3-phase, star-connected alternator supplies a delta connected load, each phase of which has a resistance of 15Ω and inductive reactance 20Ω. If the line voltage is 400V, calculate (a) the current supplied by the alternator and (b) the output power and kVA rating of the alternator, neglecting any losses in the line between the alternator and the load.

[(a) 27.71A (b) 11.52kW, 19.20kVA]

T5.33 Each phase of a delta-connected load comprises a resistance of 40Ω and a 40μF capacitor in series. Determine, when connected to a 415V, 50 Hz, 3-phase supply (a) the phase current, (b) the line current, (c) the total power dissipated, and (d) the kVA rating of the load.

[(a)4.66A (b) 8.07A (c) 2.605kW (d) 5.80kVA]

T5.34 A 440V, 3-phase a.c. motor has a power output of 11.25kW and operates at a power factor of 0.8 lagging and with an efficiency of 84%. If the motor is delta connected determine (a) the power input, (b) the line current and (c) the phase current.

[(a) 13.39kW (b) 21.97A (c) 12.68A]

T5.35 A 3-phase network is shown in Fig.T5.1 with phase voltage Van = 120∠0V. Determine (a) Ia (b) Van (c) the total power loss in the line resistances, and (d) total power input to the network

[(a) $2\angle -53\cdot 1^{0}\ A$; (b) $124.96\angle -2.9^{0}$ V; (c) 48 W; (d) 480 W]

T5.36 The star-connected load having impedance of (12-j16)Ω per phase is connected in parallel with the delta-connected load having impedance of (27+j18) Ω per phase (Fig. 19.2a), with both the loads being balanced, and fed from a three-phase, 230 V, balanced supply, with the phase sequence as R-Y-B. Find the line current, power factor, total power & reactive VA, and also total volt-amperes (VA). **[14.276A; 0.9945 lag; 5.688 KVA; 5.657 KW, 0.5925KVAR]**

T5.37 The star-connected load consists of a resistance of 15Ω, in series with a coil having resistance of 5Ω and inductance of 0.2H, per phase. It is connected in parallel with the delta-connected load having capacitance of 90μF per phase as shown in Fig.T5.2. Both the loads being balanced, and fed from a three phase, 400V, 50 Hz, balanced supply, with the phase sequence as R-Y-B. Find the line current, power factor, total power & reactive VA, and also total volt-amperes (VA). **[1.575A, 0.675lag, 737W, 805VAR]**

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Basic Electrical Engineering

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Rotating Electric Machine

Contributed By:

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Rotating Electric Machine

Chapter - 8

8.1 Introduction

The most widely used electro-mechanical device is a rotating machine, which utilises magnetic field to store energy. The main purpose of most rotating machine is a convert electro-mechanical energy i.e. to covert energy between electrical and mechanical system, either for electric power generation (as in generators) or for the production of mechanical power to perform useful tasks (as in motors). Rotating machines range in size and capacity from small motors that consume only a fraction of watt to large generators that produces several hundred megawatts. In spite of wide variety of types, sizes and methods of constructions, all such machines operate on the same principle namely, the tendency of two magnets to align themselves.

Out of all machines available induction, synchronous and DC machines are most used. Three modes of operation of a rotating machine are studies. They are motoring, generating and braking.

The Motoring mode has electrical power input and mechanical power output. The generating mode has mechanical power input and electrical power output. The braking mode has both mechanical and electrical input while all these inputs are dissipated as heat.

8.1.1 Classification of Electrical Machines

Electrical machines (both motors and generators) operates with three broad principles namely (i) principle of induction (ii) synchronous principle (iii) conventional principle (DC machines). Hence the classification can be made in the following ways.

1) Based on type of supply.

(a) DC machines

(b) AC machines

2) Based on supply to the rotor
   (a) induction machine
   (b) commutator / slipring machines
3) Based on type of ac supply
   (a) Single phase machine
   (b) Three phase machine

Taking all above classification in totality, fig. 8.1. shows the detailed classification of rotating machines.

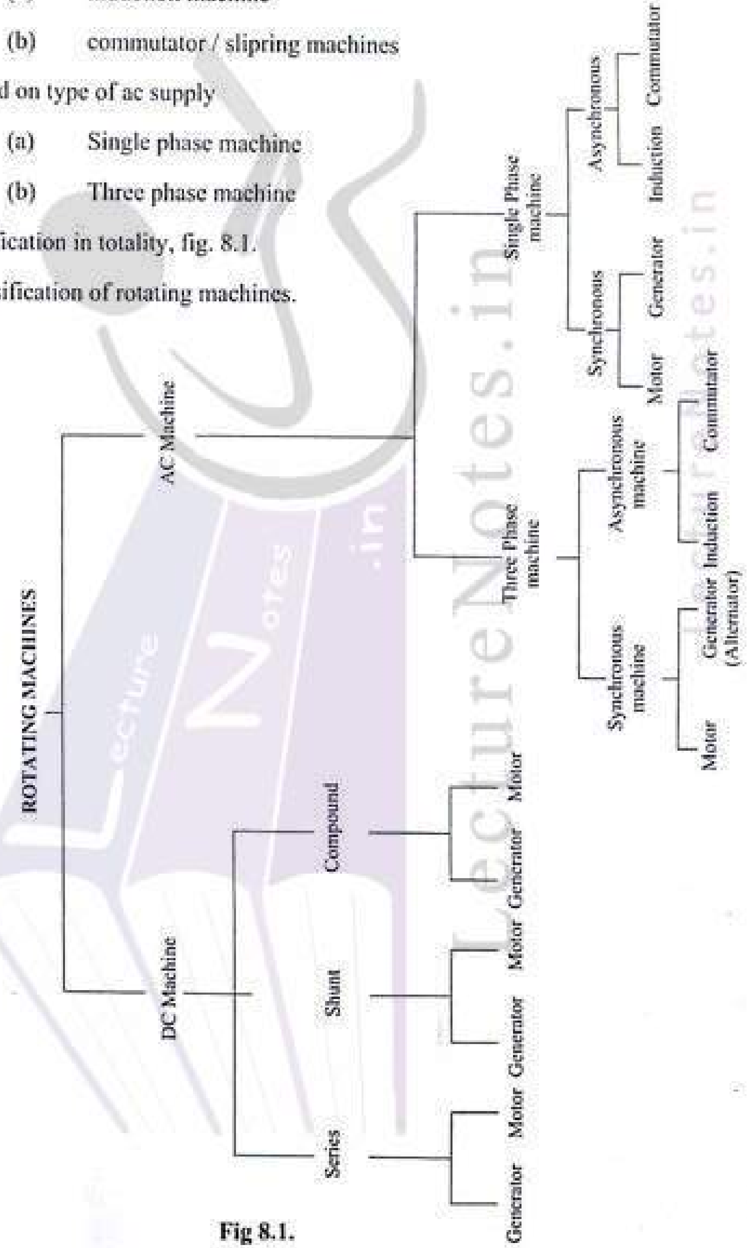

Fig 8.1.

8.1.2 Performance characteristics of Electrical Machines

Electrical machines are energy conversion devices. It is essential to calculate the energy conversion efficiency. Whether it is a motor or generator, these machines are associated with various energy losses. During the flow of energy from input to output these losses takes place in between. These losses are broadly classified as,

(i) Mechanical loss

(ii) Iron loss

(iii) Cupper loss

Mechanical Loss : These losses are also known as *rotational losses*. Which is due to friction at bearings commutators, slip rings etc. Windage loss is also part of rotational loss caused due to air friction.

Iron Loss : This is also known as core loss or *stray loss*. It consists of hysteresis loss and eddy current loss. For any machine these losses are assumed to be constant as it depends on the supply voltage.

Cupper Loss : This is also known as I^2R loss which is caused due to resistance of various windings including contact resistance between brushes and commutators. This is usually a variable loss except in some cases such as shunt field cu loss in which the current in constant.

To have an idea of the Power flow sequence in electrical machine fig 8.2. (a) and fig 8.2 (b) may be referred.

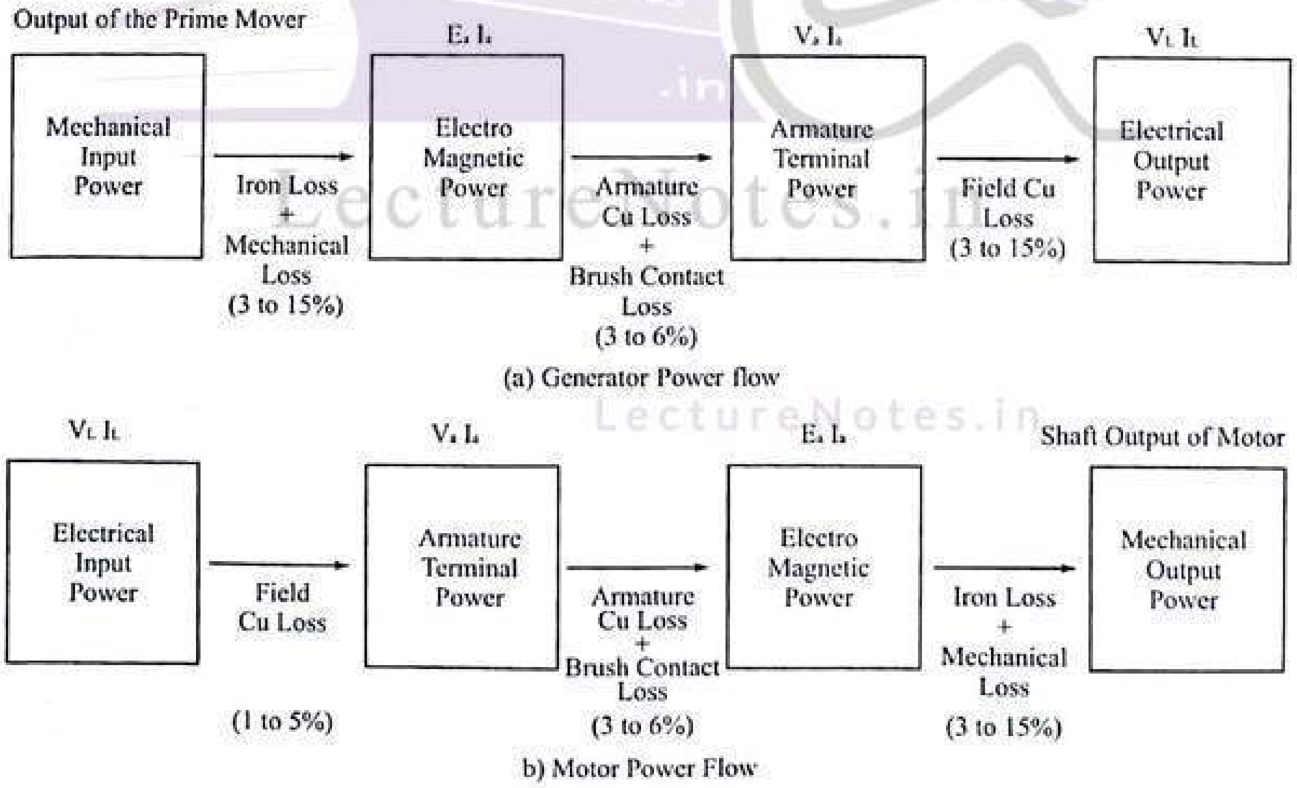

Fig 8.2

The performance of electric machine can be quantified in a number of ways. In case of motors other than efficiency the following characteristics are portrayed.

(i) Torque - Speed characteristics (for all motors)

(ii) Torque - Slip characteristics. (for induction motor only)

(iii) Torque - Current characteristics.

The torque - speed characterisitcs of a motor describe how the torque supplied by the machine varies as a function of speed of rotation of the motor for steady speed. In most engineering application, it is quite likely that the engineer required to make a decision regarding the performance characteristics of the motor best suited to a specified task.

Similarly torque current characteristics decides the torque of a motor at starting and during steady state operation.

Similarly for a generator the various performance characteristics other than efficiency are,

(i) voltage - current characteristics.

(ii) magnetisation characteristics.

The magnetisation characteristics speaks about the range of voltage in which the generator can operate without saturating the field. The voltage - current characteristics speaks about internal drop in voltage with increase in load current.

8.1.2.1 Voltage regulation

From the voltage current characteristics of any generator, it may be observed that the terminal voltage varies with increase in load current. In case of AC generator the voltage may increase or decrease depending on type of load (lagging or leading) with respect to increase in load current.

Fig 8.3 shows the terminal characteristics of a AC generator.

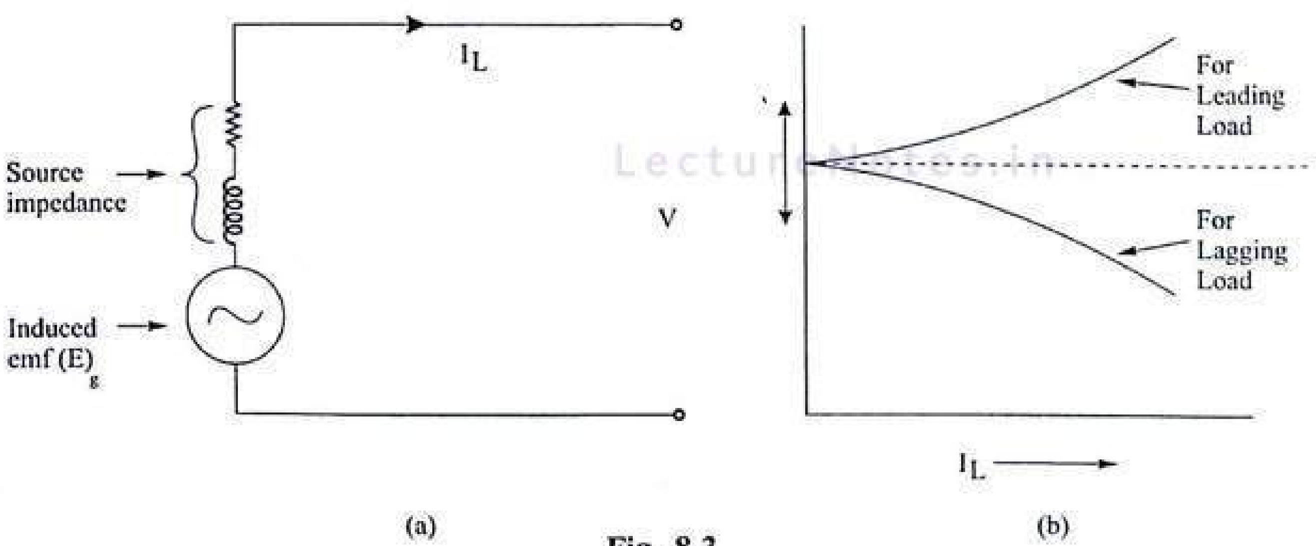

Fig. 8.3

However incase of dc generator the terminal voltage falls with increase in load current. The term ***voltage regulation is defined as change in terminal voltage from no load (V_{nl}) to full load (V_{fl}) represented in percentage of full load voltage.***

$$\text{Mathematically percentage regulation} = \frac{V_{nL} - V_{fL}}{V_{fL}} \times 100$$

8.1.2.2 Speed regulation

In case of a motor the load (torque) speed characteristics says that speed varies with variation in Load torque, except in case of a synchronous motor in which the speed remains constant irrespective of load torque. These variation in speed depends on the types of motor. Typical speed torque characteristics for various machines as shown in fig 8.4.

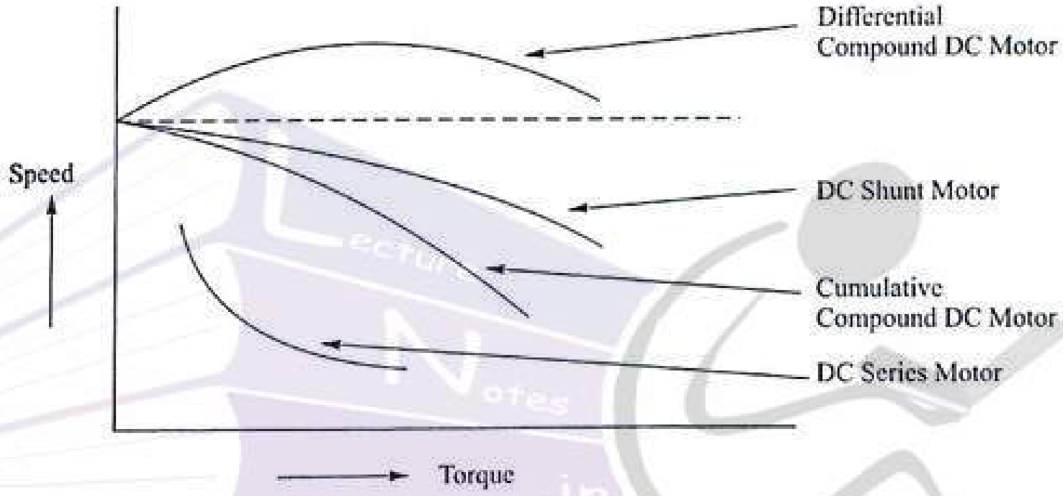

Fig 8.4

The term speed regulation of a motor can be defined as change in speed from no load $(speed_{nl})$ to full load $(speed_{fl})$ expressed in percentage of full load speed.

$$\text{Mathematically speed regulation} = \frac{speed_{nl} - speed_{fl}}{speed_{fl}} \times 100$$

8.2 DC Machines

In previous chapter it is indicated that most electrical machines operate on the basis of interaction between current carrying conductors and electro-magnetic fields. In particular generator action is based on Faraday Laws of electro-magnetic induction which implies that a voltage is induced in a conductor moving in a region having flux lines right angle to the conductor. In other words if a straight conductor of length ℓ metre moves at velocity $V m.sec^{-1}$ through a uniform magnetic field of flux density, $B\, wb.\, m^{-2}$ the conductor it self always at right angle to $B\, wb.\, m^{-2}$ then effective inducing voltage is the conductor in given by, $e = B\ell V$ volts.

Similarly a motor which operates on the principle that whenever a current carrying conductor is placed in an uniform magnetic field then a force is developed on the conductor. In other words if a straight conductor of ℓ meter carrying I ampers placed in a uniform magnetic field of B $wb.\ m^2$, a force

F will act on the conductor whose magnitude will be given by $F=BI\ell$ Newtons.

This is valid only if the direction of current is at right angles to the field. The direction of the force can be obtained from Flemings left hand rule.

In totality it can be seen that in an uniform magnetic field if a conductor is supplied with a current, a force is being exerted on it. Similarly if the same conductor is displaced by an external force in the same field, causes a current to flow in it. In the first case it is a motor while in the later case it is a generator. Hence constructionally there is no difference between a dc generator and dc motor. In general they are termed as dc machines. A dc machine can work as generator or motor depending upon the input it receives. The term dc (direct current) referred to the type of electricity supplied to or generated by it.

8.2.1 Construction of DC Machine

Fig 8.5 (a) shows some important parts of dc machine. The field poles are mounted on the stator which produce the needed flux. The field poles carry windings called field windings or field coils. Some machines carry several sets of field windings on the same pole core. The cores of the poles are built of sheet steel lamination. (because the field windings carry direct current, it is not electrically necessary to have the cores laminated). It is, however, necessary for the pole faces to be laminated, because of their proximity to the armature windings. The armature core which carries the armature windings is generally on the rotor and is made of sheet-steel laminations. The commutator is made of hard-drawn copper segments insulated from one another by mica. As shown in fig 8.5 (b)and (c)the armature windings are connected to the commutator segments over which the carbon brushes slide and serve as leads for electrical connection. The armature windings is the load carrying winding.

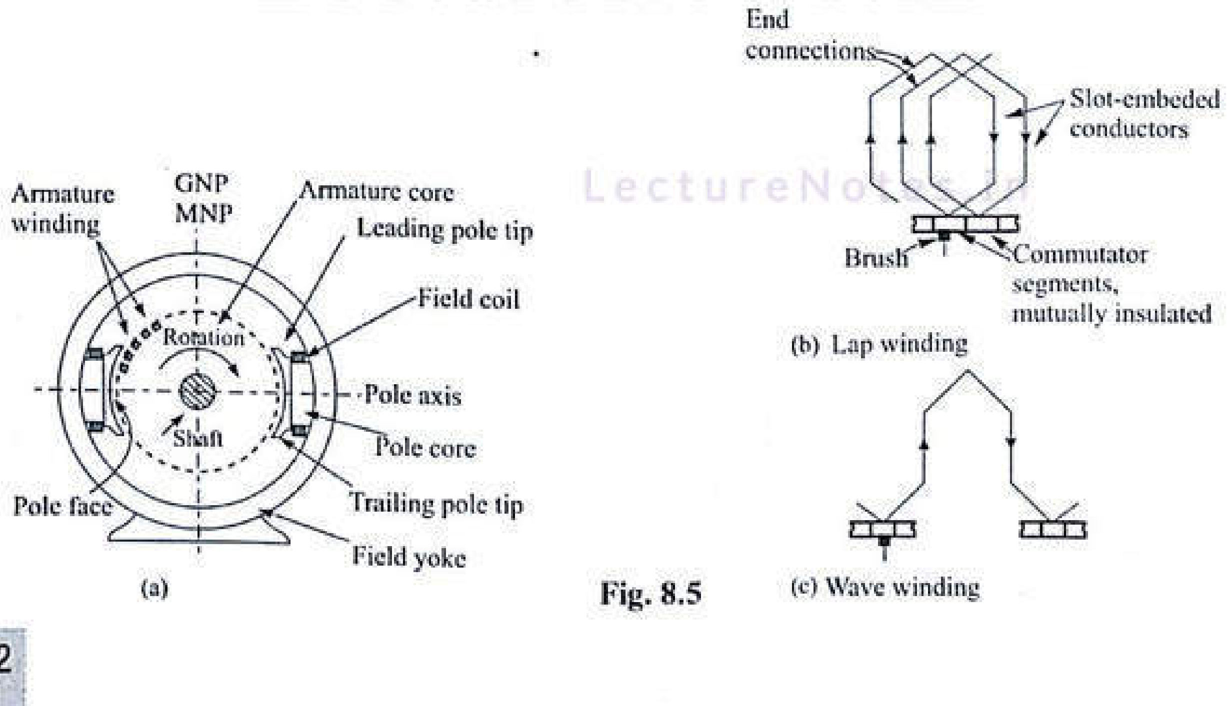

Fig. 8.5

8.2.2 Armature windings

The armature winding may be a lap winding or a wave winding are shown in fig 8.5 (b) & (c). The various coils forming the armature winding may be connected in a series - parallel combination. ***It is found that in a simplex lap winding the number of paths in parallel (A) is equal to the number of poles (P) where as in a simplex wave winding the number of parallel paths (A) is always 2.***

8.2.3 Classification of DC Machines

A DC machine consist of two electric circuits, namely (i) armature circuit (ii) field circuit. Armature circuit is responsible for receiving or supplying electric power through commutator and brushes. The field circuit or the windings wound around the field poles for generating regulated flux in the machine.

In case of a generator or motor field winding is supplied with external dc supply where as armature circuit generates electricity in case of a generator while it receives electric power in case of a motor. Depending on the field connection with the armature circuit they are broadly classified as (i) shunt (ii) series (iii) compound.

In a shunt machine, field is connected in parallel with the armature while in a series machine field is connected in series with the armature. In some machines there are two field windings wound on the same pole. one winding is connected in series and the other is connected in parallel with the armature. Those machines are known as compound machines. The fig 8.6. shows the detailed classification of dc machine.

In case of a motor if the series field flux is in phase with the shunt field flux then it is known as cumulative compound while if series flux opposes the shunt flux then it is known differential compound.

8.3 Emf Equation

To evaluate the induced emf in a dc machine (generator), let us consider the machine with the following specification.

ϕ = flux produced per pole in wb.

Z = no of armature conductors.

N = no of revolutions armature rotates in rpm

P = no of poles

A = no of parallel paths of the armature circuit (A = 2 for wave wound and A = P for lap wound)

In one complete revolution, one armature conductor passes through P no of poles. Therefore flux cut by the conductor in one revolution = ϕP wb

Time taken for completing N revolutions = 60 sec

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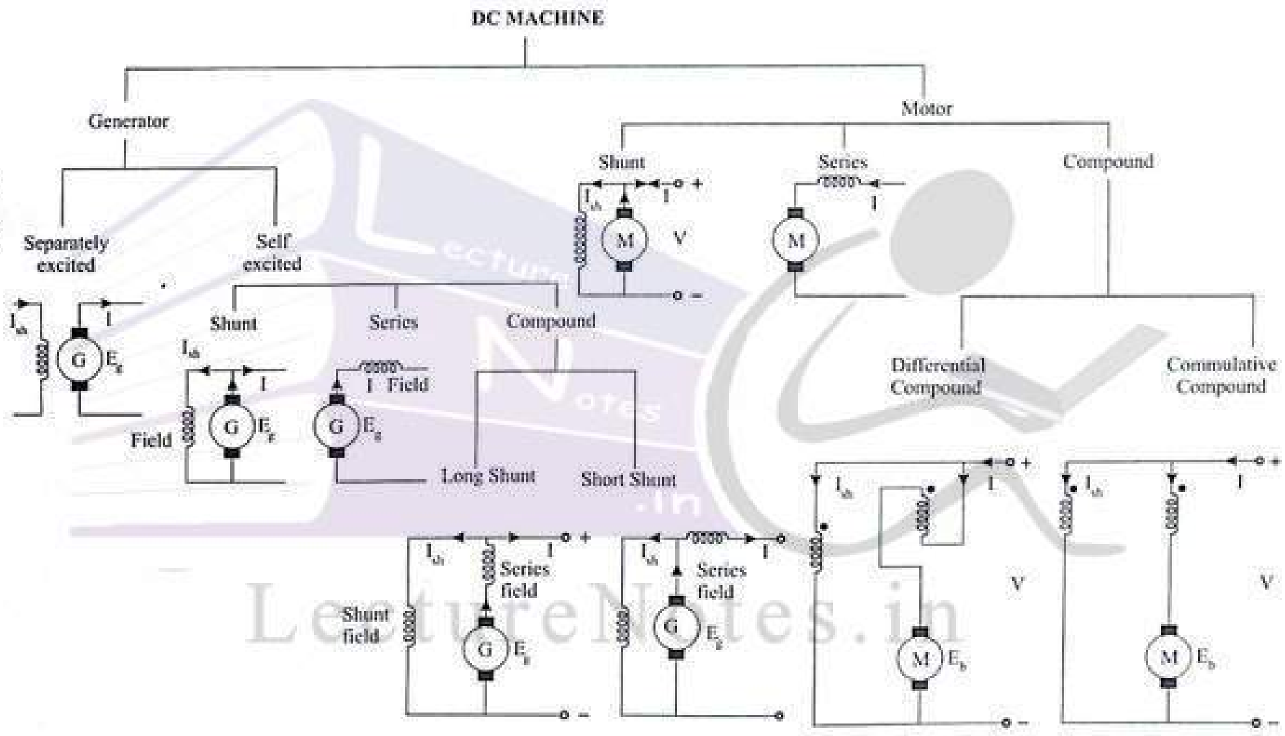
DC MACHINE
Generator
Motor
Separately excited
Self excited
Shunt
Series
Compound
Field
Long Shunt
Short Shunt
Series field
Shunt field
Differential Compound
Commulative Compound
G
M
V
I

Fig. 8.6.

Therefore time taken for one revolution = 60/N seconds

According to Faradays laws of electro-magnetic induction average induced emf in the conductor is given by,

$$e = \frac{d\phi}{dt} \text{ (considering magnitude only)}$$

Therefore in the above example emf induced per conductor is

$$\Rightarrow e = \frac{\phi P}{60/N}$$

$$\Rightarrow e = \frac{\phi PN}{60} \text{ volts.}$$

There are Z no of conductors in the armature which are arranged in A no of parallel paths i.e. there will be Z/A no. of conductors in each parallel path which are connected in series as shown in fig 8.7. Induced emf in the generator = emf per parallel path.

= emf induced per conductor × no.of conductors in each path.

Mathematically $E_g = e \times \frac{Z}{A}$

$$\Rightarrow E_g = \frac{\phi PN}{60} \times \frac{Z}{A}$$

$$\Rightarrow E_g = \frac{\phi ZNP}{60A} \text{ volts.}$$

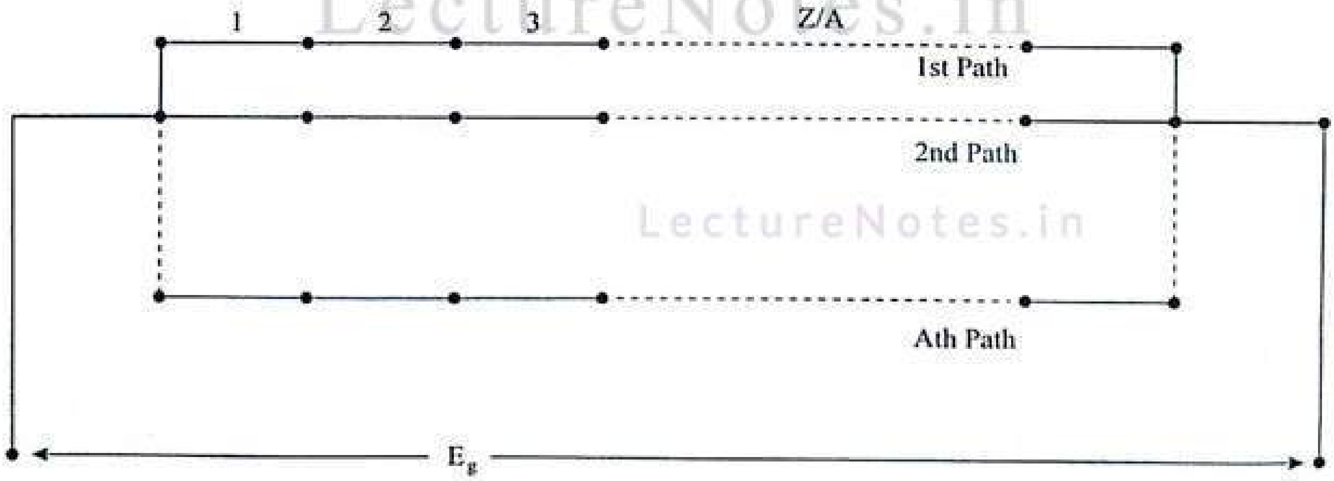

Fig. 8.7.

8.3.1 Generator Voltage and Power Equation

Like any other source, terminal voltage of a generator falls with increase in load current.

This is due to presence of resistance of armature circuit (R_a). If E_g is the generator emf, V is the terminal voltage when supplying a current of I amperes, the equivalent curcuit can be drawn as shown in fig. 8.8.

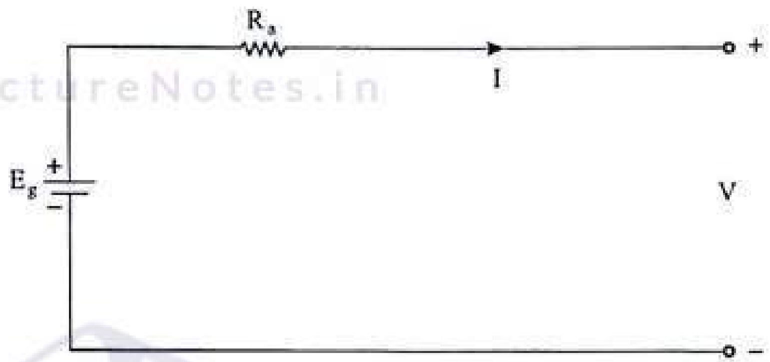

Fig 8.8

Applying KVL to the above curcuit,

$$E_g - IR_a - V = 0$$

$$\Rightarrow E_g = V + IR_a \text{ .... } (1)$$

Multiplying equation (1) by I in both sides,

$$E_g I = VI + I^2 R_a \text{ .... } (2)$$

Equation (2) represents general power equation of the generator

Where $E_g I$ = Electrical power generated in the armature.

VI = terminal output power.

I^2R_a = cu loss in the armature.

Therefore terminal output power, $P = E_g I - I^2R_a$ .... (3)

To get the condition for maximum power output, equation (3) is differentiated with respect to I and equated to zero.

Mathematically, $$\frac{dp}{dI} = 0$$

$$\Rightarrow E_g - 2IR_a = 0$$

$$\Rightarrow E_g = 2IR_a \text{ ....} (4)$$

From equation (1) $IR_a = E_g - V$ .... (5)

Substituting the value of IR_a from equation (5) in equation (4).

$$E_g = 2(E_g - V) = 2E_g - 2V$$

$$\Rightarrow E_g = 2V$$

$$\Rightarrow V = \frac{E_g}{2}$$

Thus the output power in the generator will be maximum when the terminal voltage will half of the emf generated.

8.4 Voltage and Power equation of dc Motor

Even though a motor converts electrical energy to mechanical energy in the form of rotation, it satisfies all requirements of a generator i.e. the armature conductors are rotating in an uniform magnetic field. This causes an induced emf in the armature and named as ***back emf*** (E_b) or counter emf. This is because this emf opposes the supply voltage. The magnitude is given by $E_b = \frac{\phi ZNP}{60A}$ volts.

The equivalent circuit of a dc motor is shown in fig. 8.9.

Apply KVL to above circuit,

$$V - E_b - IR_a = 0$$

$$\Rightarrow V = E_b + IR_a \text{ .... (1)}$$

Multiplying equation (1) by current I on both sides,

$$VI = E_b I + I^2 R_a \text{ .... (2)}$$

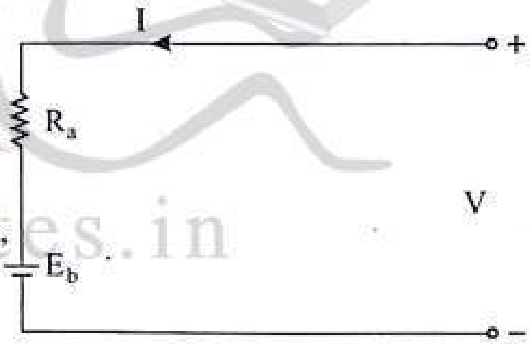

Fig. 8.9

Equation (2) represents power equation of a dc motor.

Where VI = electrical power input.

$E_b I$ = electro-magnetic power output or armature output.

$I^2R_a = C_u$ loss in the armature.

Thus output of the armature is given by

$$E_b I = VI - I^2 R_a$$

$$\Rightarrow P_a = VI - I^2 R_a \text{ .... (3)} \qquad \{\because P_a = E_b I\}$$

To get the condition for maximum power output equation (3) is differentiated with respect to I and equated to zero

Mathematically, $\frac{dP_a}{dI} = 0$

$\Rightarrow V - 2IR_a = 0$

$\Rightarrow V = 2IR_a$ .... (4)

From equation (1) $IR_a = V - E_b$ .... (5)

Substituting the value of IR_a from equation (5) in equation (4)

$$V = 2IR_a = 2(V - E_b)$$

$$\Rightarrow V = 2V - 2E_b$$

$$\Rightarrow E_b = \frac{V}{2}$$

Thus the out power in the motor will be maximum when the back emf will half of the applied voltae.

8.4.1 Torque developed in a dc motor

Before deriving the relation between the motor torque and electrical input, let us establish a relation between output power and rotational torque.

Consider a pully shown in fig. 8.10, applied by a tangential force of F newton. This causes the pully to rotate at n rps.

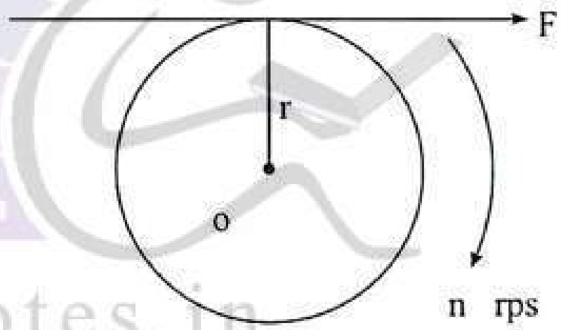

Fig. 8.10

Torque developed by the pulley = F x r Nm

work done for completing one revolution = $2\pi rF$ Joule.

Time taken for one revolution = 1/n second

Therefore power developed by the pulley,

$P = \frac{work\ done}{time\ taken}$, in one revolution.

$\Rightarrow P = \frac{2\pi rF}{1/n}$ watts.

$\Rightarrow P = 2\pi nT$ watts $\qquad \because T = Fr$

If N = revolution per minute then

$$P = \frac{2\pi NT}{60} \text{ watt. .... (1)}$$

The armature power P = $E_b I$ watts

substituting P_a for P in equation (1) motor torque T_a is given by,

$$T_a = \frac{60P}{2\pi N}\ Nm$$

$$\Rightarrow T_a = \frac{60 E_b I}{2\pi N}\ Nm$$

$$\Rightarrow T_a = \frac{60\left(\frac{\phi ZNP}{60A}\right)I}{2\pi N}\ Nm \qquad \because E_b = \frac{\phi ZNP}{60A}$$

$$\Rightarrow T_a = \frac{1}{2\pi}\frac{\phi ZPI}{A}\ Nm \text{ .... (2)}$$

Equation (2) indicates that the torque developed is directly proportional to the product of flux and current.

8.4.2 Speed of DC motor

The back emf (E_b) is given by

$$E_b = \frac{\phi ZNP}{60A}$$

$$\Rightarrow N = \frac{60 E_b A}{\phi ZP}$$

$N \alpha \frac{E_b}{\phi}$ (where A, Z and P are constant)

(i) For shunt motor ϕ is constant.

$$\therefore N \alpha E_b$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

(ii) For series motor $\phi \alpha I$

We know $N \alpha \frac{E_b}{\phi}$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_1}{I_2} \text{ \{field is unsaturated\}}$$

When the field is saturated ϕ is constant, then

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

Example 8.1 : A separately excited dc generator is characterized by the magnetisation curve is shown in fig. 8.11

(i) If the prime mover driving the generator at 800 rpm, what is the no load terminal voltage ?

(ii) If a 1 ohm load is connected to the generator what is the terminal voltage? Generator data given as 100 volt, 100 A, 1000 rpm, $R_a = 0.14\Omega$, field voltage = 100 V, field resistance = 100 Ω

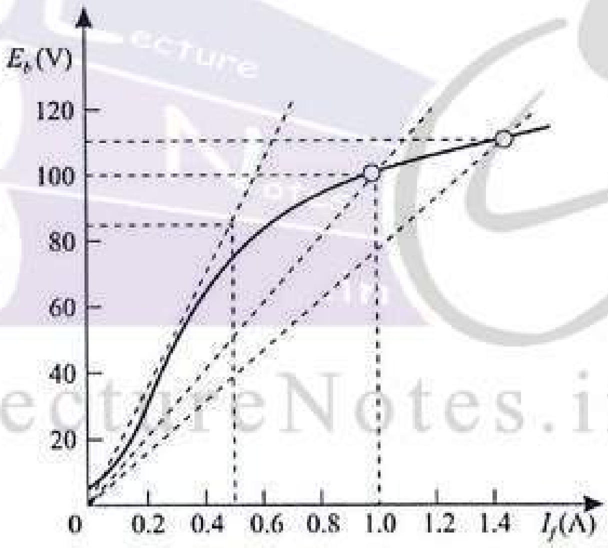

Fig. 8.11

Solution :

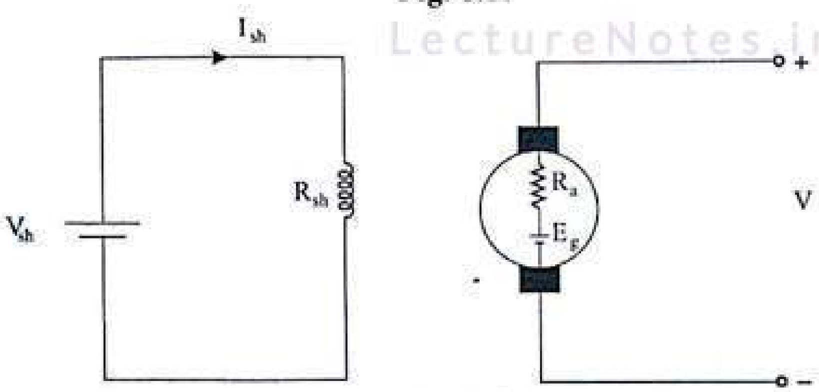

Fig. 8.12

Given that V = 100 volts

I = 100 ampers

N_1 = 1000 rpm

$R_a = 0.14\Omega$

V_{sh} = 100 volts

N_2 = 800 rpm.

$$\therefore I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{100}{100} = 1A$$

From the magnetisation curve, generated emf for the field current of 1A = 100 volts = E_{g1}. The magnetisation curve is drawn for N = 1000 rpm

From the basic emf equation $E_g \alpha \phi N$

$$\Rightarrow E_g \alpha I_{sh} N \qquad \because \phi \alpha I_{sh}$$

Since generator is separately excited I_{sh} is constant.

$$\therefore E_g \alpha N$$

$$\Rightarrow \frac{E_{g2}}{E_{g1}} = \frac{N_2}{N_1}$$

$$\Rightarrow E_{g2} = E_{g1}\left(\frac{N_2}{N_1}\right)$$

$$\Rightarrow E_{g2} = 100\left(\frac{800}{1000}\right) = 80 \text{ volts.}$$

Since generator terminal is open circuited, i.e. I = 0 (shown in fig. 8.12)

Therefore terminal voltage $V_2 = E_{g2} - IR_a$ = 80 - 0 = 80 volts.

(ii) Given $R_L = 1\Omega$

$R_a = 0.14\Omega$

$E_g = I(R_a + R_L)$

$$\Rightarrow I = \frac{E_g}{R_a + R_L} = \frac{80}{0.14+1} = 70.175A$$

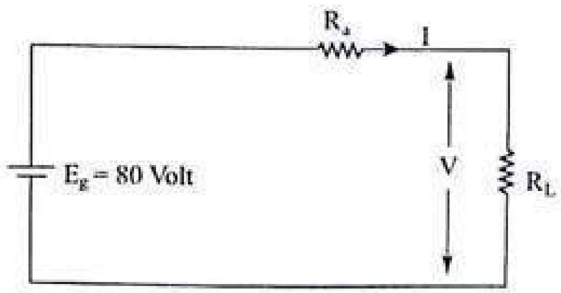

Fig. 8.13

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From Fig. 8.13, terminal voltage, $V = IR_L = 70.175 \times 1 = 70.175$ volts.

Example 8. 2 : Calculate the force exerted by each conductor, 6 inch long on the armature of a DC motor when it carries a current of 90 A and lies in a field the density of which is 5.2×10^{-4} wb per square inch.

Solution :

Given that, length $\ell = 6$ inch $= 15.24 \times 10^{-2}$m

current I = 90 A

Flux density $B = 5.2 \times 10^{-4} \dfrac{wb}{(inch)^2} = 0.806 \dfrac{\omega b}{m^2}$

$\therefore$ Force exerted on the conductor,

$F = BI\ell = 0.806 \times 90 \times 15.24 \times 10^{-2} = 11.055$ N

Example 8.3 : In a DC machine the air gap flux density is 4 $wb.m^{-2}$.

The area of the pole face is 2cm x 4cm.

Find the flux per pole in the Machine.

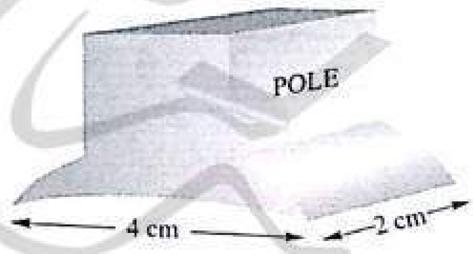

Fig. 8.14

Solution :

Given that air gap flux density $B = 4\dfrac{\omega b}{m^2}$

Area of pole face $A = 8 \times 10^{-4} \, m^2$

$\therefore$ Flux per pole $(\phi) = BA = 32 \times 10^{-4}$ ωb = 0.32 mωb

Example 8.4 : A 220 v shunt motor has an armature resistance of 0.32 and a field resistance of 110 ohms. At no load the armature current is 6A and the speed is 1800 rpm. Assume that the flux does not vary with load and calculate

(i) the speed of the motor when the line current is 62A (assume a 2 volt brush drop)

(ii) the speed regulation of the motor.

Solution :

Given that supply voltage V = 220 volts.

Armature resistance $R_a = 0.32\ \Omega$

Field resistance $R_{sh} = 110\Omega$

No load current $I_0 = 6A$

No load speed $N_0 = 1800$ rpm

Brush drop = 2 volts

Load current = I = 62 A

The brush drop (BD) is the voltage drop in the armature circuit due to contact resistance between brush and commutator along with resistance of the brushes made up of carbon. The equivalent circuit of the motor is shown in fig. 8.15

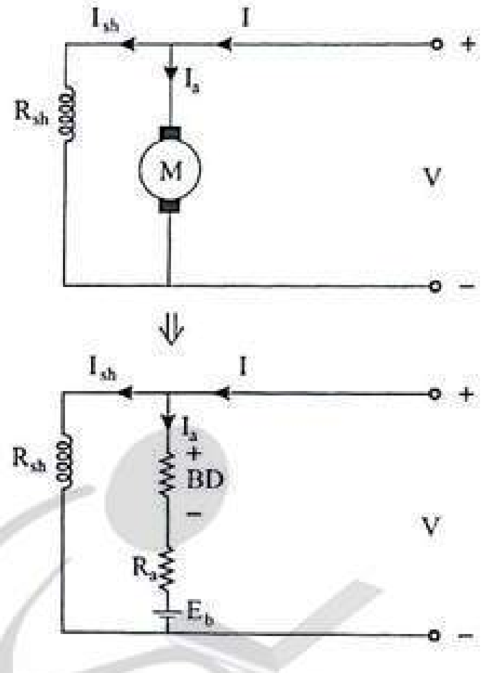

Fig. 8.15

Under no load, applying KVL to the armature circuit we get,

$$V - BD - I_a R_a - E_{b0} = 0 \text{ .... (1)}$$

But $I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2A$

Applying KCL, $I_{ao} = I_o - I_{sh} = 6 - 2 = 4A$

From equation (1) $E_{bo} = V - BD - I_{ao} R_a$

$= 220 - 2 - 4 \times 0.32$

$= 216.72$ volts

Similarly under load $E_b = V - BD - I_a R_a$

$\because I_a = I - I_{sh}$
$= 62 - 2$
$= 60A$

$= 220 - 2 - 60 \times 0 \cdot 32$

$= 198.8$ volt

(i) We have $\frac{\text{speed at no load}}{\text{speed at load}} = \frac{E_{bo}}{E_b}$ $\because$ *Flux is consta nt in both cases*

$$\Rightarrow \frac{N_o}{N} = \frac{E_{bo}}{E_b}$$

$$\Rightarrow N = \frac{E_b}{E_{bo}} N_o = \frac{198.8}{216.72} \times 1800 = 1651 \text{ rpm}$$

(ii) Speed regulation $= \frac{N_o - N}{N} \times 100 = \frac{1800 - 1651}{1651} \times 100 = 9 \cdot 01\%$

Example 8.5 : A 120V, 10 A shunt generator has an armature resistance of 0.6. The shunt field current is 2A. Determine the voltage regulation of the generator.

Solution :

Given that supply voltage V = 120 volts.

Full load current I = 10 A

Armature resistance $R_a = 0.6\ \Omega$

Shunt field current $I_{sh} = 2A$

Therefore $R_{sh} = \frac{V}{I_{sh}} = 60\Omega$

Armature current $I_a = I + I_{sh} = 10 + 2 = 12$ A

Applying KVL to the armature circuit shown in fig 8.16, we get,

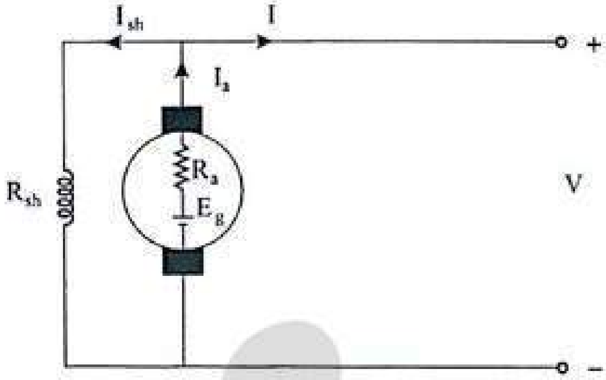

Fig. 8.16

$$V + I_a R_a - E_g = 0$$

$$\Rightarrow E_g = V + I_a R_a = 120 + 12 \times 0.6 = 127.2 \text{ volt}$$

Under no load I = 0

So $$I_{ao} = I_{sh} = \frac{E_g}{R_{sh} + R_a}$$

$$= \frac{127.2}{60 + 0.6} = 2\cdot 1A$$

E_g is being the generated emf remains constant irrespective of loading conditions.

Therefore no load terminal voltage

$$V_o = E_g - I_{ao} R_a$$

$$= 127.2 - 2.1 \times 0.6$$

$$= 125.94 \text{ volt.}$$

$$\therefore \text{Percentage regulation} = \frac{V_o - V}{V} \times 100$$

$$= \frac{125\cdot 94 - 120}{120} \times 100 = 4.916\%$$

Example 8.6 : A 50 hp, 550 volt shunt motor has an armature resistance including brushes of 0.36 ohm. When operating at rated load and speed, the armature takes 75 amp. What resistance should be inserted in the armature circuit to obtain a 20 percent speed reduction when the motor is developing 70 percent of rated torque ? Assume that there is no flux change.

Solution :

Output of the motor (P) = 50 x 746 = 37300 watts,

Supply Voltage (V) = 550 volts.

Armature resistance (R_a) = 0.36Ω

R_a includes resistance due to brush and contact.

Armature current under load, I_a = 75A

Torque under reduced load = $T_2 = 0.7\ T_1$

where T_1 is the rated torque.

Field flux assumed to be constant.

Required speed under reduced load = $N_2 = 0.8\ N_1$, where N_1 is the speed at rated load.

Let E_{b1} = back emf at rated load.

E_{b2} = back emf at reduced load.

R_{a1} = armature circuit resistance at rated load = Ra = 0.36Ω

R_{a2} = armature circuit resistance at reduced load = Ra + R

Where R = external resistance inserted in the armature circuit.

Case (i) With full load :

$$E_{b1} = V - I_{a1}R_{a1} = 550 - 75 \times 0.36 = 523 \text{ volt}$$

Case (ii) With reduced load :

$$I_{a2} = I_{a1} \times \frac{T_2}{T_1}$$

$$= 75 \times 0.7 = 52.5 \text{ A}$$

As $T \alpha \phi I_a$; ϕ being constant

$T \alpha I_a$

$\therefore \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$

$$\therefore E_{b2} = V - I_{a2}R_{a2} = 550 - 52 \cdot 5(R_{a2}) \text{ .... (1)}$$

But we have $\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$

$$\Rightarrow E_{b2} = E_{b1} \times \frac{N_2}{N_1}$$

= 523 x 0.8 = 418.4 volts.

Substituting E_{b2} in equation (1)

$$R_{a2} = \frac{550 - E_{b2}}{52 \cdot 5} = \frac{550 - 418.4}{52.5} = 2 \cdot 507\Omega$$

∴ External resistance required in the armature circuit = R = R_{a2} - R_{a1} = 2.507 - 0.36 = 2.147Ω

Example 8.7 : A 20 kw, 230 v separately excited generator has an armature resistance of 0.2Ω and a load current of 100 A.

Find (a) the generated voltage when the terminal voltage is 230 V.

(b) the output power.

Solution :

Electrical output = 20 kw = 20×10^3 watt

Full load terminal voltage V = 230 volts.

Armature resistance $R_a = 0 \cdot 2\Omega$

load current I = 100 A

Since generator is separately excited $I_a = I = 100$ A

∴ Generated emf $E_g = V + I_aR_a = 230 + 100 \times 0.2 = 250$ volt.

The output power P = VI = 230 x 100 = 23×10^3 watt.

Example 8.8 : Calculate the voltage induced in the armature winding of a 4 pole lap-wound, DC machine having 728 active conductors and running at 1800 rpm. The flux per pole is 30 mwb.

Solution :

Given that No of poles P = 4

No. of parallel path in armature circuit A = 4

No. of armature conductors Z = 728

Flux per pole $\phi = 30 \times 10^{-3}$ wb

Speed of armature N = 1800 rpm

$$\text{Generated emf } E_b = \frac{\phi ZNP}{60A} = \frac{30\times10^{-3}\times728\times1800\times4}{60\times4} = 655\cdot2 \text{ volts.}$$

Example 8.9 : A 4-pole, lap-wound armature has 144 slots with two coil sides per slot, each coil having two turns. If the flux per pole is 20mwb and the armature rotates at 720 rpm. What is the induced voltage ?

Solution

Given that no. of Poles P = 4

No of parallel path A = 4

No of armature slots = 144

No of coil sides per slot = 2

No of turns per coil = 2

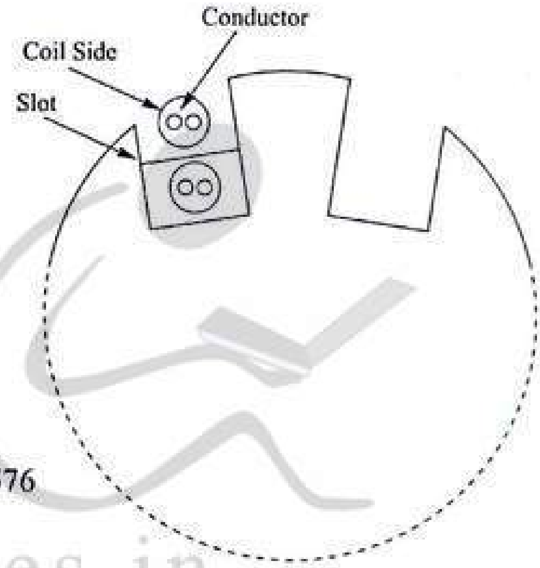

Fig. 8.17

DC windings are always double layer type, i.e. each slot consists two layers containing one coil side per layer. The coil may be single turn or multi turn type. In this example it is a two turn coil i.e. there will be two conductors per coil side. Therefore no. of conductors per slot = no of layers × no. of coil side per layer × no. of conductors per coil side = 2 x 1 x 2 = 4

∴ Total no of armature conductors Z = 4 x 144 = 576

Flux per pole $\phi = 20 \times 10^{-3}$ wb

Speed of armature N = 720 rpm.

$$\text{Generated emf } E_g = \frac{\phi ZNP}{60A} = \frac{20\times10^{-3}\times576\times720\times4}{60\times4} = 138.24 \text{ volt.}$$

Example 8.10 : A 100 kw, 230 volt, shunt generator has R_a=0.05 ohm and R_{sh}=57.7 ohm. If the generator operates at rated voltage, calculate the induced voltage at (a) full-load (b) half full-load. Neglect brush contact drop.

Solution :

Given that electrical power output = P = 100×10^3 watt

Terminal voltage v = 230 volt

Armature resistance R_a=0.05Ω

Shunt field resistance R_{sh} = 57.7Ω

(a) $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{230}{57\cdot 7} = 3.986$ A

Load current $I = \dfrac{P}{V} = \dfrac{100\times10^3}{230} = 434.78$ A

$\therefore$ Armature current $I_a = I + I_{sh} = 434.78 + 3.986 = 438.77$ A

Generated emf $E_g = V + I_aR_a = 230 + 438.77 \times 0.05 = 251.94$ volt.

(b) At half load, $I = \dfrac{P/2}{V} = \dfrac{100\times10^3}{2\times230} = 217.39$ A

$\therefore I_a = I + I_{sh} = 217.39 + 3.986 = 221.38$ A

Generated emf, $E_g = V + I_aR_a = 230 + 221.38 \times 0.05 = 241$ volts.

Example 8.11 : A 50 kw, 250 volt, short - shunt, compound generator has the following data : R_a = 0.06 ohm, R_{se} = 0.04 ohm, R_{sh}=125 ohm. Calculate the induced armature voltage at rated load and terminal voltage. Take 2 volt as the total brush - contact drop.

Solution :

Output of Generator = 50×10^3 watt.

Load voltage, V = 250 volts.

Armature resistance $R_a = 0.06\Omega$

Shunt field resistance $R_{sh} = 125\ \Omega$

Series field resistance $R_{se} = 0.04\Omega$

Brush drop = BD = 2 volts.

Connected in short shunt.

Load Current $I = \dfrac{50\times10^3}{250} = 200$ A

Therefore terminal voltage,

$V_t = V + IR_{se} = 250 + 200 \times 0.04 = 258$ volts.

$\therefore I_{sh} = \dfrac{V_t}{R_{sh}} = \dfrac{258}{125} = 2.064$ A

Armature current $I_a = I + I_{sh} = 200 + 2.064 = 202.064$ A

Generated emf $E_g = V_t + B.D + I_aR_a$

$= 258 + 2 + 202.064 \times 0.06$

$= 272.12$ volts.

Example 8.12 : A 100 kw, 230 volt shunt generator has $R_a = 0.05$ ohm and $R_{sh} = 57.5$ ohm. It has got total mechanical and core loss of 1.8 kw. Calculate (i) the generator efficiency at full load (ii) the horse power output from the prime mover to drive the generator at this load.

Solution

Given that electrical output = 100 kw

Load voltage V = 230 volts.

$R_a = 0.05\Omega$

$R_{sh} = 57.5\Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{57 \cdot 5} = 4 \text{ A}$$

$$I = \frac{P}{V} = \frac{100\times10^3}{230} = 434.78 \text{ A}$$

Armature Current $I_a = I + I_{sh} = 434.78 + 4 = 438.7\text{A}$

Generated emf Eg $E_g = V + I_a R_a = 230 + 438.7 \times 0.05 = 251.935$ volt.

Electrical power developed in the armature $E_g I_a = 251.935 \times 438.7 = 110523.88$ watt

Fig. 8.18

Therefore input to generator = Output of the Prime mover

= Armature output + core and mechanical loss

= 110523.88 + 1800

= 112323.88 watt

= 112.32 kw

(i) $$\% \text{ Efficiency} = \frac{\textit{Electrical output}}{\textit{Mechanical input}} \times 100 = \frac{100\times10^3}{112 \cdot 32\times10^3} \times 100 = 89\%$$

(ii) $$\text{H.P output of the Prime mover} = \frac{112 \cdot 32\times10^3}{746} = 150.56 \text{ hp}$$

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Example 8.13 : A 20 hp, 250 volt, shunt motor has an armature circuit of 0.22 ohms and a field resistance of 170 ohms. At no-load and rated voltage, the speed is 1200 rpm and the armature current is 3A. At full-load and rated voltage, the line current is 55A, and the flux is reduced 6% (due to the effects of armature reaction) from its value at no-load, What is the full-load speed?

Given that full load output = 20 x 746 = 14920 watt.

Load voltage V = 250 volts.

$R_a = 0.22\ \Omega$, $R_{sh} = 170\ \Omega$

No load speed, $N_0 = 1200$ rpm

No load armature current $I_{a0} = 3A$

Full load line current I = 55 A

Field flux at full load, $\phi = 0.94\phi_0$

Where ϕ_0 is the field flux at no load.

At no load, $E_{b0} = V - I_{a0}R_a$

$= 250 - 3 \times 0.22$

$= 249.34$ volts.

At full load, $I_a = I - I_{sh} = I - \frac{V}{R_{sh}} = 55 - \frac{250}{170} = 53.53$ A

$\therefore E_b = V - I_aR_a = 250 - 53.53 \times 0.22 = 238.22$ volts.

We have $\frac{N}{N_o} = \frac{E_b}{E_{bo}} \times \frac{\phi_o}{\phi}$

$\Rightarrow N = N_o \times \frac{E_b}{E_{bo}} \times \frac{\phi_o}{\phi}$ $\quad \because N \alpha \frac{E_b}{\phi}$

$\Rightarrow N = 1200 \times \frac{238 \cdot 22}{249 \cdot 34} \times \frac{1}{0.94}$

$\Rightarrow N = 1219.66$ rpm

Example 8.14 : A 10 hp, 230 volt, shunt motor takes a full-load line current of 40A. The armature and field resistances are 0.25 ohm and 230 ohm respectively. The total brush contact drop is 2 volt and the core and friction losses are 380 watt. Calculate the efficiency of the motor. Assume that stray-load loss is 1% of output.

Solution :

Mechanical output = $10 \times 746 = 7460$ watt

Supply voltage, V = 230 volts.

Full load line Current I = 40 A

$R_a = 0.25\Omega$, $R_{sh} = 230\Omega$, B.D. = 2 volts.

Core and frictional loss = 380 watt

Stray load loss = $0.01 \times 7460 = 74.6$ watt.

> Stray load losses are defined as losses that arise from the non uniform distribution of current in copper. They also encompass additional core losses that are produce in iron by the distortion of magnetic flux by a load current. It is expressed as a percentage of output power.

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1\text{A}$$

$\therefore I_a = I - I_{sh} = 40 - 1 = 39$ A

$E_b = V - I_aR_a$ - B.D. $= 230 - 39 \times 0.25 - 2 = 218.25$ volts.

$\therefore$ Electro - mechanical power developed in armature = $E_bI_a = 218.25 \times 39 = 8511.75$ watt.

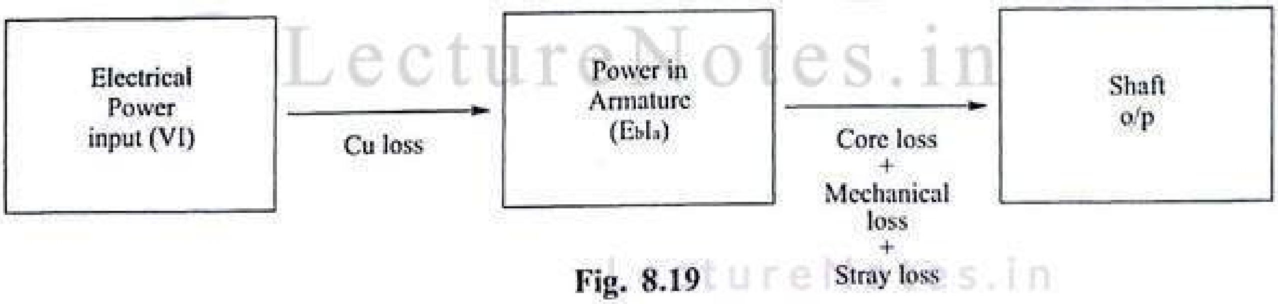


Fig. 8.19

Therefore shaft output = Armature output - mechanical and core loss - stray loss

= 8511.75 - 380 - 74.6

= 8057.15 watt.

$$\therefore \text{Efficiency} = \frac{\textit{Mechanical output}}{\textit{Electrical input}} = \frac{8057.15}{230 \times 40} = 0.876 = 87.6\%$$

Example 8.15 : A 10kw, 250 volt, shunt generator, having an armature resistance of 0.1 ohm and a field resistance of 250 ohm, delivers full-load at rated voltage and 800 rpm. The machine is now run as a motor while taking 10 kw at 250 volt. What is the speed of the motor ? Neglect brush contact drop.

Solution :

Given that, terminal voltage V = 250 volts,

$R_a = 0 \cdot 1\Omega \qquad R_{sh} = 250\Omega$

As a generator :

Electrical Power output = 10,000 watts.

Speed, N_g = 800 rpm

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{A}$$

$$I = \frac{P}{V} = \frac{10{,}000}{250} = 40 \text{ A}$$

$$I_a = I + I_{sh} = 40 + 1 = 41A$$

$\therefore E_g = V + I_aR_a = 250 + 41 \times 0.1 = 254.1$ volt,

As a motor :

Electrical input = 10,000 watts.

Line current $I = \frac{P}{V} = \frac{10{,}000}{250} = 40$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}$$

$I_a = I - I_{sh} = 40 - 1 = 39$ A

$\therefore E_b = V - I_aR_a = 250 - 39 \times 0.1 = 246.1$ volts.

We have, $\frac{Speed\ as\ Motor\,(N_m)}{Speed\ as\ generator\,(N_g)} = \frac{E_b}{E_g} \times \frac{\phi_g}{\phi_m}$

$\phi_g = \phi_m$ = flux per pole in both cases, as shunt field current is same.

$$\therefore N_m = \frac{E_b}{E_g} \times N_g = \frac{246 \cdot 1}{254 \cdot 1} \times 800 = 774 \cdot 8 \text{ rpm}$$

8.5 AC Machines

The commercial power generation is in the form of alternating current voltage. However in the early days most of rotating machines used were operating on dc voltages. But invention ac machines almost replaced the dc machines in all industrial and domestic applications. A few dc machines like dc series motor are still being used in industrial application for their unique inherent characteristics (high starting torque). AC machines have large advantages over the dc machines. They are,

(i) Since basic generation of electricity is in the form of ac voltage, no converting device (like commutator) is necessary.

(ii) Suitable for large ratings (up to more than 500 MW)

(iii) Cost of an ac machine for same power and voltage rating is less than that of a dc machine.

(iv) Since transformers are used in ac only, ac generation is must for commercial power system operation.

The AC machines are broadly classified as (i) Synchronous machines., (ii) Asynchronous machines.

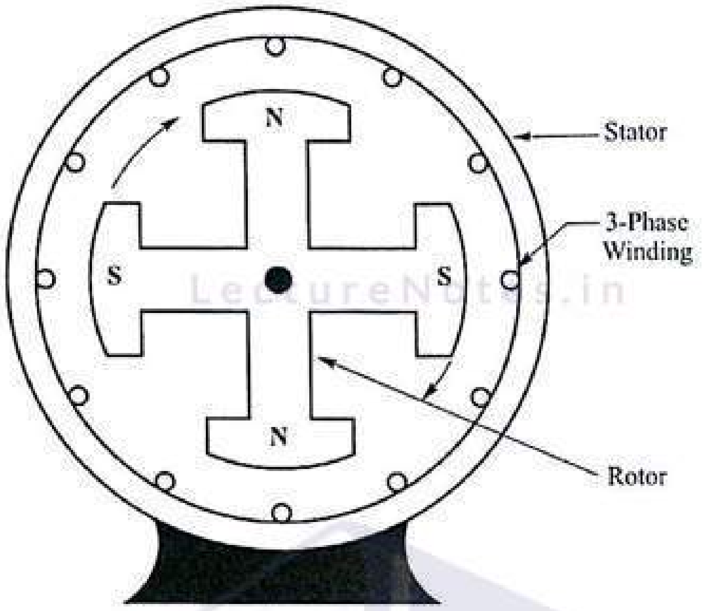
8.5.1 Synchronous Machines

Synchronous machines are so called because they operated constant speed and constant frequencies under steady state. Like most rotating machines can function either as a motor or as a generator. As a generator it is also named as alternator. Schematic end view of a salient pole type synchronous machine is shown in fig. 8.20.

The operation of a synchronous generator in based on Faradays Laws of electro-magnetic induction. It works very much like a dc generator, in which generation of emf is by relative motion of conductors and magnetic flux. However a synchronous generator does not have a commutator as does the dc generator.

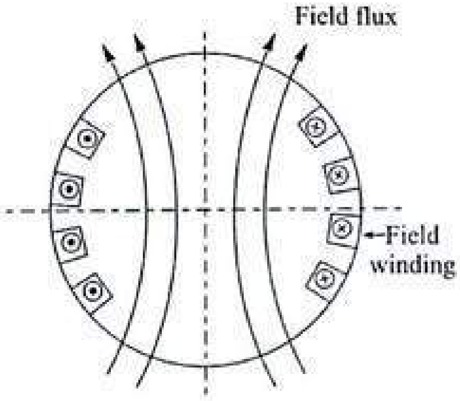
8.5.1.1. Construction

The two basic parts of a synchronous machine are field, structure carrying a dc excited winding and armature which often has a three phase winding in which the ac emf is generated. Almost all modern synchronoŭs machines have stationary armatures and rotating field structures. The dc winding on the rotating field structure is connected to an external source through slip rings and brushes or else receives brushless excitation from rotating bodies. Fig. shows shematic end view ofa salient pole type synchronous machine. In some respects, the stator carrying the armature windings is similar to the stator of a polyphase induction motor. In addition to the armature and field windings, a synchronous machine has damper bars on the rotor; these come into play during transients and start up.

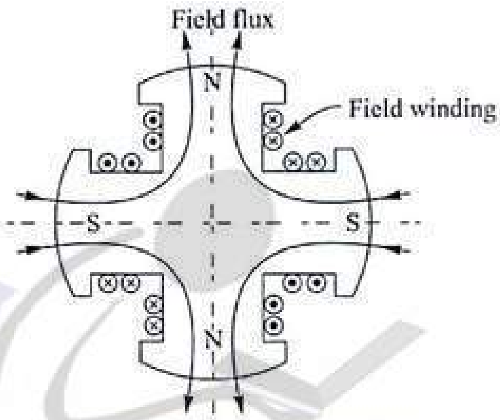


Schematic end view of a salient pole type Synchronous machine

Fig: 8.20



(a) Field winding on a round rotor



(b) Field winding on a salient rotor

Fig. 8.21

Depending upon the rotor construction, a synchronous machine may be of the round rotor type as shown in fig 8.21(a) or the salient type as shown in fig.8.21(b). The former type in used in high speed machines such as turbine generators, where as the later type is suitable for low speed, water wheel generators.

8.5.1.2. Synchronous generator

One of the most common a.c. machine is synchronous generator or alternator. All commercial alternators employes rotating type field and static armature containing 3-phase winding. This is because of the following advantages.

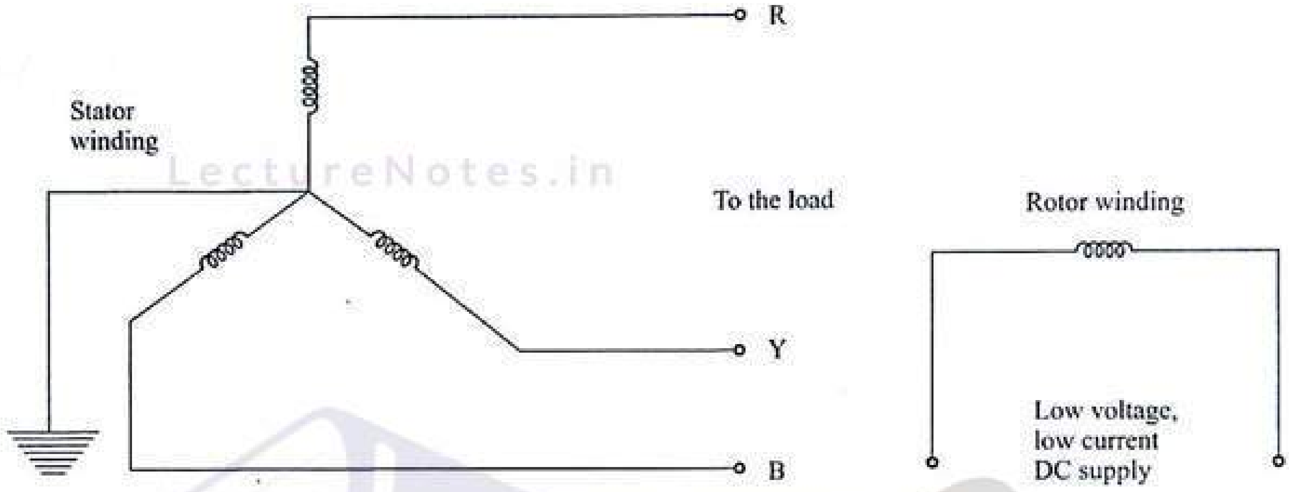
(i) rotating field is comparatively light and can run with high speed.

(ii) high voltage can be generated due to high speed up to 33KV loads.

(iii) for high voltage it becomes easy to give insulation a stator part then on a moving part.

(iv) Very little difficulty is experienced in supplying the field magnet as it requires very few amperes of current at a voltage up to 250 volts. The stator which consists of three phase winding is connected in star. In power system use the star or neutral point of the armature winding is solidly grounded and lines are drawn from the three phases for connecting to the load as shown in fig. 8.22.

Since the generation is ac voltage electrical output can be tapped out directly from the stator winding.



[Winding configuration of a 3-ϕcommercial alternator]

Fig 8.22

Frequency of output voltage :

Let P = no of rotor poles.

N = Speed of the rotor in rpm.

From the basics it can be seen that two poles (one north and one south) are responsible for generating one cycle of ac voltage in the armature winding. In one complete revolution number of poles interacted = P

Number of cycles generated in one revolution $= \frac{P}{2}$

Time taken for one revolution $= \frac{60}{N}$ seconds.

In $\frac{60}{N}$ seconds no of cycles $= \frac{P}{2}$

In one second no of cycles generated $= \frac{\frac{P}{2}}{\frac{60}{N}} = \frac{PN}{120}$

$\Rightarrow$ Frequency $(f) = \frac{PN}{120} Hz$

Emf equation :

Let P = no of poles

N = speed in rpm

T_{ph} = no of concentric turns

K_d = distribution factor.

K_c = pitch factor or chording factor.

ϕ = flux produced per pole in weber.

In one revolution flux cut by one armature conductor $= \phi P = d\phi$

Time taken for one revolution $= \frac{60}{N}$ sec. = dt.

Therefore average emf induced in the conductor is given by,

$$e = \frac{d\phi}{dt} = \frac{\phi p}{60/N} = \frac{\phi PN}{60} = \frac{2\phi PN}{120}$$

$$\Rightarrow e = 2\phi f \ volts. \qquad \because f = \frac{PN}{120}$$

Since the output voltage of an alternator is alternating in nature, average emf has no significance. For all practical purposes rms value should be calculated. For a sinusoidal ac voltage,

rms value = average value × form factor

The value of form factor is 1.11 in case of sinusoidal a.c.

Rms value of induced emf per conductor = (1.11) (e)

$= (1 \cdot 11)\ (2\phi f)$

$= 2.22\ \phi f$ volts.

Emf per turn $= 2 \times 2.22\ \phi f = 4.44\ \phi f$ volts.

(Two conductors constitute one turn)

∴ Induced emf per phase = emf per turn × no of turns per phase $= 4.44\ \phi f\ T_{ph}$ volts.

In practice the stator winding is not concentric but distributed in nature. Hence actual induced emf will be slightly less than the calculated value.

The induced emf per phase is multiplied by a factor K_d, known as *distribution factor*. (K_d varies from 0.96 to 0.98.) Similarly in practice the width of a turn known as pitch may not be full (i.e. 180° electrical). Intentionally the width of a turn is shorted due to various advantages, such windings are known as short pitched or chorded winding. The actual induced emf per turn for a chorded winding is slightly less by a factor K_c than that of a full pitched winding.

Considering all these points induced emf per phase is given by

$E_{ph} = 4 \cdot 44\phi f T_{ph} K_d K_c$ volts

The line voltage is given by,

$E_{Line} = \sqrt{3} E_{ph}$ [for star connected armature]

$E_{Line} = E_{ph}$ [for delta connected armature]

Equivalent Circuit :

The terminal voltage of an alternator changes with increase in load current. (terminal voltage may increase or decrease depending on load power factor). This is due to presence of an internal impedance known as *synchronous impedance* (Zs).

$Z_s = R + jX_s$

Where R = resistance of armature per phase.

X_s = synchronous reactance per phase.

The equivalent circuit of an alternator is shown in fig. 8.23.

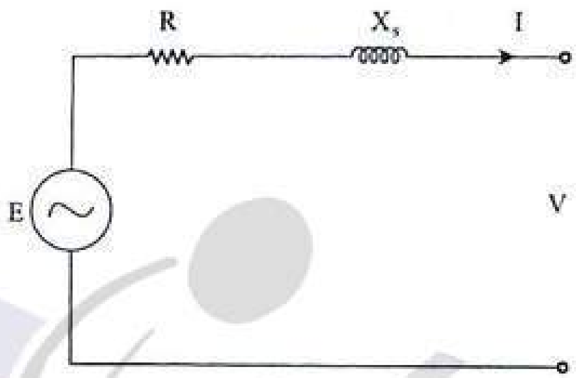


Fig 8.23

From the equivalent circuit,

$$\mathbf{V = E - IZ_s}$$

$$\Rightarrow V = E - I(R + jX_s)$$

In many practical cases R << X_s and hence R may be neglected. Thus above equation reduces to

$$\mathbf{V} = E - jIX_s.$$

8.5.1.3. Synchronous Motor

A three phase alternator when connected to a infinite bus * bar delivering electrical power to the bus, suddenly input to its prime mover is withdrawn then it continues to rotate as a motor by drawing electrical power from the bus. This synchronous machine is said to be operating as a motor known as *synchronous motor*. The salient features of a synchronous motor is described below,

(i) It is not self starting.

(ii) It rotates at constant speed irrespective of load torque. The speed at which it rotates is given by $N_s = \frac{PN}{120} rpm.$

* *Infinite bus means a power system which connected to many generators and loads maintains constant voltage and efficiency.*

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(iii) It can draw current from the supply at any power factor (lagging, leading, upf)

(iv) It needs both three phase ac supply (stator) and dc supply (rotor).

(v) It is costlier than three phase induction motor.

8.5.2. A synchronous Machines

A synchronous machine always rotates at a constant speed, but asynchronous machine operates with variable speed. One of the important asynchronous machine is induction machine. When operating in three phase it can be used as induction motor or induction generator. As induction motor, it is largely used in various applications such as from domestic air conditioning to industrial use (rolling mill etc.).

Induction generators are mostly used in wind generating system. They convert wind energy to electrical energy at constant frequency irrespective of wind speed.

Constructionally they have a stator carrying three phase windings like a synchronous machine. The stator winding may be connected in star or in delta. The rotor of an induction machine can be of two types.

(i) Squirrel cage rotor.

(ii) Slip ring or phase wound rotor.

In both type of rotor the windings are not supplied with electrical power but are short circuited. In case of a squirrel cage rotor, the rotor windings are shorted internally while in case of slip ring rotor the short circuit is done externally through three slip rings.

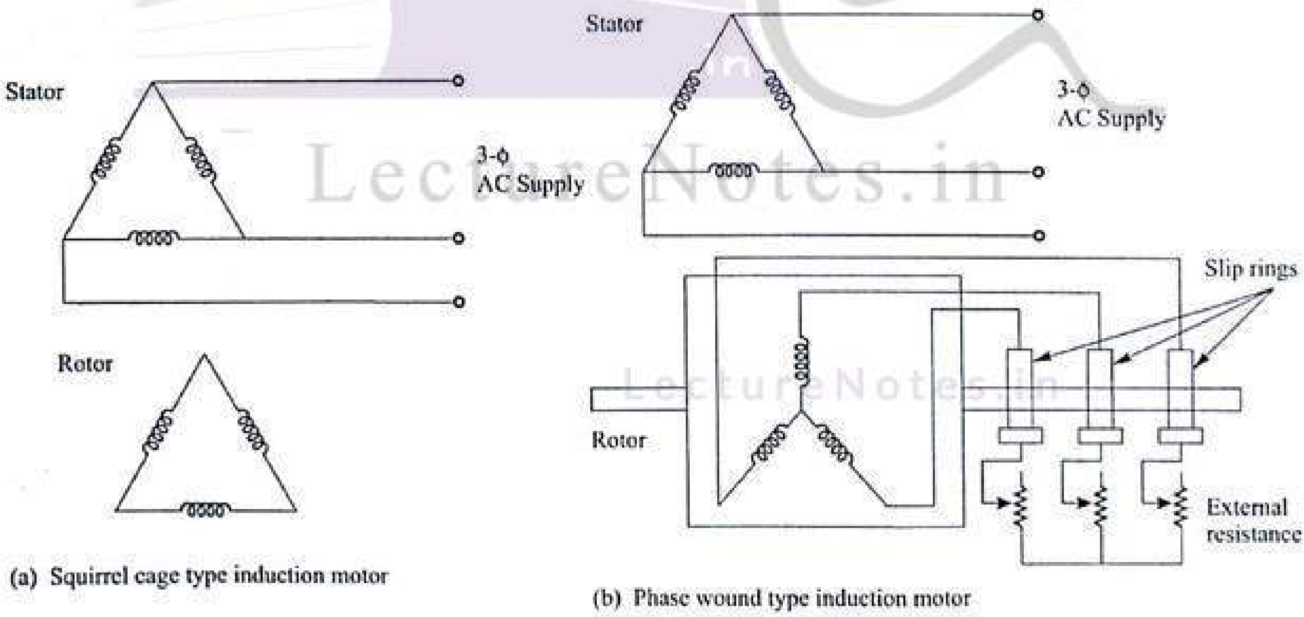


Fig 8.24

Slip ring induction motor will develop high starting torque by inserting external resistance to the rotor circuit through slip rings. The schematic diagram of a squirrel cage and phase wound type induction motor are shown in fig. 8.24.

Example 8.16 : For 60 Hz synchronous generator, list six possible combination of no of poles and speed.

Solution :

Given that, frequency of generated voltage, f = 60 Hz

We have $N_s = \frac{120f}{P}$ = speed of the synchronous generator.

The no of poles must be an even number i.e. 2, 4, 6..... Hence for various number of poles the required speed for generating 60Hz emf is shown in the table below.

Table

No. of poles	Required Speed in rpm
2	3600
4	1800
6	1200
8	900
10	720
12	600

Example 8.17 : What is the maximum speed at which,

(i) a 60 Hz

(ii) a 50 Hz

Synchronous machine can be operated.

Solution :

The minimum no. of poles that a synchronous machine that should have is equal to 2. As , $N_s = \frac{120f}{P}$, minimum no of poles will give maximum speed.

$\therefore$ For f = 60 Hz, maximum Speed = $\frac{120 \times 60}{2}$ = 3600 rpm

For f = 50Hz, maximum speed = $\frac{120 \times 50}{2}$ = 3000 rpm

Example 8.18 : A three phase, 50 Hz, 8 pole synchoronous generator with star connected winding has 72 slots and 10 conductors per slot. The flux per pole is $24 \cdot 8 \times 10^{-3}$ wb. The coils are full pitched. Find (i) the speed (ii) the line emf. Given that the distribution factor K_d=0.96.

Solution :

Given that f = 50Hz, P = 8,

No of armature conductors Z = 72 x 10 = 720

Flux per pole $\phi = 24 \cdot 8 \times 10^{-3}\ wb$

(i) Required Speed $N = \frac{120f}{P} = \frac{120 \times 50}{8} = 750$ rpm

(ii) No of turns $= \frac{Z}{2} = \frac{720}{2} = 360$

No. of turns per phase $T_{ph} = \frac{360}{3} = 120$

Since the winding is full pitched pitch factor K_c=1

$\therefore$ Induced emf per phase $= E_{ph} = 4 \cdot 44 f \phi T_{ph} K_d K_C$

$= 4 \cdot 44(50)(24 \cdot 8 \times 10^{-3})(120)(0 \cdot 96)(1)$

= 634.2 volts.

Since armature is star connected line voltage

$E_L = \sqrt{3}\, E_{ph} = \sqrt{3} \times 634 \cdot 2 =$ 1098.5 volts.

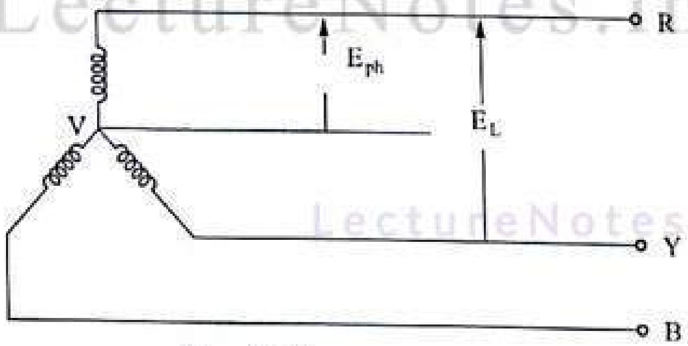


Fig. 8.25

Example 8.19 : The three phase synchronous reactance of a three phase star connected, 2.5 MVA, 6.6KV, 60 Hz turbo alternator (synchronous generator) is 10 ohms. Neglect the armature resistance and saturation. Calculate the voltage regulation when the generator is operating at full load with (i) unit power factor (ii) 0.8 power factor lagging (iii) 0.8 power factor leading.

Solution :

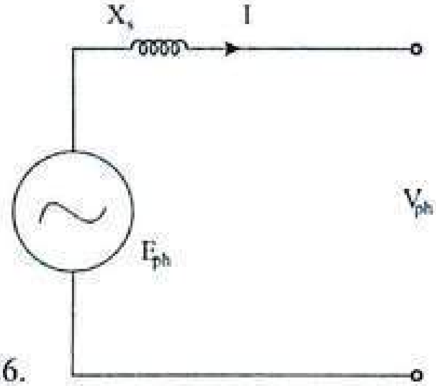


Fig. 8.26

Given that, rated three phase output (p) = 2.5 x 10^6 VA,

rated line voltage $(V_L) = 6.6 \times 10^3$ volts.

frequency f = 60Hz, synchronous reactance

(X_s) = 10 ohm, armature resistance (R_a) = 0 ohm

The per phase equivalent circuit of the alternator is shown in fig. 8.26.

Rated three phase apparent power $=\sqrt{3}V_L I_L = 2\cdot5\times10^6$ VA

$$\therefore \text{Line Current } I = I_L \frac{2\cdot5\times10^6}{\sqrt{3}V_L} = \frac{25\times10^6}{\sqrt{3}\times6.6\times10^3} = 218.7\text{A}$$

$$\text{Phase voltage } V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{6.6\times10^3}{\sqrt{3}} = 381051 \text{ volts.}$$

(1) At unit p.f. current I is in phase with V_{ph} $\therefore$ $I = 218\cdot7\angle0^0 A$

Applying KVL to the equivalent circuit, no load per phase voltage

$E_{ph} = V_{ph} + jIXs$

$= 3810.51 \angle 0 + j(218.7)10$

$= 3810.51 + j2187$

$= 4393.51 \angle 29\cdot85^0$ volts.

(Taking Load phase voltage V_{ph} as reference)

Taking magnitude only, full load voltage V_{ph} = 3810.51 Volts.

No load terminal voltage E_{ph} = 4393.51 volts.

$$\therefore \text{Voltage Regulation} = \frac{E_{ph}-V_{ph}}{V_{ph}} = \frac{4393\cdot51-3810\cdot51}{3810\cdot51} = 0.153 = 15.3\%$$

(ii) At 0.8 p.f. lagging :

$I = 218\cdot7\angle-\cos^{-1}0\cdot8 = 218\cdot7\angle-36\cdot87$ A

$E_{ph} = V_{ph} + jIX_s = 3810\cdot51 + 218\cdot7\angle-36\cdot87\times10\angle90^0$

$= 3180.51 + 2187 \angle 53\cdot13$

$= 3180.51 + 1312.2 + j1750$

$$= 4492.82 + j1750$$

$$= 4821 \cdot 61 \angle 21 \cdot 28 \text{ volts}$$

$$\therefore \text{ Voltage regulation} = \frac{4821 \cdot 61 - 3810 \cdot 62}{3810 \cdot 62} \times 100 = 26.53\%$$

(iii) At 0.8 p.f. leading :

$I = 218 \cdot 7 \angle 36 \cdot 87$ A

$E_{ph} = V_{ph} + jIX_s$

$= 3810 \cdot 62 + 218 \cdot 7 \angle 36 \cdot 87 \times 10 \angle 90^0$

$= 3810.62 + 2187 \angle 126.87$

$= 3810.62 - 1312 + j1750$

$= 2498.62 + j1750$

$= 3050 \angle 35^0$volts.

$$\text{Voltage regulation} = \frac{3050 - 3810 \cdot 62}{3810 \cdot 62} \times 100 = -19 \cdot 9\%$$

Example 8.20 : A twelve pole single phase alternator has 50 slots and 20 conductors per slot. The flux per pole is 90 mwb sinusoidally distributed. The distribution factor is 0.666 and the coil span factor is unity. If the speed is 600 rpm find (a) frequency (b) the emf generated in the alternator.

Solution

Given that flux per pole $\phi = 90 \times 10^{-3}$ wb

No of poles P = 12

Speed N = 600 rpm

No of armature conductors Z = 50 × 20 = 1000

No of turns $T = \frac{1000}{2} = 500$

$K_d = 0.666$ and $K_c = 1$

(a) Frequency $f = \frac{PN}{120} = \frac{12 \times 600}{120} = 60\,Hz$

(b) Induced emf $E = 4.44 f \phi T K_d K_c$

$= 4.44 \times 60 \times 90 \times 10^{-3} \times 500 \times 0.666 \times 1 = 7984$ volts.

Example 8.21 : A three phase 8 pole 576.6 volts, star connected alternator runs at 900 rpm. Calculate the no of turns in the stator winding if flux per pole is 25 mwb. Assume $K_d = 0.96$

Solution :

Given that line voltage E_L=576.6 volts.

Speed N = 900 rpm.

Flux per pole $\phi = 25\times10^{-3}$ wb.

$K_d = 0.96$

Assuming full pitch winding $K_c = 1$

Since frequency is not given assuming initial conditions f = 50Hz.

We have induced emf per phase E_{ph}=4.44 $f\phi T_{ph} K_d K_C$

$$\therefore T_{ph} = Turns\ per\ phase = \frac{E_{ph}}{4\cdot44 f\phi K_d K_c}$$

$$= \frac{576\cdot6/\sqrt{3}}{4\cdot44\times50\times25\times10^{-3}\times0\cdot96\times1} = 62$$

$\therefore$ Total no of turns = 3 T_{ph} = 3 x 62 = 186

Example 8.22 : Find the BHP of a diesel engine driving an ac generator supplying a load current 70 A at 400 V in a 3-ϕ supply system. Assume p.f. of the supply is 0.84 and the efficiency of the alte rnator is 84%.

Solution :

Given that load current I_L = 70 A.

Load voltage V_L = 400 V.

P.f. = cos ϕ = 0.84

Efficiency of alternator (η) = 0.84

Output of the alternator = $\sqrt{3} V_L I_L \cos\phi = \sqrt{3}$ x 400 x 70 x 0.84 = 40737 watt.

Fig. 8.27

Output of the diesel engine = input of the alternator

$$= \frac{alternator\ output}{\eta}$$

$$= \frac{40737}{0 \cdot 84} = 48497 \text{ watt.}$$

$\therefore$ BHP of the engine $= \frac{48497}{746} = 65$ HP.

Example 8.23 : The current in each phase of a star connected alternator is 1000 A when the generated phase voltage & p.f. are 5000 V and 0.5 respectively. Calculate the line voltage adn total power output of the alternator.

Solution :

Given that phase voltage $V_{ph} = 5000$ volts.

Line Current $I_L = 1000$ A

P.f. = $\cos\phi = 0.5$

Since the armature is star connected line

Voltage $V_L = \sqrt{3} V_{ph} = \sqrt{3} \times 5000 = 8660$ volts.

$\therefore$ Power output $P = \sqrt{3} V_L I_L \cos\phi$

$= \sqrt{3} \times 8660 \times 1000 \times 0.5$

$= 7.5 \times 10^6$ watts.

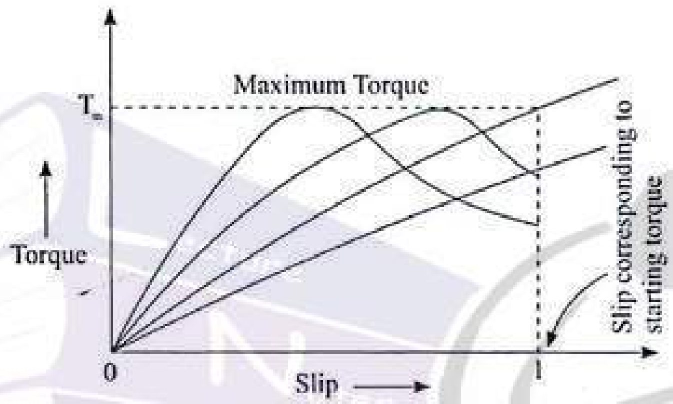
$= 7.5$ MW.

1. A dc shunt motor is running at 1500 rpm. How can you obtain a speed of 1000 rpm?

(1st semester 2003)

Solution : Speed 1000 rpm can be done by armature voltage controlled.

2. Sketch slip - torque characteristics of a 3-phase induction motor. *(1st semester 2003)*



3. A dc motor runs on a 200V supply and has negligible armature resistance. What happens to its speed if the armature voltage is reduced by 20% and the field current is increased by 30%.

(1st semester 2004)

Solution : V = 200 volts.

Since R_a is zero, $E_b = V$

Given that $E_{b2} = 0\cdot 8 E_{b1}$

Field current $I_{f2} = 1\cdot 3 I_{f1}$

As flux $\phi \alpha I_f$, $\frac{I_{f2}}{I_{f1}} = \frac{\phi_2}{\phi_1}$

$$\Rightarrow \frac{\phi_2}{\phi_1} = 1\cdot 3$$

As $N \alpha \frac{E_b}{\phi}$ So $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$

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$$\Rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2} = 0\cdot 8 \times \frac{1}{1\cdot 3} = 0\cdot 615$$

$\therefore \; N_2 = 0\cdot 615\, N_1 = (1 - 0\cdot 385)\, N_1$ i.e. the speed will reduce by 38.5%.

4. What happens to the rotor speed and rotor current when the mechanical load on an induction motor increases ?

(1st semester 2004)

Solution : Rotor Speed decreases

5. A dc shunt generator generates an emf of 520 V at a speed of 1200 rpm. It has 2000 armature conductors and flux per pole is 0.013 wb. The armature winding has 4 parallel paths. (i) Determine the number of poles. (ii) Find the generated voltage, if the armature winding is wave connected.

(1st semester 2004)

Solution : E = 520 volts, N = 1200 rpm, Z = 2000

ϕ = 0.013 wb. A = 4.

(i) $E = \frac{P\phi ZN}{60A} \;\Rightarrow\; P = \frac{60\,AE}{\phi ZN} = \frac{60(4)(520)}{(0\cdot 013)(2000)(1200)} = 4$

(ii) $E = \frac{P\phi ZN}{60A} = \frac{4(0\cdot 013)(2000)(1200)}{60(2)} = 1040\,\text{volts.}$

Where $A = 2$, for wave connected.

6. A dc shunt motor rotating at 1500 rpm is fed by a 120 V dc source. The line current drawn by the motor is 51 A, and the shunt field resistance is 120Ω. Find (i) the back emf (ii) the mechanical power and torque developed by the motor. Take armature resistance 0.1Ω.

(1st semester 2004)

Solution : V = 120 volts, $I_L = 51A$, $R_{sh} = 120\,\Omega$, $R_a = 0\cdot 1\Omega$

$$V = I_{sh} R_{sh}$$

$$\Rightarrow I_{sh} = \frac{V}{R_{sh}} = \frac{120}{120} = 1A$$

$$I_L = I_a + I_{sh}$$

$$\Rightarrow I_a = I_L - I_{sh} = 51 - 1 = 50A$$

(i) Back emf $E_b = V - I_a R_a = 120 - 50(0\cdot 1) = 115$ volts.

(ii) Mechanical power output $= E_b I_a = (115)(50) = 5\cdot 75\,Kw$

Mechanical power, $P = T\omega$

$$\Rightarrow T = \frac{P}{\omega} = \frac{E_b I_a}{2\pi f} = \frac{5750}{2\pi\left(\frac{1500}{60}\right)} = 36\cdot 6\,Nm$$

7. A 208V, 10 hp, 4 pole, 60 Hz, 3 phase star connected induction motor has a full load slip of 5 percent. Determine (i) the rotor speed and shaft torque at the rated load. (ii) rotor frequency and rotor induced voltage at $\frac{1}{4}$th the rated load. ***(1st semester 2004).***

Solution : V = 208 volts, $P_{out} = 10\,hp = 10 \times 746 = 7460W$

Pole, $P = 4,$ $f = 60\,Hz,$ Slip, $S = 0.05$

(i) $N_s = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800\,rpm$

But $S = \frac{N_s - N}{N_s}$

$\Rightarrow 0 \cdot 05 = \frac{1800 - N}{1800}$

$\Rightarrow N = 1710\,rpm$

Torque, $T = \frac{P_{out}}{2\pi f} = \frac{7460}{2\pi\left(\frac{1710}{60}\right)} = 41 \cdot 68\,\text{Nm}.$

(ii) For an induction motor, when the load torque in working range, it is given by $T \,\alpha\, S$.

$\Rightarrow \frac{T_2}{T_1} = \frac{S_2}{S_1} = \frac{1}{4}$

$\therefore$ $S_2 = \frac{1}{4} S_1 = \frac{1}{4}(0 \cdot 05) = 0 \cdot 0125$

Therefore rotor frequency $f^1 = sf = 60(0 \cdot 0125) = 0 \cdot 75\,Hz.$

Assuming unity turn ratio, $E_2 = E_1 = V = 208$ volts.

Rotor emf $E_2^1 = SE_2 = 0 \cdot 0125(208) = 2.6$ volts. (between slip rings)

8. A 500 V dc shunt motor has a speed of 1200 rpm, the line current being 5A. Find the speed when line current increases to 30 A. The shunt field resistance and armature resistance are 250Ω and 1.1Ω respectively. ***(2nd semester 2004)***

Solution : Given that V = 500 volts, $N_1 = 1200$ rpm

$I_1 = 5A,$ $R_{sh} = 250$ ohms, $R_a = 1.1$ ohms. $I_2 = 30A$

$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A$

$I_{a1} = I_1 - I_{sh} = 5 - 2 = 3A$

$E_{b1} = V - I_{a1} R_a = 500 - 3(1 \cdot 1) = 496 \cdot 7$ volts.

Similarly $I_{a2} = I_2 - I_{sh} = 30 - 2 = 28A$

$E_{b2} = V - I_{a2}R_a = 500 - 28(1.1) = 4692$ volts.

For a shunt motor, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$

$$\Rightarrow N_2 = N_1 \frac{E_{b2}}{E_{b1}} = 1200 \times \frac{469 \cdot 2}{496 \cdot 7} = 1133 \cdot 56 \text{ rpm.}$$

9. A 120V, dc shunt motor has an armature resistance of 0.2Ω and field resistance of 60Ω. It runs at 1800 rpm when it is taking a full-load current of 40 A.

(i) Find the speed of the motor, when it is operating with half the full-load.

(ii) The mechanical torque developed at half load. *(Supplementary Exam 2004)*

Solution : V = 120 volts, $R_a = 0 \cdot 2\Omega$, $R_{sh} = 60\Omega$

$N_1 = 1800$ rpm, Full load current $I_1 = 40A$

(i) Current drawn at half full load $I_2 = \dfrac{40}{2} = 20A$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2A.$$

$$I_{a1} = I_1 - I_{sh} = 40 - 2 = 38A$$

$$E_{b1} = V - I_{a1}R_a = 120 - 38(0 \cdot 2) = 112.4 \text{ volts.}$$

$$I_{a2} = I_2 - I_{sh} = 20 - 2 = 18A$$

$$E_{b2} = V - I_{a2}R_a = 120 - 18(0 \cdot 2) = 116.4 \text{ volt}$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$\Rightarrow N_2 = N_1 \times \frac{E_{b2}}{E_{b1}} = 1800 \times \frac{116 \cdot 4}{112 \cdot 4} = 1864 \text{ rpm.}$$

(ii) Mechanical Power developed at half load

$$P_2 = E_{b2}I_{a2} = 116 \cdot 4 \times 18 = 2095 \cdot 2 \text{ watt}$$

$$\therefore \quad \text{Torque developed } I_2 = \frac{P_2 \times 60}{2\pi N_2} = \frac{2095 \cdot 2 \times 60}{2\pi(1864)} = 10 \cdot 739 \text{ Nm.}$$

10. A 3 phase, 50 Hz induction motor has 8 poles. If the full load slip is 2.5%, determine (i) synchronous speed (ii) Rotor speed (iii) Frequency of the rotor current.

(Supplementary Exam 2004)

Solution f = 50 Hz, Pole (P) = 8, Full load slip = 0.025

(i) Synchronous speed $N_s = \frac{120f}{P} = \frac{120(50)}{8} = 750\,\text{rpm}.$

(ii) Rotor speed $N = N_S(1-S) = 750(1-0\cdot025) = 731\cdot25\,\text{rpm}.$

(iii) Frequency of the rotor current $f^1 = sf = 0\cdot025(50) = 1\cdot25\,\text{Hz}.$

11. A 4 pole DC generator running at 1500 rpm has 540 conductors. The flux per pole is 10 mwb. Determine the induced emf if the armature winding is (i) Lap connected and (ii) wave connected. ***(1st semester 2005)***

Solution P = 4, N = 1500 rpm, Z = 540, $\phi = 10\times10^{-3}$ wb.

(i) A = P = 4, for Lap connected.

$$E = \frac{\phi ZNP}{60A} = \frac{10\times10^{-3}\times540\times1500\times4}{60(4)} = 135\text{ volts.}$$

(ii) A = 2, for wave connected.

$$E = \frac{\phi ZNP}{60A} = \frac{10\times10^{-3}\times540\times1500\times4}{60(2)} = 270\text{ volts.}$$

12. In a 4 pole DC generator, the flux per pole is 6 mwb, there are 96 conductors and they are wave connected. What is the induced voltage if the armature rotates at a speed of 1500 rpm.

(2nd semester 2005)

Solution :

Pole (P) = 4, Flux per pole $(\phi) = 6\times10^{-3}$ wb

Z = 96, A = no of parallel path = 2

N = 1500 rpm.

Induced Voltage $(E) = \frac{\phi ZNP}{60A} = \frac{6\times10^{-3}\times96\times1500\times4}{60\times2} = 28\cdot8$ volts.

13. A 3-phase induction motor develops 25 kw output power at an efficiency of 88% at a p.f. of 0.85 lagging, when connected to a 3-phase 415V, 50 Hz supply. The windings of the motor are delta connected. Calculate both the phase currents and line currents drawn by the motor from the supply.

(1st semester 2006)

Solution

Output $P = 25\times10^3$ watt, $\eta = 0\cdot88$, $\cos\phi = 0\cdot85$

$V_L = 415$ volts, f = 50 Hz.

Input Power drawn by the motor $P_{in} = \frac{output}{efficiency} = \frac{25\times 10^3}{0\cdot 88} = 28409$ watts.

$P_{in} = \sqrt{3} V_L I_L \cos\phi$

$\therefore$ Line current drawn from the supply

$$I_L = \frac{P_{in}}{\sqrt{3} V_L \cos\phi} = \frac{28409}{\sqrt{3}\times 415\times 0\cdot 85} = 46\cdot 49 A$$

Since the windings are delta connected, the current in phase winding $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{46\cdot 49}{\sqrt{3}} = 26\cdot 841 A$

14. Calculate the slip of a 4 pole induction motor running at 1470 rpm while being connected to a 50 Hz three phase source. *(1st semester 2007)*

Solution : N = 1470 rpm, P = 4, f = 50 Hz

$$\therefore \quad N_S = \frac{120f}{P} = \frac{120\times 50}{4} = 1500 \text{ rpm}$$

$$\therefore \quad \text{Slip (s)} = \frac{N_s - N}{N_S} = \frac{1500 - 1470}{1500} = 0\cdot 02$$

15. A three phase, 4 pole induction motor runs at 1% slip when supplied from a 400 V, 50 Hz three phase AC supply. Calculate rotational speed of the revolving magnetic field and running speed of the motor. *(1st semester 2009)*

Solution : P = 4, S = 0.01, f = 50 Hz, V_L = 400 volts.

Speed of the revolving magnetic field $= \frac{120f}{P} = \frac{120(50)}{4} = 1500$ rpm

Running Speed of the motor $N = (1-S)N_S = (1-0\cdot 01)(1500) = 1485$ rpm.

16. A DC shunt motor develops 200 V on no load, while running at 1200 rpm. If the machine has 4-poles and 100 lap wound armature conductors, calculate the flux per pole. Also calculate the shunt field current if the resistance of the shunt field is 200 ohms.

(1st semester 2009)

Solution : $E_b = 200$ volts, N = 1200 rpm, P = 4

Z = 100, A = P = 4, $R_{sh} = 200$ ohms

$$\therefore \quad \text{Flux per pole } (\phi) = \frac{60AE_b}{ZNP} = \frac{60(4)(200)}{(100)(1200)(4)} = 0\cdot 1 wb$$

Neglecting armature resistance supply voltage

$V = E_b = 200$ volts.

$\therefore$ Shunt field current $I_{sh} = \frac{V}{R_{sh}} = \frac{200}{200} = 1A$

17. Calculate the slip of a 6-pole induction motor running at 960 rpm while drawing power from a 50 Hz three phase balanced source.

(2nd semester 2009)

Solution : N = 960 rpm, P = 6, f = 50 Hz

$$N_S = \frac{120f}{P} = \frac{120(50)}{6} = 1000 \text{ rpm}$$

$$\text{Slip (s)} = \frac{N_S - N}{N_S} = \frac{1000 - 960}{1000} = 0 \cdot 04$$

Do Your Self

T8.1 Calculate the force exerted by each conductor, 6 in long on the armature of a DC motor when it carries a current of 90A and lies in a field the density of which is $55 \cdot 2 \times 10^{-4}\ Wb / m^2$.

[11.06N]

T8.2 In a DC machine, the air-gap flux density is 4 Wb/m^2. The area of the face is 2cm x 4 cm. Find the flux per pole in the machine.

[0.32m Wb]

T8.3 A 220V shunt motor has an armature resistance of 0.32Ω and a field resistance of 110 ohms. At no load the armature current is 6A and the speed is 1,800 rpm. Assume that the flux does not vary with load and calculate. (a) The speed of the motor when the line current is 62 A (assume a 2-volt brush drop)., (b) The speed regulation of the motor. **[1657 rpm, 8.65%]**

T8.4 A 120 V, 10 A shunt generator has an armature resistance of 0.6Ω. The shunt field current is 2A. Determine the voltage regulation of the generator. **[4.9%].**

T8.5 Calculate the voltage induced in the armature winding of a 4-pole, lap wound, dc machine having 728 active conductors and running at 1800 rpm. The flux per pole is 30 mWb. If the armature is designed to carry a maximum line current of 100A, what is the maximum electromagnetic power developed by the armature ? **[655.2V, 65.5 KW]**

T8.6 A 100 kW, 230 V, shunt generator has Ra = 0.05Ω and Rf = 57.5Ω. If the generator operates at rated voltage, calculate the induced voltage at (a) full-load and (b) half full-load. Neglect brush contact drop. **[252V, 241V].**

T8.7 At what speed, in rpm must be armature of a dc machine run to develop 572 KW ata torque of 4605 N - m ? **[1187 rpm]**

T8.8 The armature of a dc machine running at 1200 rpm carries 45 A in current. If the induced armature voltage is 130V, what is the torque developed by the armature ? **[46.5 N.m]**

T8.9 A self-excited shunt generator supplies a load of 12.5 KW at 125 V. The field resistance is 25 Ω and the armature resistance is 01.Ω. The total voltage drop because of brush contact and armature reaction at this load is 3.5V. Calculate the induced armature voltage. **[139 V]**

T8.10 A 6-pole, lap-wound armature, having 720 conductors, rotates in a flux of 20.35 mWb per pole, (a) If the armature current is 78 A, what is the torque developed by the armature ? (b) If the induced armature voltage is 420 V, what is the motor speed ?

[(a) 181.9 N.m (b) 1720 rpm].

T8.11 A 50-hp, 550 volt shunt motor has an armature resistance, including brushes, of 0.36 ohm. When operating at rated load and speed, the armature takes 75 amp. What resistance should be inserted in the armature circuit to obtain a 20 percent speed reduction when the motor is developing 70 percent of rated torque ? Assume that there is no flux change. **[2.15Ω]**

T8.12 A 20 kW, 230 V separately excited generator has an armature resistance of 0.2Ω and a load current of 100 A. Find

(a) The generated voltage when the terminal voltage is 230 V.

(b) The output power. **[250V, 23KW]**

T8.13 A 10 KW, 120V DC series generator has an armature resistance of 0.1Ω and a series field resistance of 0.05Ω. Assuming that it is delivering rated current at rated speed, find (a) the armature current and (b) the generated voltage (c) voltage across the brushes.

[83.33 A, 132.5V, 128.33V]

T8.14 The armature resistance of a 30 KW, 440 V shunt generator is 0.1Ω. Its shunt field resistance is 200 Ω. Find

(a) The power developed at rated load.

(b) The load, field and armature currents.

(c) The electrical power loss. **[31.471 KW, 62.8A, 2.2A, 70.4A, 1464W]**

T8.15 A four-pole, 450 kW, 4.6 kV shunt generator has armature and field resistances of 2Ω and 333Ω. The generator is operating at the rated speed of 3,600 rev/min. Find the no-load voltage of the generator and terminal voltage at half load. **[4820.4V, 4810.7V]**

T8.16 A shunt DC motor has a shunt field resistance of 400Ω and an armature resistance of 0.2Ω. The motor name plate rating values are 440V, 1,200 rev/min, 100 hp, and full-load efficiency of 90 percent. Find

(a) The motor line current.

(b) The field and armature currents.

(c) The counter emf at rated speed.

(d) The output torque. **[188.4A; 1.1A, 187.3A; 402.5A; 593.5N-m]**

T8.17 A 30 kW, 240 V generator is running at half load at 1,800 rev/min with efficiency of 85 percent. Find the total losses and input power. **[2.647 kW, 17.64Kw]**

T8.18 A 240 volt series motor has an armature resistance of 0.42Ω and a series-field resistance of 0.18Ω. If the speed is 500 rev/min when the current is 36 A, what will be the motor speed when the load reduces the line current to 21 A ? (Assume a 3-volt brush drop and that the flux is proportional to the current.) **[893 rpm]**

T8.19 A 220V DC shunt motor has an armature resistance of 0.2Ω and a rated armature current of 50 A. Find

(a) the voltage generated in the armature.

(b) The power developed. **[210V, 10.5 KW]**

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T8.20 A 550 volt series motor takes 112 A and operates at 820 rev/min when the load is 75 hp. If the effective armature-circuit resistance is 0.15Ω, calculate the horsepower output of the motor when the current drops to 84 A, assuming that the flux is reduced by 15 percent.

[56.7 hp]

T8.21 A 200 V DC shunt motor has Ra=0.1Ω & Rf=100 Ω. When running at 1,100 rev/min with no load connected to the shaft, the motor draws 4A from the line. Find E and the rotational losses at 1,100 rev/min (assuming that the stray-load losses can be neglected).

[199.8V; 399.6W]

T8.22 A 200V DC shunt motor with an armature resistance of 0.1Ω and a field resistance of 100 Ω draws a line current of 5A when running with no load at 955 rev/min. Determine the motor speed, the motor efficiency, the total losses (i.e., rotational and I^2R losses), and the load torque (i.e. T_{sh}) that will result when the motor draws 40 A from the line. Assume rotational power losses are proportional to the square of shaft speed.

[100 r/s; 85.84%; 1133W; 69.9N-m]

T8.23 A shunt motor operates at a flux of 25 mWb per pole, is lap-wound, and has 2 poles and 360 conductors. The armature resistance is 0.12Ω and the motor is designed to operate at 115V, taking 60 A armature current at full load, (a) Determine the value of the external resistance to be inserted in the armature circuit so that the armature current shall not exceed twice its full-load value at starting, (b) When the motor has reached a speed of 400 rpm, the external resistance is cut by 50%. What is the armature current then, at this speed ? (c) The external resistance is completely cut out when the motor reaches its final speed; the armature current is then at its full-load value. Calculate the motor speed.

[(a) 0.838 ft. (b) 102 A; (c) 718.6 rpm]

T8.24 A 230 V shunt motor, having an armature resistance of 0.05 ft. and a field resistance of 75Ω, draws a line current of 7 A while running light at 1120 rpm. For a load at which the line current is 46 A, determine (a) the motor speed, (b) motor efficiency, and (c) total core and mechanical losses.

[(a) 1110.5 rpm; (b) 83.9%; (c) 903.9W]

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